Examine how types of statistical variability recommended in GAISE can be taught alongside the data displays recommended in CCSSM.

ROYALTY, RACING, AND ROLLING PIGS

ZURIJETA/THINKSTOCK

Randall E. Groth

One morning, a statistician sat down for breakfast with a bowl of her favorite cereal, Crunchy Oats and Raisins. As she enjoyed the meal, several questions raced through her mind:

- How much does the largest raisin in my bowl weigh? How much does the smallest weigh? How different in weight are all the rest of the raisins?
- How could I even weigh the raisins accurately? Since they are so tiny, would a scale be sensitive enough to give the same reading each time, or would the readings differ slightly each time I set a raisin on it?
- Crunchy Oats and Raisins is an organic cereal. Would the raisins become plumper if farmers used a special chemical treatment on their grape fields?

Each question asks about a different part of describing and making the cereal, but the common thread is that all of them are concerned with a single, uniting concept: variability in data.

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Some statisticians have pointed out that the field of statistics essentially exists to study the variability we see in everyday life. Snee put it in the following terms: "If there was no variation, there would be no need for statistics and statisticians" (1999, p. 257). Likewise, Cobb and Moore stated that the need for statistics "arises from the omnipresence of variability" (1997, p. 801). Because variability is so foundational, the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE 2007), published by the American Statistical Association (ASA), recommend that teachers help students understand the nature of variability and its origins during their earliest experiences learning statistics.

Given the importance of early work with statistical variability, elementary school teachers who are implementing the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) may face a dilemma: The word *variability* is not mentioned at all in K–grade 5 CCSSM. Instead, the K–grade 5 CCSSM data and measurement standards focus on constructing line plots, picture graphs, and bar graphs. After constructing these displays, students are to perform such tasks as finding the difference between the highest and lowest observation and determining how many more or less one category may contain than another (see **table 1**). Although such activities may set the stage for later study of different types of variability, how to engage students in studying sources of variability in elementary school under CCSSM without overburdening the curriculum with too many topics is not immediately obvious. I faced the CCSSM variability dilemma in a summer math camp for students in grades 2–5. I wanted to provide experiences to help them succeed with the measurement and data standards in CCSSM, but I also wanted the students to understand the nature and importance of statistical variability. I found that by selecting problem contexts carefully, I could provide experiences with all types of variability recommended for young learners in the GAISE report while addressing the graphing-focused standards of CCSSM. Separate lessons for graphing and variability were unnecessary. Within the span of three lessons, students worked with three types of variability from GAISE—natural, measurement, and induced—while also working with the types of statistical graphs required in CCSSM.

Lesson 1: Natural variability

Natural variability is described in the following manner in the GAISE report:

> Variability is inherent in nature. Individuals are different. When we measure the same quantity across several individuals, we are bound to get differences in the measurements. Although some of this may be due to our measuring instrument, most of it is simply due to the fact that individuals differ. (Franklin et al. 2007, p. 6)

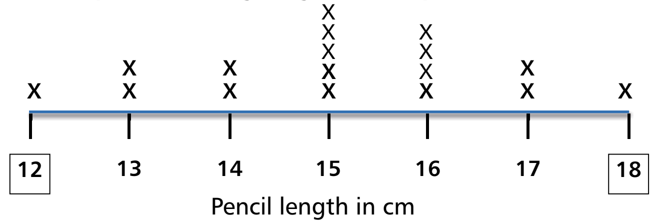
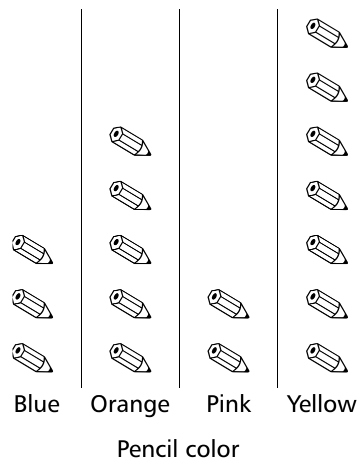
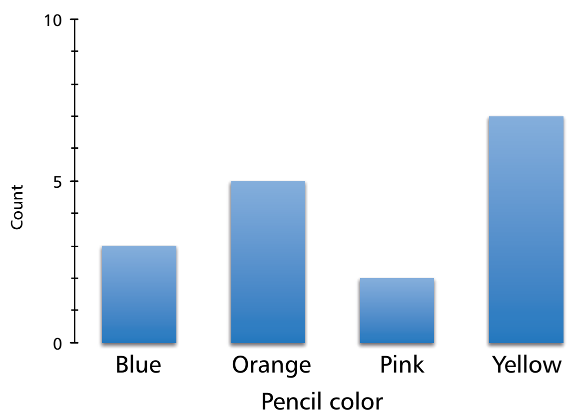
Many contexts for gathering data provide opportunities to observe natural variability. Contexts that connect to children's experiences allow them to explore and graph intriguing data sets.

