J. Matt Switzer

Which lake is *bigger*?

NCTM's ***Principles to Actions:*** *Ensuring Mathematical Success for All* (NCTM 2014) outlines eight teaching practices for effective teaching and learning of mathematics. One of the teaching practices, to pose purposeful questions, states that—

> effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. (p. 10)

Solving our problem this month, students will respond to questions purposefully designed to assess and develop key ideas related to measurement.

Problem scenario

Brenna and Shakira are preparing for a trip to Okoboji, Iowa, for their summer vacation. They are trying to decide if they should rent a cabin on the shore of West Okoboji Lake or Spirit Lake. [Refer students to the map shown on the student **activity sheet** on **page 212.**] *Because they plan to go for long hikes and boat rides during their vacation, they want to choose the biggest lake. Brenna thinks that West Okoboji Lake is bigger than Spirit Lake. Shakira is not sure which lake is bigger. Brenna and Shakira need your help to determine which lake is bigger.*

See the **activity sheet** for the questions.

Classroom setup

Before using the task with your students, solve the problem on your own. In addition, anticipate the multitude of strategies, appropriate and inappropriate, that your students might use to represent and solve the problem. You will also need to gather some materials:

- Copies of the activity sheet (**p. 212**) for each group or pair of students
- Copies of the full-size map (**online**) for each group or pair of students
- A means for each pair or group of students to share their solutions (e.g., a document camera, a large piece of paper and markers, etc.)
- A variety of nonstandard and standard measuring tools (e.g., square tiles, grid paper, string, rulers, Unifix® cubes, patty paper)
- A digital camera or smartphone, or a tablet with a camera.

Organize your students into pairs or small groups. Launch the task by showing students the map of Okoboji that is available **online**. Have students share what they notice about the map and what information they can determine from it. Present the problem scenario by pointing out West Okoboji Lake and Spirit Lake on the map. Explain that the task is to determine which lake, West Okoboji Lake or Spirit Lake, is *bigger*. Allow pairs or small groups a few

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minutes to discuss which lake they think is bigger. Have each pair or group of students share with the class which of the two lakes they believe is bigger. After each pair or group of students has shared, ask those who selected West Okoboji Lake to explain their choice. Next, ask those who selected Spirit Lake to explain their choice. Through the students' sharing, the question of what makes one lake bigger than another should arise and be left open for students to explore. Distribute the activity sheet and/or paper for students to record their strategies, solutions, and justifications.

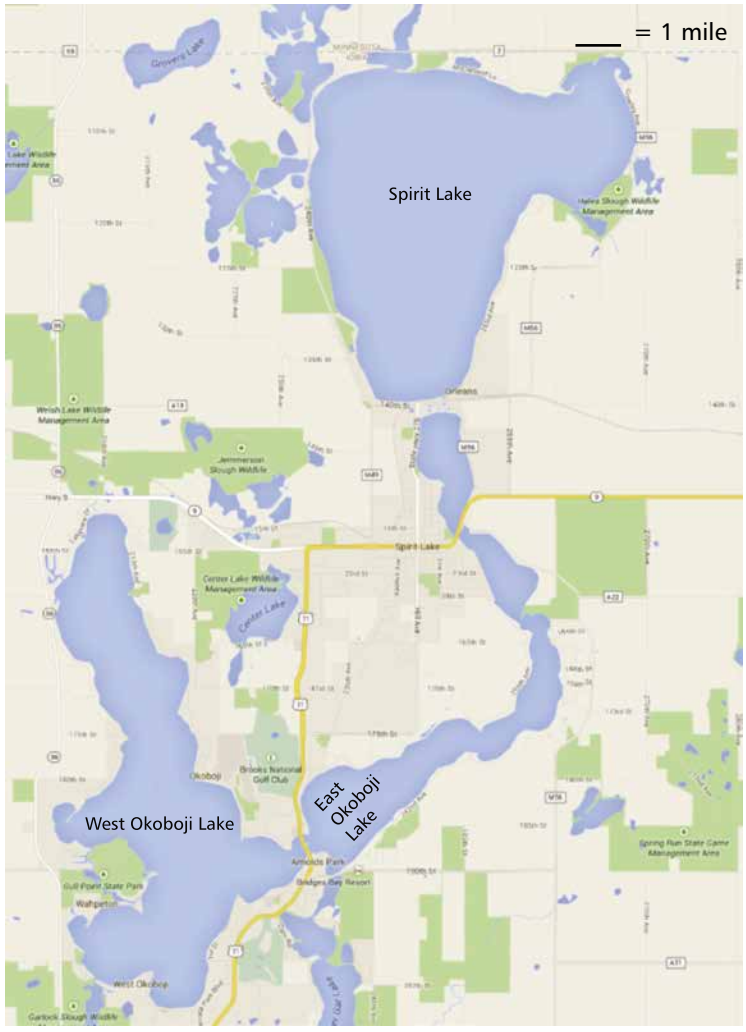
As students work, walk around the classroom and monitor the strategies that they use to solve the problem. You may want to take some pictures with a digital camera, a smartphone, or a tablet to help gather evidence of student thinking during the solution process. To maintain the task's high level of cognitive demand, resist telling students how to solve the problem. Instead, ask questions that build on the ways students are thinking about the problem. Some example questions include the following:

