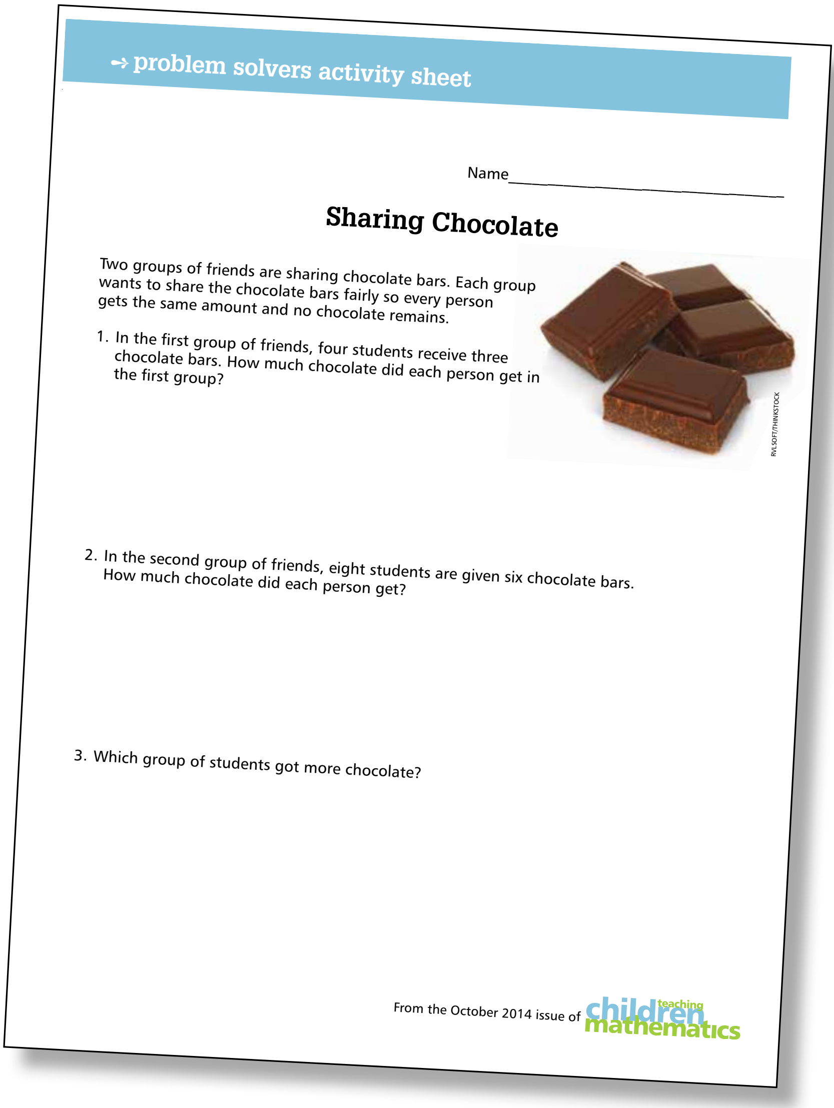
Sharing chocolate

The October 2014 problem scenario offers students an opportunity to divide whole chocolate bars into fractional amounts. The investigation focuses on student understanding of the partitioning of a whole into different fractional amounts, on comparing these amounts, and on the ability to develop and defend their thinking. Go to **http://www.nctm.org/tcm**, All Issues to access the full-size activity sheet.



→ problem solvers activity sheet

Name____________________________

Sharing Chocolate

Two groups of friends are sharing chocolate bars. Each group wants to share the chocolate bars fairly so every person gets the same amount and no chocolate remains.

1. In the first group of friends, four students receive three chocolate bars. How much chocolate did each person get in the first group?

2. In the second group of friends, eight students are given six chocolate bars. How much chocolate did each person get?

3. Which group of students got more chocolate?

From the October 2014 issue of teaching children mathematics

Two groups of third graders at Campbell Elementary School in Arlington, Virginia, investigated how to represent the fair sharing of chocolate bars when there are fewer chocolate bars than people who want to share them. This investigation occurred before any formal teaching of fractions, and it revealed student understanding of partitioning a whole into equal pieces, fractional language and notation, using fractions to represent division, and equivalent fractions.