I designed a lesson involving natural variability using the book *How Big Is a Foot* (Myller 1990). The story is set in a kingdom where standard units of measurement had not yet been invented. One day, a king instructs a servant to build a bed three feet wide and six feet long. On receiving his orders, the servant uses his own feet to measure the length and width of the bed, not knowing that the king's feet are much bigger than his own. When the bed ends up being much too small, the king throws the servant in jail. As the servant sits in jail, he realizes that the king's foot must be a different length than his own. The servant finally obtains a marble copy of the king's foot, uses it to measure the correct dimensions for the bed, and gets himself released from jail. The marble copy of the king's foot then becomes the standard foot length measurement for the entire kingdom.

After reading *How Big Is a Foot*, I asked students why the servant got into trouble and

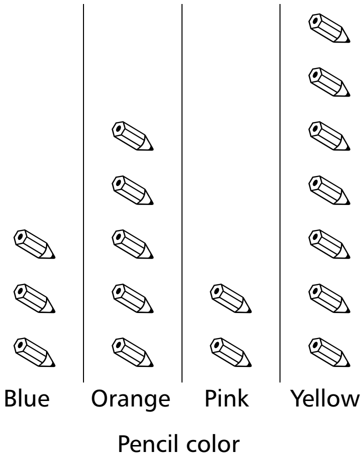
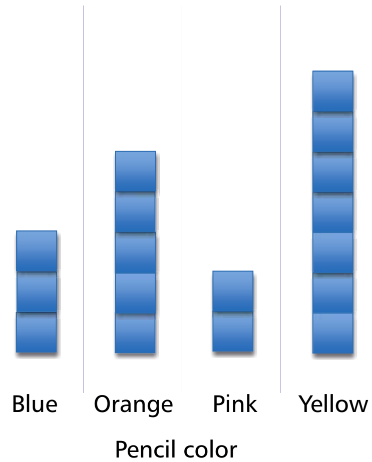
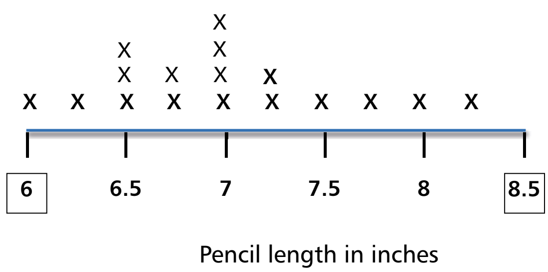
TABLE 1

Although such activities as those in the Common Core content standards for statistical graphs in grades 2–5 may lay a foundation for later study of different types of variability, how to engage students in studying sources of variability in elementary school is not immediately obvious.

<table>
<tr><th rowspan="2">Grade level</th><th colspan="2">CCSSM measurement and data content standards for representing and interpreting data</th></tr>
<tr><th>Standard</th><th>Graphs corresponding to the given standards</th></tr>
<tr><td rowspan="3">2</td><td>CCSS.Math.Content.2.MD.D.9
Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</td><td>(a) Line plot (showing lengths of 17 pencils)
</td></tr>
<tr><td rowspan="2">CCSS.Math.Content.2.MD.D.10
Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.</td><td>(b) Picture graph (showing colors of 17 pencils)
</td></tr>
<tr><td>(c) Bar graph (showing colors of 17 pencils)
</td></tr>
</table>

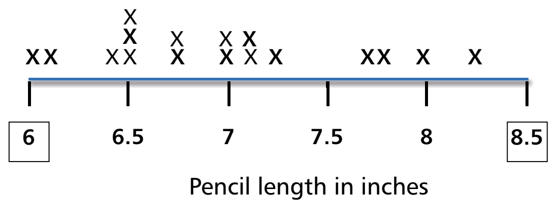
(**Table 1** *is continued on the next page*.)

