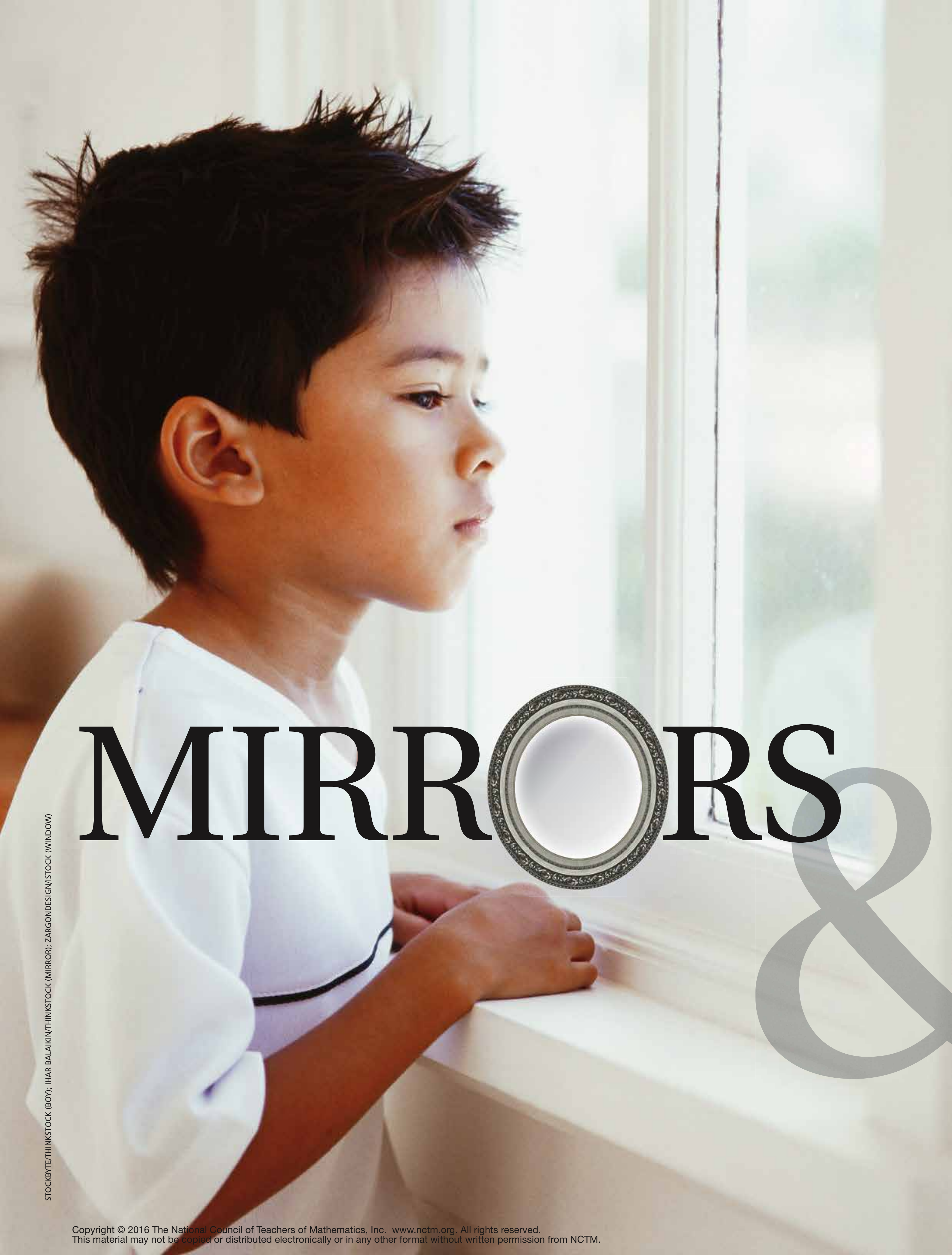


MIRRORS &

STOCKBYTE/THINKSTOCK (BOY); IHAR BALAIKIN/THINKSTOCK (MIRROR); ZARGONDESIGN/ISTOCK (WINDOW)

WINDOWS INTO STUDENT NOTICING

Learn how to help culturally diverse and socioeconomically disempowered students use problem posing to reflect their knowledge and gain insight into new knowledge.

