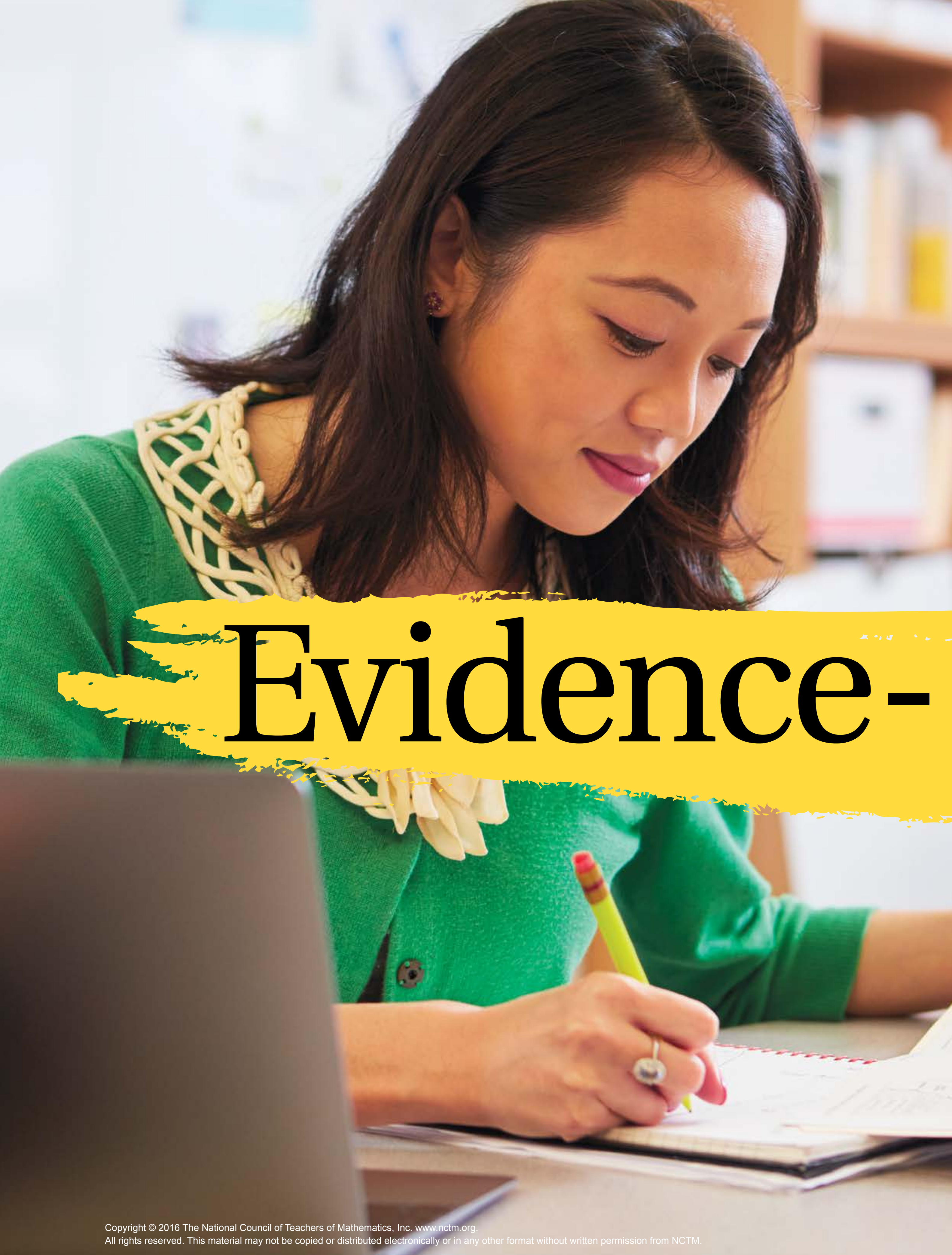

# Evidence-

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**Comparing two fractions gives a context for exploring students' flexibility with and understanding of mathematical ideas.**

# Centered Assessment

Kimberly Morrow-Leong

The importance of developing mathematical fluency is increasingly gaining recognition in mathematics education. Students should develop fluency in stages. For example, students in grade 1 should be able to add and subtract within 20 using objects, drawings, and equations but by the end of grade 2 know them with automaticity (CCSS1 2010, p. 19). By distinguishing between "fluently" adding and subtracting within 20 and knowing these facts from memory, the Common Core State Standards for Mathematics (CCSSM) defines fluency as broader than quick and accurate fact recall.