TABLE 1 (cont.)

Grade level	CCSSM measurement and data content standards for representing and interpreting data Standard	 Graphs corresponding to the given standards
3	**CCSS.Math.Content.3.MD.B.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	**(d) Scaled picture graph (showing colors of 85 pencils)**  *Graph key: Each icon above represents 5 pencils*
		(e) Scaled bar graph (showing colors of 85 pencils)  *Graph key: Each square above represents 5 pencils*
	CCSS.Math.Content.3.MD.B.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	**(f) Line plot (showing lengths of 17 pencils)** 

(**Table 1** *is continued on the next page*.)

TABLE 1 (cont.)

Grade level	CCSSM measurement and data content standards for representing and interpreting data Standard	Graphs corresponding to the given standards
4	**CCSS.Math.Content.4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	**(g) Line plot (showing lengths of 17 pencils)** 
5	**CCSS.Math.Content.5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	

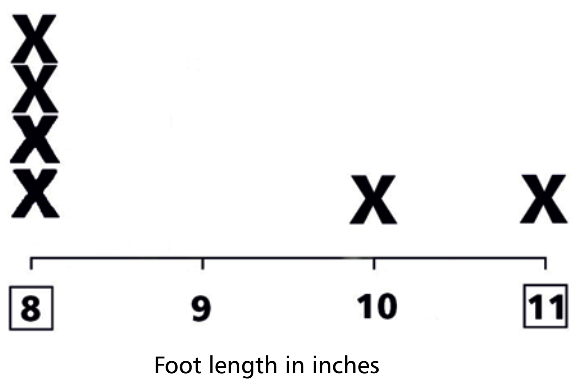
how he redeemed himself. Students said that the foot size of the king and the servant were different. I used this opportunity to introduce the term *natural variability*, pointing out that it exists because individuals differ in many ways. I then asked students to imagine that our classroom was also a kingdom where standard units of measurement had not yet been invented. Since we had no "king" in our classroom, I suggested that we find the typical foot length in our class to decide what the standard length of a "foot" should be.

As students began measuring foot lengths, an interesting discussion began. Some students were measuring their feet with their shoes on, and others with their shoes off. Without my prompting, they recognized that this would cause a problem with the data. Ultimately, students decided it would make more sense to measure length with shoes off for the sake of accuracy. The GAISE report highlights the importance of attending to such issues by recommending that students talk about measurement protocols like this during their early experiences of collecting data. These types of discussions may be implied in the sixth of the Common Core's Standards for Mathematical Practice (SMP) 6, "Attend to precision," but they are not explicitly mentioned in the CCSSM content standards. It is important to keep sight of having such discussions, as they do not detract from addressing the content standards. In fact, in this case, students felt more confident in the data as a result of the discussion.

After students agreed on a method for measuring foot length, I had each of them record their measurement on a sticky note. We placed the sticky notes on the whiteboard and organized them into columns. This organizational scheme lent itself readily to producing a line plot showing the measurements we had obtained (see **fig. 1**). I did not require students to measure halves and quarters of an inch; doing so is required in the grades 3–5 CCSSM but not in grade 2. Teachers working exclusively with students in grades 3–5, however, could make this small adjustment to more fully address the CCSSM line plot requirements. Our discussion of the class line plot brought some interesting student ideas to light. When I asked what the foot length measurement for our

FIGURE 1

Students agreed on a method for measuring foot length, recorded their measurement on a sticky note, and organized the notes into columns on the whiteboard. We then produced a line plot of students' foot lengths (measured to the nearest inch).



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"classroom kingdom" should be, some immediately responded "twelve inches" without even referring to the data or line plot. Instead, they justified the decision by saying they already knew that there are twelve inches in a foot. This response gave me an opportunity to direct students' attention back to the original question, our reason for collecting data, and the data itself. As I did so, some students began to focus instead on where the data appeared to be clustered on the graph, leading them to suggest eight or nine inches as a reasonable length. GAISE highlights the importance of drawing students' attention back to the context and original question by emphasizing that context is what gives data meaning in statistics. This sort of activity may be implied in SMP 1, "Make sense of problems and persevere in solving them," and SMP 6, "Attend to precision," but it is not explicitly mentioned in the Common Core's content standards. Directing students' attention back to the context in this case was a natural reaction to students' thinking, and did not detract from addressing CCSSM graphing content standards. On the contrary, doing so seemed to enhance students' abilities to make sense of the class line plot (see **fig. 1**). **Figure 2** summarizes the guiding questions used to motivate students and facilitate construction and analysis of the class line plot.