Getting started

Math coach Anne Oliveira drew representations of the two groups sharing chocolate on chart paper in preparation for the student investigation. As she introduced the problem, Oliveira was careful to avoid using the word *fractions* and emphasized that the chocolate in each situation needed to be shared equally.

> Two groups of friends are sharing chocolate bars. Each group wants to share the chocolate bars equally so every person gets the same amount and there is no chocolate left over. In the first group of friends, four students were given three chocolate bars. In the second group of friends, eight students were given six chocolate bars. How much chocolate did each person get in the first group? How much chocolate did each person get in the second group? Which group of students got more chocolate?

By not mentioning the word *fractions*, the teacher did not give away the representation she was hoping that students would use. This would also provide Oliveira with an authentic diagnostic assessment of her students' understanding of fractions without the suggestion from the teacher to use fractions in their solution.

After checking to make sure that students understood the task, Oliveira organized them into pairs and provided materials they needed to solve the problem.

Observations, reflections, and next steps

As the pairs of third-grade students were working, Oliveira circulated around the room, observing students and recording their initial strategies. She noticed that most students began with the same strategy of drawing models of the chocolate and people. Most students accurately represented the number of chocolate bars and people; however, some students confused the number of chocolate bars with the number of people or incorrectly drew the number of people sharing the chocolate.

When students had completed the task, Oliveira made the general conclusion that everyone understood the task and that all were able to demonstrate some success with the investigation. After collecting student solutions, Oliveira sorted the work into three categories that she labeled as *partial understanding*, *understanding*, and *advanced understanding*.

Partial understanding

At the partial understanding level, Oliveira identified students who—

- divided the chocolate bars into halves; and
- were unable to correctly distribute all the chocolate to all the friends.

Oliveira commented that most students at this level provided a solution for the first question by distributing one-half of a chocolate bar to each friend and ignoring or discarding the extra chocolate bar pieces. A student described the pieces as "leftovers." Another suggested that they "split [the bars] in half and save the rest." Another child circled the extra halves and wrote, "Give it to someone." Taking a

FIGURE 1

Showing partial understanding of partitioning a whole, this student partitions each chocolate bar into halves, provides three friends with two halves, and leaves the fourth friend with no chocolate.

Two groups of friends are sharing chocolate bars. Each group wants to share the chocolate bars fairly so every person gets the same amount and no chocolate remains.

1. In the first group of friends, four students receive three chocolate bars. How much chocolate did each person get in the first group?

FIGURE 2

This student solution shows an understanding of partitioning a whole into equal amounts, dividing the chocolate bars into halves and fourths and distributing one-half and one-fourth to each friend.

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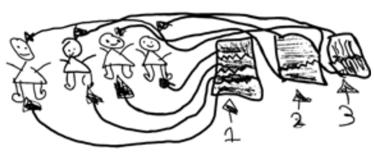

2. In the second group of friends, eight students are given six chocolate bars. How much chocolate did each person get?

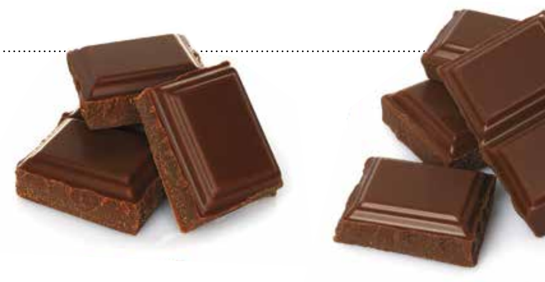

FIGURE 3

This student shows advanced understanding of portioning a whole, representing three chocolate bars divided into fourths and six chocolate bars divided into eighths. He correctly describes the amount that friends in each group received, using the terms *quarters* and *halves of quarters*.