*Principles and Standards for School Mathematics* indicates that students exhibit fluency when they "demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (NCTM 2000, p. 152). In one of her NCTM President's Messages, Linda Gojak emphasized flexibility and fluidity when she said that—

> focusing on efficiency rather than speed means valuing students' ability to use strategic thinking to carry out a computation without being hindered by many unnecessary or confusing steps in the solution process. (Gojak 2012)

Kling and Bay-Williams (2014) summarized these definitions of fluency into four properties that include *flexibility*, *accuracy*, *efficiency*, and the *appropriate use of strategies*.

A broader definition of fluency requires us to consider different approaches to the assessment of student learning and challenges us to look carefully at evidence of student understanding in their written work. And although written work cannot reveal the full range of student understanding, it gives teachers the most common opportunity they have to closely examine individual student thinking in depth. In this article, I propose guidelines for the close examination of student work. These guidelines focus on classifying the work that students do, shifting perspective to search for what is right about the student work, and homing in on evidence to make inferences about student thinking, and in this case, embrace the broader view of fluency for comparing fractions.

The sixth-grade student work featured in this article is taken from a short checkup in the Connected Math Project's "Bits and Pieces" unit (Lappan et al. 2006). Only four items are the focus of my assessment of student thinking because I believe that these four alone offered enough information to not only assess individual students' understandings but also broadly understand the progress of the whole class. Take a moment to compare these four sets of fractions and make a note of the first strategy you thought about. Afterward, try at least one other strategy that you think students might use to compare the fractions.

a. $\frac{2}{4}$ ____ $\frac{7}{12}$

b. $\frac{5}{8}$ ____ $\frac{6}{10}$

c. $\frac{8}{12}$ ____ $\frac{10}{15}$

d. $\frac{3}{8}$ ____ $\frac{3}{12}$

If you are reading this article as part of a collaborative team, ask each member of the group to share his or her strategies. Think about which comparison strategies in the group are the same and which are different and why.

## Strategies or representations?

While examining students' solution strategies, I named and listed the strategies and began to group them. If you did the grouping of strategies as suggested above, you may have noticed that it was a challenging task. I found that characteriz-

**FIGURE 1**

Rose drew pictures to compare two fractions that have the same numerator, which she partitioned to show the different sizes.

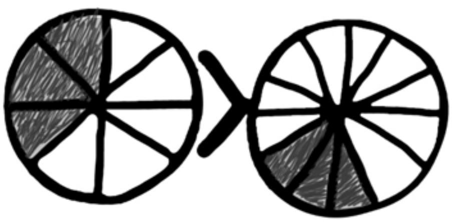


**FIGURE 2**

Harry used words to carefully describe the size difference between the two pieces while maintaining the same number of pieces.

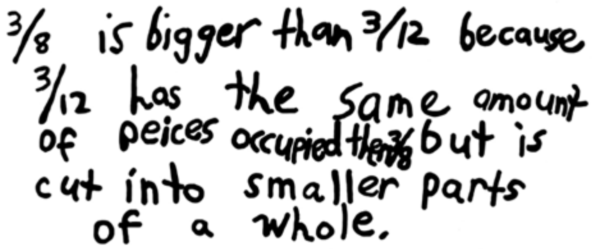


TABLE 1

The teacher inventoried students' strategies, then tallied them in a table. The total is greater than 36 because some students displayed more than one strategy.