CCSSM for grades 2–3 also recommend that students learn to construct graphs for categorical data. Distinct from quantitative data, categorical data consist of categories rather than numerical quantities. For example, in a classroom survey, students might ask their classmates to name their favorite flavor of ice cream. Responses to this question would be categories like "vanilla," "chocolate," and "strawberry," rather than numbers. I extended the "classroom kingdom" context from our *How Big Is a Foot* lesson to prompt students to generate categorical data. They filled out a questionnaire asking what some of the official symbols of our classroom kingdom should be. Students provided opinions on the official animal, color, food, sport, plant, insect, football team, and beverage. This gave us a great deal of categorical data that we then used to construct bar graphs to be used to make decisions about the official symbols.

The line plots and bar graphs we produced provided opportunities to solve "put-together, take-apart, and compare" problems required by CCSSM. For example, I asked,

- "What is the difference between the shortest and longest foot length in our class?"
- "In a different classroom, Pat's foot is three-and-one-half inches longer than Joe's. Joe's foot is seven inches long. How long is Pat's foot?"
- "If everyone's shoes were put end-to-end, how long would the line of shoes be?"
- "How many more people preferred red than blue?"

Hence, we were able to satisfy CCSSM measurement and data standards while also addressing subtle but important points about learning statistics highlighted in GAISE.

Lesson 2: Measurement variability

The GAISE report describes *measurement variability* in the following manner:

> Repeated measurements on the same individual vary. Sometimes two measurements vary because the measuring device

FIGURE 2

Guiding questions were used to motivate students and facilitate construction and analysis of the class line plot.

Launching the foot measurement investigation

- Why did the servant in the story get into trouble? How did he redeem himself? Imagine that standard units of measurement have not yet been invented. We could use a typical foot length as a standard unit. What is the typical foot length of students in our classroom?

During the foot measurement investigation

- Why is it important for all of us to measure in the same way?
- What are some different ways people might measure foot length? Which way is best? Why?

After producing a line plot from the foot measurement investigation

- What is the typical foot length for students in our class? How does the line plot help us see it?

> produces unreliable results, such as when we try to measure a large distance with a small ruler. At other times, variability results from changes in the system being measured. For example, even with a precise measuring device, your recorded blood pressure could differ from one moment to the next. (Franklin et al. 2007, p. 6)

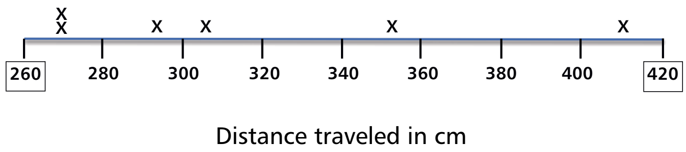
CCSSM for grade 2 briefly allude to the idea of measurement variability by stating that students should make repeated measurements of the same object, although the idea of measurement variability is not further developed explicitly in subsequent grade levels.

To connect the idea of measurement variability to something of relevance to my students, I had them measure the distance traveled by a pull-back toy car. I explained that I purchased several of the same type of pull-back car at the store without knowing how far they might go forward after being pulled back and released. I asked students if they believed a given car would go the same distance every time. No one believed that it would. This provided motivation to take repeated measurements of the distance traveled on several different trials.

As students worked in pairs with the pull-back cars, opportunities to explore measurement issues once again emerged. Students discussed how far they should pull the car back before letting it go. They could measure this distance by listening to the number of clicks the car made when pulled back. Students also discovered that the cars tended to go in circles when placed on a hard surface but ran straighter on carpet. This observation led them to conduct trials and do measurements on the carpeted floor. Even on the carpet, however, the cars sometimes curved a bit off a straight-line course. In such trials, measuring the total distance traveled required deciding where the car veered off course and moving the tape measure in the appropriate direction. Additionally, sometimes the cars would travel farther than the length of one tape measure. Such cases required iteration of the tape measure to determine total distance.

