teaching children mathematics

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2015 Focus Issue: Between the Wholes: Promoting Fraction and Decimal Understanding

Welcome to the 2015 *Teaching Children Mathematics* (*TCM*) Focus Issue, which is all about examining the mathematics that lies "between the wholes" (as in whole numbers). The knowledge of fractions and decimals that children develop in the elementary grades provides an essential foundation for the study of algebra and more advanced mathematics. Yet it is widely recognized that developing deep and generative understanding of fractions and decimals poses considerable challenge to learners and their teachers.

Last year, we asked you for the approaches, classroom activities, and ideas that you use to help children make sense of fractions and decimals as numbers, benchmarks, measures, quotients, or operators. In response, we received over fifty manuscript submissions describing a wide variety of ways that teachers facilitate children's understanding of fractions and decimals. We appreciate that these authors went beyond the typical pizza and brownie fare to help us visualize alternative models for fractions and decimals.

We constructed this Focus Issue to present a developmental series of instructional ideas in fractions and decimals. Arriving at the collection of articles that comprise this issue was a difficult process, but we hope you will agree that we have assembled a strong set of informative, interesting, influential ideas for the classroom.

We open with "Engaging Students with Multiple Models of Fractions" (p. 138) by Xiaofen Zhang, M. A. (Ken) Clements, and Nerida F. Ellerton. This article addresses the importance of children's understanding of unit fractions as a framework for future success and presents a wide variety of activities to develop unit fraction understanding using varied representations and models.

Ron Tzur and Jessica Hunt's article, "Iteration: Unit Fraction Knowledge and the French Fry Tasks" (p. 148), also presents ideas integral to the development of unit fractions but emphasizes an iterative approach. The authors argue that such an approach is useful in helping children transition beyond a part-whole understanding of fractions to a view of fractions as a multiplicative relation in which a fraction such as three-fourths is thought of as three copies of the unit fraction one-fourth.

In "Uncovering Students' Ideas about Fractions" (p. 158), Rebecca M. Lewis, Lynsey K. Gibbons, Elham Kazemi, and Teresa Lind demonstrate how a formative assessment task involving equal sharing can be used to elicit and understand students' ideas about the meaning of fractions. By listening and examining children's oral, written, and symbolic representations of their thinking, the teacher in this article is able to assess his students' current levels of understanding and pose an appropriate next problem to support and further their thinking.

We have included two articles addressing fraction multiplication and division. Both of them focus on ideas for teaching these operations in ways that emphasize conceptual understanding. In "Units Matter" (p. 170), authors Ji Yeong I, Barbara J. Dougherty, and Zaur Berkaliev address a common classroom occurrence in which children can use the repeated addition strategy to solve such multiplication problems as 4 × 3

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and 2 × 1/3 but that does not work when trying to solve 3/4 × 4/5. To develop an understanding of these problems, the authors present a "units of units" approach and explicitly describe the "intermediate unit," which can be useful in developing children's multiplication strategies beyond repeated addition.

A final fraction article, "Unpacking the Division Interpretation of a Fraction" (p. 178), by Rebecca C. Poon and Priscilla Eide Lewis, addresses the process and meaning behind fraction division as well as the important idea of the referent unit as it pertains to interpreting a remainder. The authors present several examples addressing the importance of multiple units and the role they play in developing an understanding of our algorithmic approach to fraction division.

Our last article introduces a framework for interpreting children's thinking about decimals. "5 Indicators of Decimal Understandings" (p. 186)—by Kathleen Cramer, Debra Monson, Sue Ahrendt, Karen Colum, Bethann Wiley, and Terry Wyberg—was developed through extensive collaboration with classroom teachers and presents a set of signs of rich conceptual understanding of decimals. Teachers can use the framework to monitor student learning as well as to guide future instruction.

We believe that these articles, along with others that will appear in *Teaching Children Mathematics* during the remainder of this volume year, will support teachers who are interested in whetting their students' appetites for understanding and computing the numbers that lie "between the wholes."

On behalf of the *TCM* Editorial Panel,
Wendy Bray, Lisa Englard, and
Andrew Tyminski,
TCM Focus Issue Editors

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COACHES' CORNER

Comparing fractions sensibly

BY ROBYN SILBEY, PD AND CAMPUS CONSULTANT

Elementary school students find fractions to be one of the most compelling and challenging areas of study. Assist teachers in helping students make sense of comparing and ordering fractions. For the deepest understanding and the most sense making, students might learn comparisons in the following order:

Same denominator, different numerator

- What do you notice about the fractions 3/8 and 7/8? (They have the same denominator.)
- How would you begin to draw each fraction? Why? (Draw two same-size wholes, each with eight same-size parts because the denominators are both eight.)
- How many parts would you shade for each fraction? Why? (Shade three parts for 3/8 and seven parts for 7/8; the numerators tell the number of parts shaded.)
- Which is greater? How do you know? (7/8 is greater. The parts of both fractions are the same size, and seven parts is more than three parts. Simply put, comparing 3/8 and 7/8 is similar to comparing three apples and seven apples; seven apples is greater.)

Same numerator, different denominator

- What do you notice about the fractions 1/2 and 1/4? (They have the same numerator, but different denominators.)
- How would you begin to draw each fraction? Why? (Draw two same-size wholes. Divide one into two same-size parts, and the other into four same-size parts because the denominator tells how many parts make the whole.)
- How many parts would you shade for each fraction? Why? (One; the numerator is one in both fractions.)
- Which is greater? How do you know? (1/2 is greater because only two, rather than four, pieces are needed to make the whole.)

Different denominator, different numerator

- What are some ways to name 0, 1/2, and 1? (Answers will vary, but names for 0 have 0 in the numerator; names for 1 have matching numerators and denominators, and names for 1/2 have numerators exactly half the value of the denominators.)
- How can you use 0, 1/2, and 1 to help you compare fractions? (Compare the fractions to 0, 1/2, or 1. For example, 2/7 is less than 1/2; 3/5 is greater than 1/2. Therefore, 2/7 is less than 3/5.)

By guiding teachers to help students use sense making to compare fractions, we guide other fraction concepts toward a solid, logical foundation.

Questions? Comments? Contact the author: **robyn@robynsilbey.com**.