Higinio Dominguez

In many classrooms, students solve problems posed by others—teachers, textbooks, and test materials. These problems typically describe a contrived situation followed by a question about an unknown that students are expected to resolve. Unsurprisingly, many students avoid reading these problems for meaning and instead engage in a suspension of sense making (Schoenfeld 1991) characterized by rule-following behavior (Boaler 1998) and keyword searches (English 2009). Problems in everyday situations, however, do not come preformulated. Instead, these problems and the reasoning that they instantiate develop simultaneously as problem solvers informally question the situation (Why is this happening?) and begin to formulate conjectures and possible pathways for solving these problems (How can I begin to solve this problem?). For example, posing an equal-sharing problem exclusively on an equal-amount-per-sharer basis ignores how children may notice fairness experientially, including who is hungrier, who prefers to eat less, or who likes the shared product the most.

In everyday situations, problems do not predate the person-world interaction; rather, problems are defined and redefined by problem solvers. In fact, many situations in daily life are not necessarily problems. Consider the following example.

> Luis had some candies. His sister gave him 7 more candies.

After all, ending up with more candies is hardly a problem for children. Therefore, an agreement between students and teacher regarding what constitutes a "problem" seems to be foundational for fostering student problem posing.

A windows-and-mirrors framework for investigating student noticing

Word problem difficulties have been investigated primarily by looking at what students do or do not do, including looking at student errors (Newman 1977) or students' failure to use linguistic knowledge (Greeno 1985). For English language learners (ELLs), evidence points to vocabulary and other language-related skills that have not yet been acquired, which often leads to suggestions about simplifying the vocabulary and linguistic structures of word problems (Abedi 2001). Alternatively, good problem solvers are those who use schemata—meaning structures that exist in the mind (Fischbein 1999; Hershkovitz and Nesher 2003). These explanations, although valid, leave the mathematical problem formulation untouched, suggesting that students, not the problems, need to change. Gutiérrez (2002) has questioned why students must adapt to school mathematics and not the other way around. Silver (1994) also noted that "students are rarely, if ever, given opportunities to pose in some public way their own mathematics problems" (p. 19).

I suggest using a windows-and-mirrors framework for encouraging students to be problem posers. Like a window, a problem should be an opportunity for students and teachers to look out for what makes sense to solve the problem. Simultaneously, like a mirror, a problem should be an opportunity for students and teachers to look into what students notice as relevant for solving a problem. Is the window too wide, too open or, on the contrary, too narrow; or is it just right? For the same matter, does the mirror allow students to recognize in the situation something that they know and that can be useful for solving a problem? Readjusting the window and/or the mirror—as it is recognized in word problems—signifies reformulating a problem. Expert problem solvers spend considerable time formulating and reformulating problems (Silver and Marshall 1989). This reformulating is important because asking students to solve preformulated problems has shown that students do not make sensible connections of mathematics to situations (Silver and Shapiro 1992). When such experiences repeat every day, students develop self-perceptions that they are not good at math.

Blas, who used to be bad in math

Working with a first-grade bilingual Latino student who used to think of himself as not good in math reminded me of the importance of problem posing in racially and socioeconomically diverse classrooms. I met Blas in a school that offers 100 percent reduced-price or free lunch to a large population of Latino/a and African American students. Their parents are predominantly working class, and many have been unemployed for a long time. In an interview administered to forty-three elementary students in this school, Blas, like many other students, identified himself as not being good in math. To challenge this self-perception and the premises on which it may have been constructed, I gave Blas the following separate, change-unknown problem.

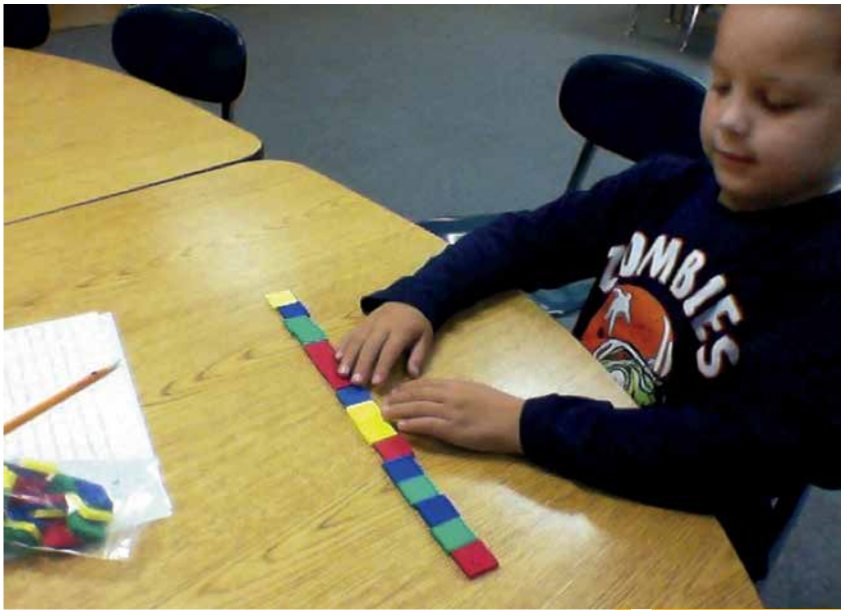
> Juan had 16 toys, but he lost some. Now he has 9 toys left.