- How can you start to solve this problem?
- With respect to the two lakes, what does *bigger* mean to you? How did you determine which lake is bigger?
- Is there another way to measure which lake is bigger? Why or why not? [*Students could use a different unit of measure for the same attribute they measured, or they could measure a different attribute.*]
- What is another way to find (or justify) your solution?

As you monitor the class at work, select particular strategies for pairs or groups to share with their peers, and carefully consider the order in which students will present the strategies. While making these instructional decisions, keep in mind the purpose of the questions, namely, to assess and develop students' use of standard and nonstandard units of measure to compare the size of the two lakes. (Refer to the **Where's the math?** section for more details about the development of student reasoning with respect to this purpose.) You could sequence the strategies, from sharing students' initial use of nonstandard units of measure to a transitional phase of using models of standard units to using standard units of measure. For instance, one pair or group of students might use a pipe cleaner to compare the lengths of the shorelines, resulting in lengths of x number of pipe cleaners. Students who have developed a deeper understanding of measure and standard units might lay a length of string along a ruler, mark inches on the string, and then use the string to determine the lengths of the shorelines in inches. You could also sequence strategies from sharing students' misconceptions or partial conceptions to sharing correct

FIGURE 1

Ask students what information they can deduce from the map. The full page version is available online.



conceptions about measure. For example, one group might try to partition the surface area of the lake into smaller units by drawing and counting the number of "squares" on the map, but the "squares" are of different sizes. In contrast, another group might use square tiles to cover the lake on the map and attend to the need to count partial tiles that extend beyond the shoreline of the lake.

As pairs or groups share their strategies, ask questions that help students make connections among the various strategies and build the mathematical understandings underlying the task:

- Which attributes were used to determine which lake was "bigger"? Why?
- What did you use to determine the lakes' [*distance, perimeter, area*]?
- How did different pairs or groups compare West Okoboji Lake and Spirit Lake?
- How are the various strategies that you used to determine the [*distance, perimeter, area*] of the lakes similar and different?
- Did everyone decide that the same lake was bigger? Do you agree? Why or why not?
- Why do you think everyone did not get the same [*distance, perimeter, area*] for the lakes?
- Do you need to know the actual [*distance, perimeter, area*] of the lakes to determine which is bigger? Why or why not?

TABLE 1

This table shows the reported area and perimeter of the lakes. (Adapted from Iowa Great Lakes Association 2015)

Name of lake	Miles of shoreline	Square miles of surface area
West Okoboji	19.00	5.77
East Okoboji	17.10	12.89
Spirit Lake	15.33	8.88

On the board or chart paper, maintain a running record of group contributions to provide students the opportunity to make connections between strategies, representations, and mathematical ideas.

Extensions and modifications

Opportunities for differentiation are incorporated into the nature of the task. Some students may need objects of the same size to use as their unit of measure to completely cover the area or shoreline of the lakes; others may be able to iteratively use a single unit of measure. As indicated in the sample prompts, encourage students who finish early to consider other ways to find or justify their solutions and to articulate some of the big ideas of measurement (e.g., comparison of the same attribute, repetition of the same unit of measure, inverse relationship between measures and sizes of units, relative meaning of *bigger*) (Van de Walle, Karp, and Bay-Williams 2016).

For students who have yet to develop an underlying understanding of measurement and who still solely rely on how big the lakes look, make the map larger so that students may use repeated units of larger physical models. For students with a more advanced understanding of measure, provide the scale for the map and encourage them to determine the length of the shorelines (in miles) and surface area (in square miles) of the lakes (see **table 1**). To extend the task even further, ask students to make (and then test) predictions about how East Okoboji Lake compares to West Okoboji Lake and Spirit Lake.

Share your students' work

Try this problem in your classroom. We are interested in how your students responded to the problem, which problem-solving strategies they used, and how they explained or justified their reasoning. Send your thoughts and reflections—including information about how you posed the problem, samples of students' work, and photographs showing your problem solvers in action—by **January 15, 2016**—to Problem Solvers department editor **Sarah Quebec Fuentes**, Texas Christian University, TCU Box 297920, Fort Worth, TX 76129; or email her at **s.quebec.fuentes@tcu.edu**. Selected submissions will be published in a subsequent issue of

TCM and acknowledged by name, grade level, and school unless you indicate otherwise.