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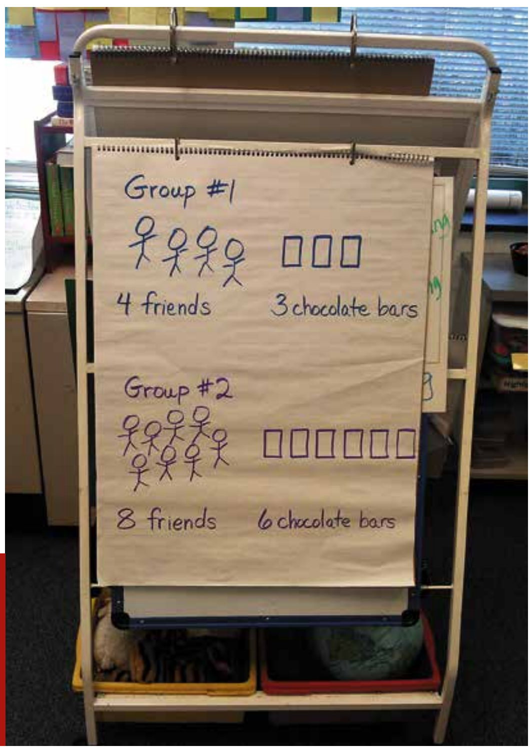

Students were presented with a visual representation of the situation as the teacher introduced the problem.

somewhat different approach, a classmate gave three people two pieces of chocolate each and nothing to the fourth person. Many students at this level also showed confusion in their labels between the parts and the wholes, referring to pieces interchangeably as “chocolate bars” or “pieces.”

Oliveira felt that these students would need more experiences going forward in third grade with fraction concepts, such as identifying parts of a region that represent fractions for fourths. These students may well be able to identify fourths but may be uncomfortable with partitioning a whole into fractions beyond one-half.

Understanding

At the understanding level, Oliveira identifies students who—

- divided the chocolate bars into halves and fourths; and
- equally distributed all the chocolate to the friends.

Oliveira noted that students at this level made pictures of chocolate bars divided into halves and fourths and typically drew a lot of lines connecting the pieces to the friends in each group. Most of the students in this level did not use fractional language, but they were able to articulate that in each group students received an equal amount of chocolate.

Many of these students began by partitioning the chocolate bars into halves, but unlike students in the first group, they recognized that the third bar would need to be divided into four pieces to ensure a fair sharing of the chocolate. Oliveira felt that these students would benefit from the modeling of fractional language and written representations of fractions in future instruction.

Advanced understanding

At the advanced understanding level, Oliveira identifies students who—

- equally distributed all pieces of the chocolate to the friends using halves,

fourths and eighths;

- used fractional language to describe the chocolate bars that friends received; and
- articulated that in each group each friend received the same amount.

Oliveira reflected that although some of these students did not name the fractions as three-fourths and six-eighths, the depth of their understanding surpassed their fraction language. One pair of students mentioned that in group 1, the friends received larger pieces of chocolate but that in group 2, the friends received more pieces of chocolate. Another student wrote next to his model of fourths and eighths, "I know 3/4 = 6/8 because you just need to double 3/4. So you do 3 + 3 = 6 and 4 + 4 = 8 and that equals 6/8." Oliveira felt that students in this group were ready for further investigations of equivalent fractions and modeling of fractional language and representations.

Oliveira used this investigation to find out what her students knew, with regard to not only their understanding of fractions but also their use of problem-solving strategies and the methods they used to represent and organize their thinking. Oliveira combined these data with her understanding of the continuum of models, strategies, and important ideas in fraction instruction to identify a variety of starting points for future teaching on the basis of the range of understanding demonstrated during the investigation.

Edited by **Ed Enns**, **ed_enns@wrdsb.on.ca**, who works as an elementary school learning services consultant with the Waterloo Region District School Board in Kitchener, Ontario, Canada. In his work with elementary school mathematics teachers, he emphasizes conceptual understanding and is exploring effective strategies for teaching mathematics through problem solving. Each month, this section of the Problem Solvers department discusses the classroom results of using problems presented in previous issues of *Teaching Children Mathematics*. Find detailed submission guidelines for all *TCM* departments at **http://www.nctm.org/tcmdepartments**.