| Student strategies | | | | |
|---|---|---|---|---|
| | $\frac{2}{4}$ —— $\frac{7}{12}$ | $\frac{5}{8}$ —— $\frac{6}{10}$ | $\frac{8}{12}$ —— $\frac{10}{15}$ | $\frac{3}{8}$ —— $\frac{3}{12}$ |
| **Task number** | **1a** | **1b** | **1c** | **1d** |
| Cross multiplication | 1 | 1 | 1 | 0 |
| Common denominator | 24 | 27 | 28 | 19 |
| Common numerator | 0 | 0 | 0 | 11 |
| Benchmarking to 1 or 1/2 | 8 | 3 | 1 | 1 |
| Partitioned line or area | 3 | 4 | 4 | 4 |
| Conversion to a decimal | 0 | 1 | 1 | 0 |
| None given | 1 | 2 | 2 | 1 |
| **Total** | **37** | **38** | **37** | **36** |

ing student strategies was problematic because looking only at "representations" did not fully describe students' thinking. For example, I created a category for pictorial representations that compare each fraction to one-half (a benchmark strategy). I then wondered if a student who used a written explanation but not a picture for the same benchmarking-to-one-half approach demonstrated the same strategy. The two approaches looked very different on paper, but the thinking seemed to share many traits. Similarly, five students drew pictures to compare three-eighths to three-twelfths, each showing three regions of different sizes. Another six students verbally described the different-size regions. Consider Rose's and Harry's work: Rose drew pictures (see **fig. 1**) to compare two fractions that had the same numerator; she carefully partitioned the fractions to show the different sizes. Harry used words (see **fig. 2**) to describe the size difference between the two pieces, while maintaining the same number of pieces. Because the approaches have traits in common, I found it useful to refer to them as the same *strategy* expressed with different *representations*. A *strategy* is the teacher's best interpretation of a student's thinking; a *representation* is the outward evidence of that thinking. For this data, I focus primarily on written evidence to infer student strategies, believing these to be the reliably accurate reflections of student thinking but also acknowledging that knowing exactly what a child was thinking is impossible.

Although the modes of *representation* in this data set included number lines, fraction strips, circle fractions, and symbolic representations, students also used many *strategies* to compare. For example, comparing by using a common numerator (with different denominators) is an example of a strategy that can be represented various ways. In the examples above, the strategy is represented by the circular fraction model showing the relative size of the pieces, as well as by a verbal description of the student's reasoning about the relative sizes. Looking at students' strategies, not just at answers, allows a teacher to pair students who are thinking alike or, conversely, pair those who are thinking differently.

## What is *right*?

Categorizing student strategies (David Wees, "Categorizing Student Work," Blogarithm [blog], *MTMS*, June 8, 2015) is not the same as separating right answers from wrong answers. Wrong answers could come from an ineffective strategy or even a normally effective strategy derailed by a simple error. The goal of strategy categorization is to understand how the whole class, as well as the individual student, is making sense of the skill being assessed. The goal of this focused look at student work was to formatively assess students' understanding of not just equivalency but also fraction fluency in general. The student work I examined had an overall success rate of 92 percent, typically not data that warrants a detailed analysis. However, because our definition of fluency also includes efficiency, flexibility, and appropriate use of strategies, I was encouraged to look deeper at student responses.

The first step was to inventory each student strategy and keep a tally (see **table 1**). Look at your own comparisons and categorize them. You may have additional strategies that these

FIGURE 3

Michael's pictorial model, like many of his peers' models, had an incorrect representation because a couple of his partitions lack precision. He identified the correct number of partitions. His caption is notable.

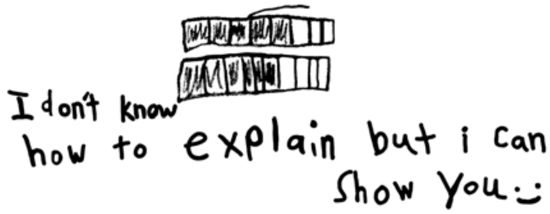



FIGURE 4

Patrick's orderly, accurate work *seems* to show precision and fluency, but it lacks *flexibility* because he used a single strategy for every exercise on the page.

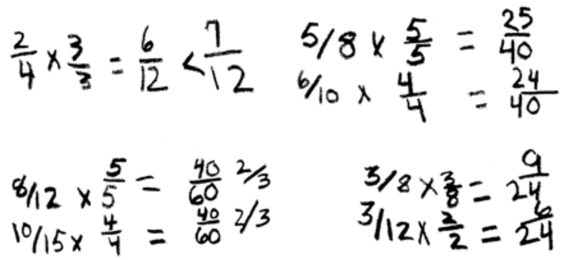



FIGURE 5

Kellie's work shows her ability to compare fractions with common denominators and may also reveal an efficient benchmarking strategy.