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Access **http://www.nctm.org** to find the full-size map online. This is a members-only benefit.

Where's the math?

When learning to measure, students must develop an understanding of the various attributes of an object that can be measured (i.e., weight, length, mass, area, volume, and time) as well as the process of measuring. For each attribute, the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) has a progression of standards. For example, some of the content standards for the attribute of length are—

- **K.MD.A.1:** Describe measurable attributes of objects, such as length or weight.
- **K.MD.A.2:** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.
- **1.MD.A.2:** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
- **2.MD.A.1:** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, metersticks, and measuring tapes.

The grade 3 standards for the attribute of area have a similar progression.

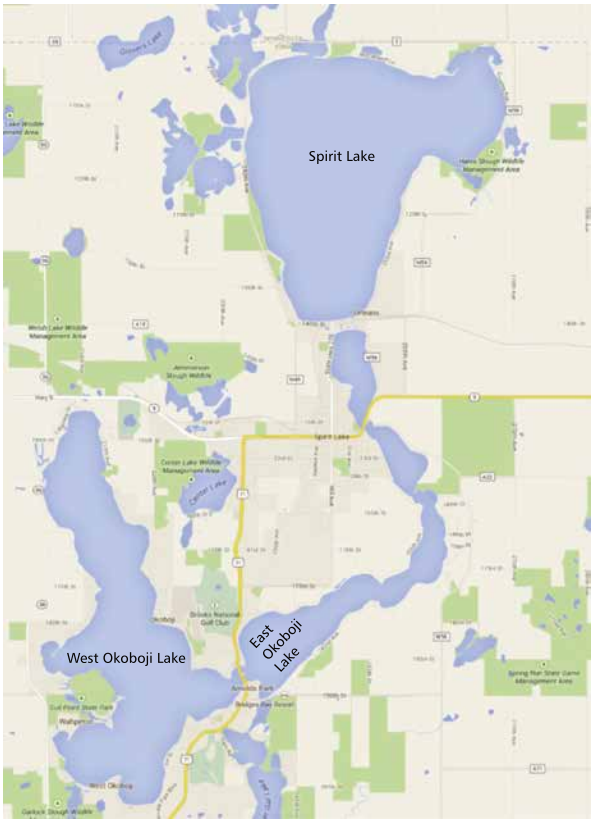
The following standards reflect recommendations made by Van de Walle and his colleagues (2016) with respect to the process of measuring and an instructional sequence for measurement. The process of measuring has three stages:

1. **Decide on the attribute to be measured.**
2. **Select a unit that has that attribute.**
3. **Compare the units—by filling, covering, matching, or using some other method—with the attribute of the object that is being measured. The number of units required to match the object is the measure (p. 454).**

To develop an understanding of measurement, Van de Walle and his colleagues outline a sequence of experiences for measurement instruction. First, students need experiences making comparisons so that they understand the attribute that they are measuring. After students understand the attribute being measured, they are then ready to use "physical models of measuring units to fill, cover, match, or make the desired comparison of the attribute with the unit" (2016, p. 455). Students should begin with nonstandard units before progressing to the use of standard units. Finally, after developing proficiency in using standard units, students should progress to the use of formulas or measuring tools. The fifth of CCSSM's Standards for Mathematical Practice (SMP 5), to *use appropriate tools strategically* (CCSSI 2010), is particularly relevant in measurement instruction. The tools that students use to measure are indicators of students' understanding of the various attributes and should reflect students' current developmental stage with respect to measurement.

The aforementioned process of measuring and instructional sequence should be repeated for all attributes. Students need multiple learning opportunities to progress through and develop an understanding of measurement and the process of measuring. The problem scenario presented herein provides a context, in the form of a decision about which of two lakes is *bigger*, that addresses the attributes of length and area and is accessible to students at various stages in the developmental process of understanding measurement.

Name_____



Which Lake Is *Bigger*?

Brenna and Shakira's families are going to Okoboji, Iowa, for their summer vacation. They are trying to decide if they should rent a cabin on the shore of West Okoboji Lake or Spirit Lake. Because they want to go for long hikes and boat rides during their vacation, they want to choose the biggest lake. Brenna thought that West Okoboji Lake was bigger than Spirit Lake. Shakira was unsure which lake was bigger. Brenna and Shakira need your help to determine which lake is bigger.

1. Using the map, find West Okoboji Lake and Spirit Lake. By just looking at the map, explain why Brenna might think West Okoboji Lake is bigger than Spirit Lake.

2. By just looking at the map, explain why someone might think Spirit Lake is bigger than West Okoboji Lake.

3. Measure the two lakes. Record the two measurements below. Which lake is bigger?

_____	_____
Spirit Lake	West Okoboji Lake

4. Measure using a different attribute or unit that would show that the lake you found to be bigger in question 3 is the smaller of the two lakes.

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