To invite Blas into problem posing, I asked, "What's the problem here?"

He replied, "That he lost some toys."

"Right!" I said, adding, "It's a problem when people lose things, isn't it?"

Blas reciprocated with a possible scenario about *how* Juan could have lost his toys, as he conjectured, "Maybe he was playing with his toys in his room and some ended up under the bed," suggesting that "he would have to clean his room, because some toys may be under the bed, and some may be behind his dresser or something."



HIGINIO DOMINGUEZ

Posing meaningful problems increases student engagement and the likelihood of successful problem solving.

Silver (1994) explained that—

> problem formulation represents a kind of problem-posing process because the solver transforms a given statement of a problem into a new version that becomes the focus of solving. . . . If the source of the original problem is outside the solver, the problem posing occurs as the given problem is reformulated and "personalized" through the process of reformulation. (pp. 19–20)

Clearly, Blas was not noticing—at least not yet—the mathematics embedded in a problematic situation but, instead, the problematic situation embedded in the mathematics that I wanted him to learn. In talking about how it is problematic to lose something, Blas and I opened a window that was just right to see the mathematics, and in so doing, he recognized his own experiences with losing toys.

Blas successfully solved this problem by first creating a set of sixteen counters. He then separated a subset, saying, "He lost some!" thus demonstrating understanding of the concept of an unknown. Next Blas turned to count the known set, explaining that he wanted that set to have nine. Seeing that this set had only seven, he moved two counters from the unknown set to make nine. Finally he counted those that Juan had lost and got seven.

I asked, "How come you said earlier that you were not good in math?"

Blas did not say a word. Instead, he looked down slightly with a smile on his face that combined modesty, joy, discovery, and pride.

Inviting students to problem pose

Several strategies—all research-based—can support all students in problem posing, particularly those whose perspectives do not figure in classroom instruction. For example, problem-solving interviews have been used as tools for understanding a child's mathematical reasoning. Teachers can recalibrate this tool to develop cognitive empathy with children—particularly racially and socioeconomically diverse students—and learn more about what matters in their world (Dominguez 2011). We could enter the child's mind (Ginsburg 1997), but we could also look for ways to enter the child's world of experiences, funds of knowledge, or simply what matters in the child's world by deliberately inviting students to pose problems.

Problem posing can also be promoted with problems describing situations in which contexts are unfinished, thus creating opportunities for students to engage meaning and interpretation (Carraher and Schliemann 2002). Finally, research on children's mathematical thinking (Carpenter et al. 1999) highlights how children attend to action words in problems (e.g., *give, lose, share*), presenting opportunities to promote problem posing by inviting students to customize these action words. Following are four practical problem-posing strategies.

1. Let students specify some quantities.
If students are familiar with the number of items included in a set (e.g., crayons, gum, cards), leaving that quantity unspecified can serve as a mirror into what students know. For example, I used the following problem with two groups of bilingual Latino/a third graders (Dominguez 2011):

> The cafeteria cook needs to make scrambled eggs for breakfast for 270 children. How many egg cartons does the cook need to open?

Students initially reacted, "But we don't know how many eggs are in a carton!"

When I asked, "Really?" they remembered having seen various sizes at the grocery store, such as 12, 24, or even 36 eggs per carton. Using this knowledge, they began problem posing by arguing that if the school had 270 children, using the largest carton would make more sense. Students also considered how many eggs per breakfast, with some using only one egg, and others using two eggs. This problem scenario invited these young problem solvers to use what they knew about egg cartons and about how many eggs they like to eat for breakfast. But the problem also prompted students to open multiple mathematical windows as they engaged in various negotiations that were saturated with meanings and interpretations. For example, some students remarked, "If we use 12, it's going to take longer to divide 270 by 12; so let's use a bigger carton."

