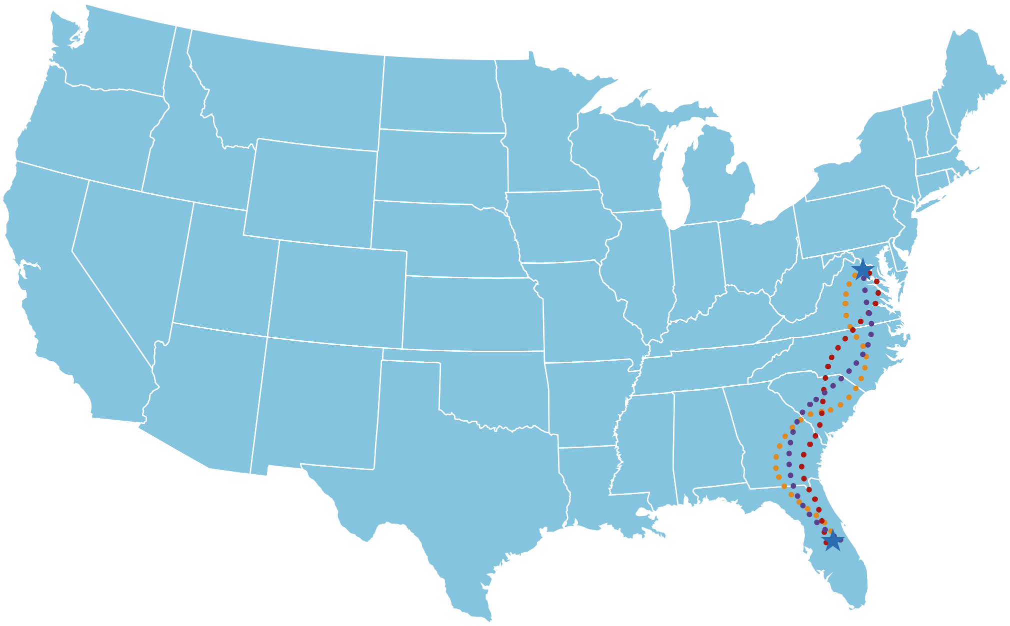
Sarah Quebec Fuentes

Family vacation



NCTM's ***Principles to Actions:*** *Ensuring Mathematical Success for All* outlines eight teaching practices for effective teaching and learning of mathematics (NCTM 2014). One of the teaching practices, "Establish mathematics goals to focus learning," states,

> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses goals to guide instructional decisions. (p. 10)

In this problem scenario, students will engage in a task, which has multiple access points, with the ultimate goal of comparing fractions using number sense.

Problem scenario

For vacation, several families are traveling by car from the Washington, D.C., area to Orlando, Florida—a distance of 840 miles. Determine which family is closer to Orlando at different points during the trip.

See the student **activity sheet** on **page 131** for the questions.

Classroom setup

Before using the task with your students, solve the problem on your own. In addition, anticipate the multitude of strategies, appropriate and inappropriate, that your students might use to represent and solve the problem. Gather some materials before you present the problem to your class:

- Copies of the activity sheet (see **p. 131**) for each group or pair of students
- A way for each group or pair of students to share their solutions (e.g., a document camera, a large piece of paper and markers, etc.)
- A map of the east coast of the United States
- A digital camera or a smartphone or tablet with a camera

Organize your class into pairs or small groups of students. Launch the task by showing them a map of the east coast of the United States. Present the problem scenario, pointing out Washington, D.C., and Orlando, Florida, on the map and explaining that several families will be traveling from the Washington

Where's the math?

Much of students' number and operation sense about fractions builds on the idea of a unit fraction. The Common Core State Standards for Mathematics introduces the domain of Number and Operations–Fractions in third grade with the following standard about unit fractions:

> Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b* (CCSSI 2010, 3.NF.A.1).

Cramer, Wyberg, and Leavitt (2009) outline categories of student reasoning strategies for comparing fractions, two of which build on an understanding of unit fractions and are highlighted in the current problem scenario.

- **Same numerator (e.g., 3/8 and 3/10):** Eighths are larger than tenths, because a whole divided into eight equal-size pieces will yield bigger pieces than the same whole divided into ten equal-size pieces. Three of the bigger pieces will be greater than three of the smaller pieces; therefore, 3/8 is greater than 3/10.
- **Residual (e.g., 5/6 and 7/8):** Both fractions are a unit fraction from a whole; that is, 5/6 is 1/6 away from the whole, and 7/8 is 1/8 away from the whole. Eighths are less than sixths. Therefore, 7/8 is greater than 5/6 because 7/8 is closer to the whole.