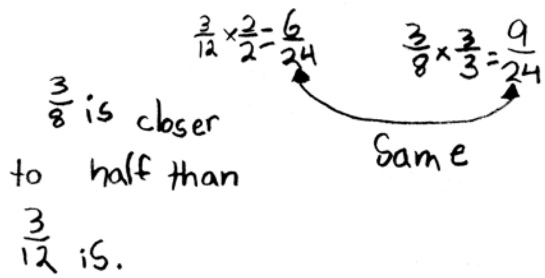
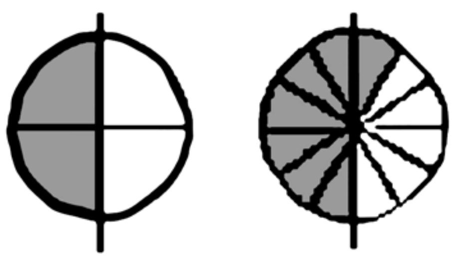



FIGURE 6

This computer-generated facsimile of Viktoria's work shows that she may be using one representation to understand another.



students did not use. In this sample, the least effective representation overall was the pictorial model. For most students, the lack of precision in partitioning resulted in an incorrect representation, as Michael's work shows (see **fig. 3**). However, incorrect responses are far more valuable than a single judgment of right or wrong. Michael's representation shows mostly accurate partitioning of two bars into eighths and tenths. There is also evidence that he is able to correctly identify the correct number of partitions. His strategy is not the most efficient for comparing fractions, but his teacher has learned a great deal about his capacity to represent fractions on a number line or in an area model. Furthermore, the caption that accompanies the solution is very telling, showing that Michael can fairly accurately partition an area, even if it is not accurate enough to compare a difference of 0.025. However, he cannot explain his process. On the other hand, Patrick's fraction comparison work seems like an ideal combination of precision and fluency (see **fig. 4**). On his neatly presented paper, he was able to efficiently find a common denominator for each problem on the assessment, and his work was entirely accurate. However, Patrick's work, like the work of many others, lacked *flexibility*. Patrick uses a single efficient strategy for every exercise on the page, regardless of the numerical relationships within and between the fractions. Sometimes extreme precision can hide a lack of flexibility as a student blindly executes a procedure without regard for the value of the numbers in the exercises.

Michael's work shows no evidence that he is capable of Patrick's efficient comparison strategies. But similarly, Patrick's work shows no evidence that he is able to partition a number line and accurately locate one fraction's position in relation to another's. However, the information gleaned from an in-depth examination of what is *right* in both students' work offers the teacher an opportunity to group these students together and address each student's area of growth.

Often the evidence of student thinking can focus instructional decisions for whole-class planning. In the case of Patrick and Michael, the teacher has evidence of contrasting understandings that provide opportunities for rich conversations. But sometimes evidence-centered assessment yields detailed information about a single student. Kellie's work shows not only

FIGURE 7

Viktoria's tape diagram is mathematically unsupported, lacks precision, and prompts an incorrect answer. In comparison with her other representation (see **fig. 6**), this answer shows less flexibility.

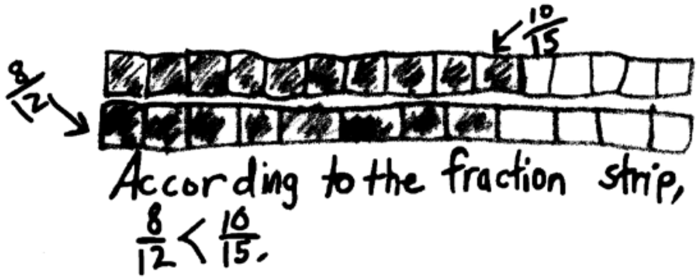
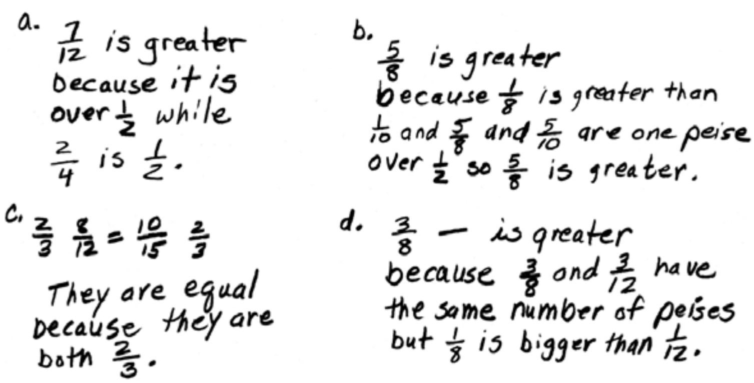