Others said, "If the cook opens smaller cartons, it's going to take longer to make the breakfast; and the line at the cafeteria is going to be longer, and kids would have to wait forever!"

2. Let students frame problem questions. A group of practicing teachers (grades 1, 2, and 4) and preservice teachers (K–grade 5), who were interested in eliciting student noticing, replaced the question at the end of a problem with open-ended questions similar to those I had used with Blas. Consider the following task.

> Juan had 16 toys, but he lost some. Now he has 9 toys.

Instead of asking, How many toys does Juan have left? teachers asked, What's the problem here? thus eliciting the children's personal—therefore meaningful—interpretations of the problem situation. Like Blas, many students confidently said, "That he lost some toys!" As they continued asking open-ended questions, teachers listened hermeneutically—without judging or evaluating (Davis 1996)—to children's problem posing.

Students often changed the problem situation. For example, one student grabbed two pencils to animate his own story:

> OK, let's say this [*holding one pencil*] is a good guy who had sixteen dollars in his wallet, and this one [*using another pencil*] is a bad guy who stole some dollars from him. Now he [*waving the pencil representing the good guy*] needs to find out how many dollars the bad guy stole!

Sometimes the "mirror" in these problems reflected so many details that students focused more on the details than on the mathematics. A gentle turn toward a "window" so that students could see what the teachers wanted them to learn was all it took to refocus most students. For example, the teachers asked, "Instead of Juan cleaning his room to find the toys, do you think you could help Juan solve his problem with just math?" Teachers validated the idea of cleaning the room, but at that point, asking children to use math was a challenge that these problem posers were prepared to take on, because they had transformed the problem from "He lost some toys" to "How many toys did he lose?"

None of these students had received instruction on separate, change-unknown problems (e.g., 16 – ? = 9). More important, many of them had self-identified as not good in math. However, nearly all of them solved the problem using meaningful math strategies that stemmed from equally meaningful problem posing. In addition, students are more likely to problem pose when they interpret a situation as problematic, for example, when they lose something. Certain situations, however, may not be perceived as problematic at all. Consider this example:

> Diego has fifty books, and he wants to donate thirteen to the public library.

In this case, such action words as *donate* (or *share*, *give*, etc.) are all voluntary actions that students may not perceive as problematic. Open-ended questions are still possible for a task like this; for example, "What do you think is going to happen after Diego donates thirteen books to the library?" Such open-ended questions reflect an interest in eliciting how the child thinks about situations that others have considered as "problematic."

3. Promote problem posing at various points.

Many students are not used to—and often resist—problem posing. Like Blas, these students have developed a view that being good in math means quickly getting right answers, and they tend to see problem posing as interfering with this view. The aforementioned preservice teachers found that children across K–grade 5 often did not respond well to their invitations to problem pose. However, teachers also noticed that those who problem posed reported their solutions with more confidence than those who did not, locating the evidence for this claim in the students' voice intonation. For example, unlike their peers who reported answers with raising intonation indicating doubt, problem posers reported their answers without such voice inflexion.

Teachers can help students restore the mirrors and open the windows of mathematics. For example, I often ask students, "What if Juan had not sixteen but twenty-three toys and after losing some, he had nine left? Do you think your answer to the original problem could help you solve this new situation?"

For students who find the original problem challenging, the "what if" prompt could be "What if Juan had fifteen toys, but he loses some, and now he has ten?" Changing to easier number combinations can help students develop the habit of reformulating problems. Once students can solve these reformulated problems, I give them the original problems again, asking them to use their work and ideas while they are fresh from their problem-posing activity.

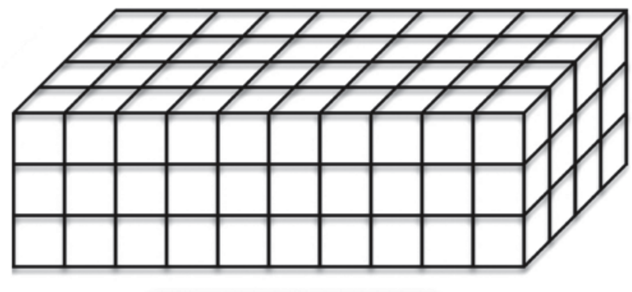
4. Invite students to interpret representations.