The goal is for students to generate these comparison strategies. To do so, students may need multiple opportunities with pairs of fractions that align with each strategy (Van de Walle and Lovin 2006). By *"look*[*ing*] *for and express*[*ing*] *regularity in repeated reasoning"* (CCSSI 2010, Standards for Mathematical Practice 8), students can learn to identify the nature of a pair of fractions and then use an appropriate reasoning strategy to compare them.

The Common Core State Standards for Mathematics also advocates developing an understanding of fractions through the use of the number line:

> Understand a fraction as a number on the number line; represent fractions on a number line diagram. (CCSSI 2010, 3.NF.A.2)

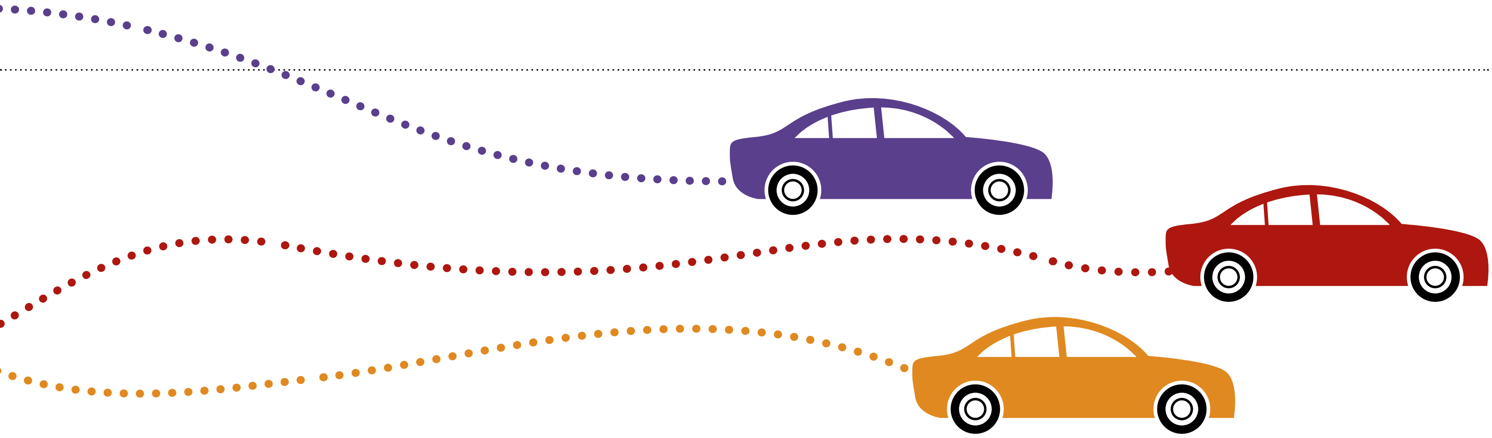
Students demonstrate struggles (related to the unit, tick marks, and partitions) when representing fractions on a number line (Cramer, Ahrandt, and Monson 2015). In response to these challenges, Cramer and her colleagues found that adding a context to a task helped students understand how to represent fractions on a number line. The problem scenario presented herein provides a context, in the form of a family vacation, which lends itself to using a number line to represent the distance traveled.

metropolitan area to Orlando for vacation. Students are to determine which family, of a pair of families, is closer to Orlando at certain junctures during the trip. Emphasize to students that each question on the activity sheet refers to *a different pair of families*. Distribute the activity sheet and/or paper for students to record their strategies and solutions.

As students work, walk around the classroom and monitor the strategies that students use to solve the problem. You may want to take some pictures with a digital camera, smartphone, or tablet to help gather evidence of student thinking during the solution process. To maintain the task's high level of cognitive demand, try not to tell students how to solve the problem. Instead, ask questions that build on the ways that students are thinking about the problem. Some example questions follow:

- How can you start to solve this problem?
- How did you determine which family is closer to Orlando?
- How can you use what you learned in question 1 to help you figure out question 2?
- How can you use what you learned in question 1 to help you figure out question 3?
- What is another way to find (or justify) your solution?

As you monitor students at work, select particular strategies for pairs or groups to share with the class, and carefully consider the order in which students present the strategies. While making these instructional decisions, keep in mind the lesson goal: comparing fractions using number sense. (Refer to the *Where's the math?* section for more details about student reasoning with respect to this goal.) The strategies could be sequenced from sharing misconceptions to sharing correct conceptions about the size of fractions. For example, one pair or group may use whole-number thinking and believe that one-fifth is greater than one-fourth because five is greater than four. In contrast, another pair or group may view each fraction as a number and understand that when a whole is divided into a greater number of equal-size parts, the size of the parts, as represented by unit fractions, decreases.