FIGURE 3

This is the line plot of one group's measurements for distance traveled by a pull-back car (measurements taken in centimeters).



To wrap up the repeated measures activity, I asked one group to share the line plot for their measures on the board (see **fig. 3**). At this point, I wanted to emphasize that we had done the activity not only for the sake of making line plots but also for understanding how a pull-back car would behave. So, I pointed out that the packaging for the cars did not say how far we could expect it to go. I asked students to think about what a reasonable claim would be to put on the package. Just as in the *How Big Is a Foot?* scenario, this question helped prompt them toward examining where the data seemed to be clustered. In **figure 3**, for example, most data were clustered between 270 cm and 310 cm. The process of looking for central clusters of data helps lay groundwork for the grade 6 Common Core standards, when students are to relate measures of center, such as mean and median, to characteristics of data distributions.

The Pull-Back Car lesson blended the ideas of measurement variability and natural variability. Measurement variability came into play, as noted, when students took repeated measurements of the same attribute (distance traveled) for the same object (a pull-back car). One could also think of variation in distances traveled as a type of natural variability, because the pull-back cars varied in their built-in capabilities for this attribute. Differences in measurement procedure from group to group might be considered another source of variability that is present in the lesson. Key questions help students get the most out of the lesson (see **fig. 4**).

Lesson 3: Induced variability

The third type of variability that GAISE recommends students encounter early in their learning of statistics is *induced variability*. Induced variability is often at the core of

FIGURE 4

The Pull-Back Car lesson blends measurement variability and natural variability. Key questions for the lesson 2 class line plot help students get the most benefit from the lesson.

Launching the pull-back car investigation

- Here is a pull-back car I just purchased. Do you think the car will travel the same distance forward each time it is pulled back? Why or why not?
- The company that makes these pull-back cars wants to put a label on the package to say how far you can expect it to go forward after you pull it back. How could we help them decide what to put on the label?

During the pull-back car investigation

- How far should you pull the car back before letting it go each time? Why?
- Does the type of surface you run the car on matter? Why or why not?
- What should you do to measure the distance traveled if the car travels longer than the length of your tape measure?

After gathering and plotting distance measurements

- Let's look at group 1's line plot. How can we use their graph to help the company decide what to put on the label that will say how far we can expect the car to travel?
- Do any groups have line plots that differ from group 1's? If so, how do they differ? Why do they differ?

FIGURE 5

The class began exploring induced variability by playing a game called Pass the Pigs.

Launching the pig-rolling investigation

- Here is a small toy pig. If I toss the pig, do you think it has a better chance of landing on its feet or on its side? Why?
- If you tossed the pig 100 times, how many times do you think it would land on its snout and ear? Explain.

During the pig-rolling investigation

- What do you think would happen if we taped a bead to the side of the toy pig? Would it help you get higher scores? Why or why not?

After producing line plots for scores from using both the original pig and the modified pig

- Which pig do you think is higher scoring, the normal pig or the modified pig? Use the line plots we made to help explain your answer.

statistical investigations. For example, we may be interested in studying whether cornfields treated with fertilizer have greater yields than comparable untreated fields. Both the treated and untreated fields will have a degree of natural variability, but differences between treated and untreated fields may be induced by application of fertilizer. Similarly, the usefulness of a medical treatment is often established by comparing its effects on groups receiving the treatment to comparable groups not receiving it.

My class started its exploration of induced variability by playing a modified version of the game Pass the Pigs. The game involves tossing a small toy pig in the air and observing how it lands (see **the activity sheet** on **p. 228**). Essentially, the less likely the outcome, the more points gained. A pig is less likely to land on all four feet, for example, than on its back. The probability of landing on all four feet is approximately 9 percent, and the probability of landing on its back is approximately 22 percent (Kern 2006).

As students played several rounds of the game, they recorded their scores. When they finished, I had each group graph its scores in a line plot. Doing so provided additional practice constructing this CCSSM-required representation. I then asked students if the line plots would look different if we taped a bead to the side of the pig (see **fig. 6**). Some students thought this would not make a difference in the scores obtained. Others thought it would increase the scores because the pig would be less likely to land on its side. To explore the issue, students played several more rounds of the game, this time with a bead taped to the side of the pig.