Often, well-intended representations of mathematical concepts (e.g., shaded parts in a circle for fractions) generate unintended student misinterpretations emerging from the inherent ambiguity of static representations. For instance, are students supposed to notice the shaded or unshaded parts for naming a fraction in a circular model? In my current work with teachers, I encourage them to invite students to scrutinize these representations before they begin the problem-solving process. For example, a third-grade bilingual Latina misinterpreted the intended three-dimensionality of a 4 × 3 × 10 rectangular prism on a benchmark test (Dominguez 2014). As a result, she counted only the faces of the one cubic centimeter on the three visible faces of the model, resulting in an incorrect answer that she found as part of the multiple choices on the test (see **fig. 1**). Refusing to believe that she could not correctly compute

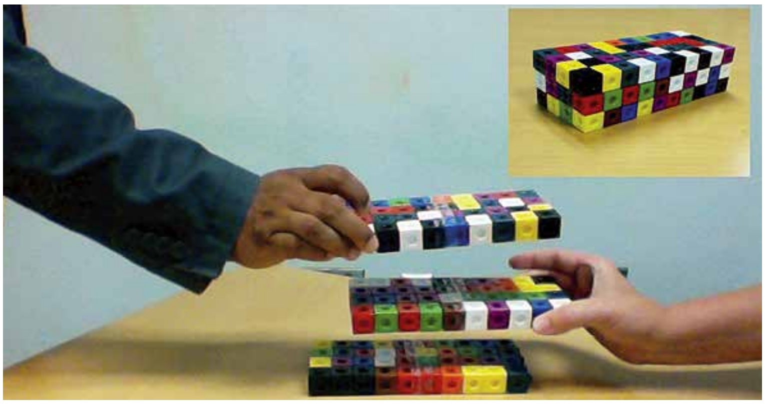
FIGURE 1

Reinterpreting the picture of a 3 × 4 × 10 rectangular prism on a test, a fourth-grade bilingual Latina corrected her answer for a volume problem. The initial partial surface area of eighty-two little squares (produced by counting only the squares on the visible faces) was revised and re-envisioned as the student first invented successive composite units of ten connected cubes and then the larger composite unit of forty connected cubes.

(a) El siguiente modelo está hecho con cubos de 1 centímetro.



(b) ¿Cuál es el volume del modelo?



HIGINIO DOMINGUEZ

the volume, I asked her to use connected cubes to reconstruct the drawn model. This reformulation allowed her to shift her attention back and forth from one "window" (the model drawn on the test) to another "window" (the emerging model in front of our eyes) as she noticed the dimensions of length, width, and height on both models. This student also kept opening larger and larger "windows" as she invented two progressively complex composite measurement units. First she invented the composite unit of ten (ten connected cubes). She subsequently came up with a unit of forty by looking at three layers of forty cubic centimeters each.

Empowering all students through problem posing

Questions that can elicit student noticing by restoring experiential mirrors and opening mathematical windows include the following:

- What is the problem here?
- What do you know about this situation that is not mentioned in this task?
- To make this task easier for you, would you change the story or the numbers?
- What do you think it is going to happen next in this situation?
- If you do not think this is a problem, can you create a true problem either with a different situation or different numbers or both?
- What would the solution be if the situation involved different quantities?
- Would the previous solution support you in finding the new solution?
- Could we represent this given model in an easier way?

It is in this noticing that students can see reflections of what they already know and opportunities to see what else they can know as they continue learning mathematics.

Common Core Connections
SMP 1
SMP 4

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Let's continue this chat

TCM has a new way for the journal audience to interact with authors and fellow readers.

On **Wednesday, February 10, at 9:00 p.m. EST**, we will expand Higinio Dominguez's conversation "Mirrors and Windows into Student Noticing" (p. 358). Join us at #tcmchat. We will also Storify the conversation for those who cannot join us live.

We anticipate tweeting about a feature from each future *TCM* issue.

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Higinio Dominguez, **Higinio@msu.edu**, an assistant professor in the department of teacher education at Michigan State University in East Lansing, focuses his research on supporting teachers to elicit student noticing as a continuous, self-sustained source of ideas for developing common resources for teaching and learning mathematics.