The strategies could also be sequenced from concrete to abstract. For instance, some pairs or groups might use the total distance to determine which family is closer to Orlando; others might resolve the problem using only the fractions. Similarly, some pairs or groups might use a number line (with the total distance and/or fractions); whereas others do not need the number line to reason about the distance or about the size of the fractions. As pairs and groups share their strategies, ask questions that help students make connections between the various strategies and build the mathematical understandings underlying the task:

- How are the various strategies similar and different?
- If the total distance is changed, do the answers to each question change? Why or why not?
- Do you need to know the total distance to answer the questions? Why or why not?
- How could you answer the questions without knowing the total distance?
- How do you compare pairs of fractions like those in question 2?
- How do you compare such pairs of fractions as those in question 3?

Maintain a running record of pair/group contributions on the board or on chart paper to provide students with the opportunity to make connections among strategies, representations, and mathematical ideas.

Extensions and modifications

Opportunities for differentiation are incorporated into the nature of the task. Some students may need to use the total distance to answer the questions; others are able to reason using only the fractions. As indicated in the sample prompts, encourage students who finish early to consider other ways to find or justify solutions and to make generalizations about comparing pairs of fractions on the basis of their attributes (e.g., unit fractions, same numerator, and a unit fraction distance from the whole).

Further, the pairs of fractions in each question can be modified to meet your students' current level of reasoning about fractions. For students who have yet to develop the underlying understanding of unit fractions, the pairs of fractions in questions 2 and 3 can be changed to unit fractions (e.g., 1/8 and 1/10; 1/6 and 1/8). For students with a developing understanding of unit fractions, the pairs of fractions in questions 2 and 3 can be altered so that students can build on their reasoning for question 1 (e.g., 3/4 and 3/5; 3/4 and 4/5). For students with a better developed understanding of fractions, the pairs of fractions can remain as is, and the information about the total distance of the trip can be omitted.

Share your students' work

Try this problem in your classroom. We are interested in how your students responded to the problem, which problem-solving strategies they used, and how they explained or justified their reasoning. Send your thoughts and reflections—including information about how you posed the problem, samples of students' work, and photographs showing your problem solvers in action—by **December 1, 2015**—to Problem Solvers department editor **J. Matt Switzer**, Texas Christian University, TCU Box 297920, Fort Worth, TX 76129; or email him at **j.switzer@tcu.edu**. Selected submissions will be published in a subsequent issue of *TCM* and acknowledged by name, grade level, and school name unless you indicate otherwise.

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Sarah Quebec Fuentes is an associate professor of mathematics education at Texas Christian University in Fort Worth. She is interested in the teaching and learning of mathematics for understanding. Her research projects focus on classroom discourse, teacher knowledge, mathematics curriculum materials, teacher self-efficacy, collaboration, and developing fraction sense. Edited by **J. Matt Switzer**, an assistant professor of mathematics education at Texas Christian University.

Each month, this section of the Problem Solvers department features a new challenge for students. Readers are encouraged to submit problems to be considered for future columns. Receipt of problems will not be acknowledged; however, those selected for publication will be credited to the author. Find submission guidelines for all departments at **http://www.nctm.org/tcmdepartments**.

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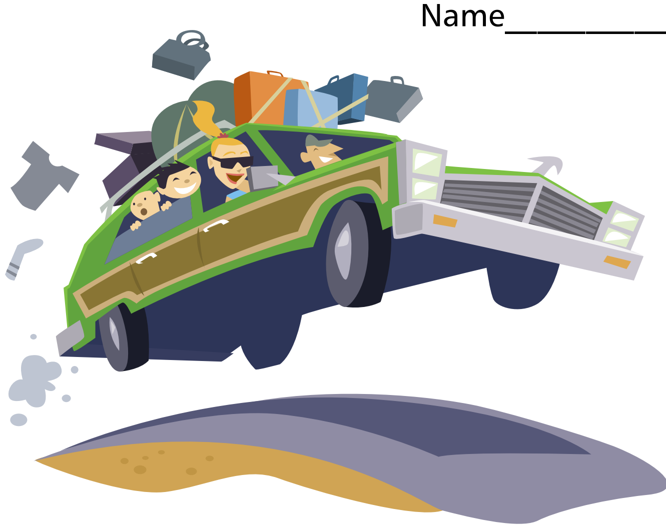

Family Vacation

For vacation, several families are traveling from the Washington, D.C., area to Orlando, Florida, a distance of 840 miles. Determine which family is closer to Orlando at different points during the trip.

1. Before stopping for lunch, Julia's family traveled $\frac{1}{4}$ of the distance, and Robert's family traveled $\frac{1}{5}$ of the distance. Who is closer to Orlando? Explain.

2. Two other families stopped after a day of driving and stayed at a hotel. Harlee's family drove $\frac{3}{8}$ of the distance, and Derek's family drove $\frac{3}{10}$ of the distance. Who is closer to Orlando? Explain.

3. On their second day of traveling, two other families stopped at a rest area. Jackson's family traveled $\frac{5}{6}$ of the total distance, and Ana's family traveled $\frac{7}{8}$ of the total distance. Who is closer to Orlando? Explain.

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