Students quickly noticed that taping a bead to the side of the pig increased the scores. The bead made it not only less likely for the pig to land on its side but also much more likely for it to land on its snout and ear. No one in the first round of the game had attained the snout-and-ear landing. Such an outcome has a probability of only about 3 percent with a nonmodified pig (Kern 2006). The line plots of scores that students produced for the second round of the game helped confirm what they observed as they carried out trials: Marks on the line plot were much farther to the right along the horizontal axis. Modifying the pig by taping a bead to its side led to higher scores, just as applying fertilizer might lead to greater yields, or giving a medical treatment

might have positive effects. Again, key questions help guide students during the lesson (see **fig. 5**).

Making sense of graphs

By the end of the three-lesson sequence, students had accumulated a great deal of experience working with line plots and bar graphs, as required by CCSSM. They also engaged in the Common Core's SMP of making sense of problems, persevering in solving them, and attending to precision. These practices were apparent as students discussed measurement protocols and analyzed data by thinking about the context in which the data were generated. While students engaged with these Common Core standards, they experienced all three types of variability recommended in GAISE, which I found especially exciting.

In their work, statisticians do not make graphs just for the sake of producing them. Instead, graphs serve the purpose of revealing characteristics of data sets and patterns within them. By working with the three variability contexts, my students not only practiced constructing graphs but also used them to make sense of situations involving natural, measurement, and induced variability. This, in turn, helped foster the idea that producing statistical graphs is not an end in itself, but a way to help us represent and analyze different types of variability we observe regularly in the world around us. By carefully choosing the problem contexts we use in class, we can address the Common Core measurement and data standards at the same time as we build students' intuitions about different types of variability described in the GAISE report.

Common Core Connections
2.MD.D.9
2.MD.D.10
3.MD.B.4

FIGURE 6

To study induced variability, students tossed a modified pig die into the air and recorded how it landed (on its feet, on its back).

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Name________________________________

Pig Toss Rules and Score Sheet

To start the game, toss the pig as if rolling a number cube. Record your score on the score sheet. Then give the pig to the next player. The person with the highest score after ten tosses wins the game.

If the pig lands on its—	You get—
Side	0 points
Back	5 points
Feet	10 points
Snout	15 points
Snout and ear	20 points

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Game 1

Toss	1	2	3	4	5	6	7	8	9	10
Points										

Total score for game 1: _________

Game 2

Toss	1	2	3	4	5	6	7	8	9	10
Points										

Total score for game 2: _________

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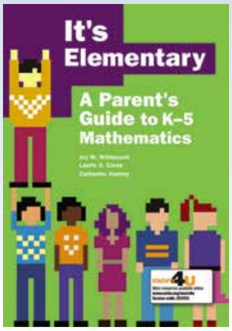

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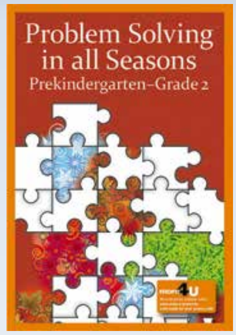

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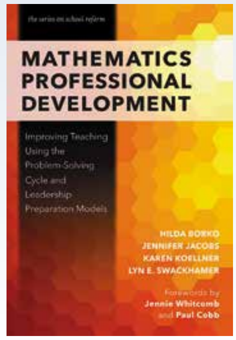

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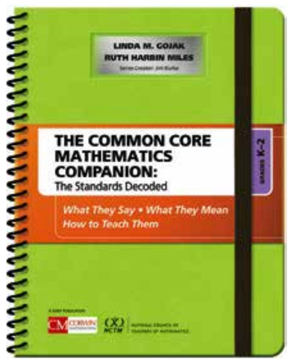

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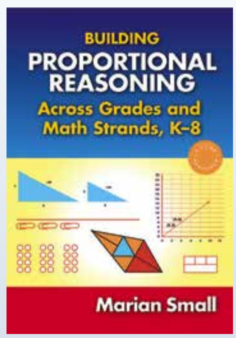

BY MARIAN SMALL

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A resource providing examples and questions that introduce students in the early grades to the basic ideas of proportional reasoning, helping them develop skills for working with ratios, percentages, and complex representations.

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