FIGURE 8

Ariel's work (reproduced here for clarity) shows that she altered partitions within the whole to accommodate three extra pieces and erased three extra partitions, evidence that she recognized the importance of the unit whole.

FIGURE 9

No other student demonstrated as much flexibility and productive use of the relationships between and within the two fractions as Jack's use of four different strategies.




her ability to compare fractions with common denominators but also her efficient benchmarking strategy. She compares each fraction to one-half, indicating that three-eighths is closer to one-half than three-twelfths is (see **fig. 5**). Similarly, Viktoria drew models that highlight one-half, using them to compare two-fourths to seven-twelfths (see **fig. 6**). This same thinking is represented in her written text, where she again highlights each fraction's relation to the one-half benchmark. In Viktoria's work, I see evidence that she is making some connection between her drawings and her written words. For example, her circle sketches show that she carefully identified one-half marks on each of the drawings: Seven-twelfths is shown as one unit fraction more than one-half. This same thinking is represented in her written text, where she again highlights each fraction's relation to the one-half benchmark. This is an example of two very different representations modeling the same mathematical idea.

Viktoria's work on **figure 6** hints that she may be using one representation to understand another. In contrast, Viktoria did not support her tape diagram (see **fig. 7**) with a substantive mathematical comment. Her tape diagram lacks precision, and perhaps because she could not refer to known benchmark values, this particular representation prompts an incorrect response. The benchmark fraction awareness may have helped her connect the drawing directly to her description in the first example, but the written words in the second example appear to be restating what the fraction strip already describes, and it offers no new evidence for her answer. A lack of coordination between her written words and her drawn representation in the second sample (**fig. 7**) demonstrate less flexibility. Both observations yield important assessment information about her mathematical thinking.

Generally, a close examination of the details of a student's work, including erasures, subtle marks of emphasis, or overwritings can also give us clues about a student's thought process. A close look at Ariel's work (see **fig. 8**) revealed erasures that demonstrate adjustments to her partitions of the fifteenths tape diagram. Beyond the whole is a shadow of three extra partitions that have been erased, and other soft lines show that the partitions within the whole have been adjusted to accommodate the three extra pieces, albeit unequally. This adjustment shows evidence of Ariel's recognition of the importance of the unit whole as she adjusted the fifteenths bar to be the same size as the twelfths bar. On the other hand, Jack's work is decisive: He had few if any hesitations (see **fig. 9**). He used four different strategies, one for each of the four items in the task. His work shows evidence of using benchmark fractions when comparing two-fourths and seven-twelfths as well as using

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Taking time to sort and classify student work according to common solution srategies garners powerful insights into student understanding.

equivalence structures (common denominator) to compare eight-twelfths and ten-fifteenths. He clearly shows that he is able to reason about the fractions as numbers in comparing five-eighths and six-tenths, and he also responded to the common numerator cue with three-eighths and three-twelfths. Jack’s work exhibits flexibility because he not only appears to readily recognize the equivalence of eight-twelfths and ten-fifteenths but also used a benchmark strategy, which is seen in his insightful observation that five-eighths and six-tenths are both greater than one-half by one unit fraction. In contrast to some other students, Jack’s flexibility is with the variety of strategies that he is able to present to compare these fractions, even if his use of representations exhibits less variability. Many students in this class correctly compared each of the four sets of fractions, but no other student demonstrated as much flexibility and productive use of the relationships between and within the two fractions as Jack did in selecting strategies. Evidence-centered assessment, which focuses on subtle details in student work, highlights Jack’s fluency.

## A “What’s right?” lens

Assessing student understanding is a critical part of a teacher’s routine. Most assessments are reviewed with a quick eye, but the evidence-centered assessment strategy encourages us to slow down and look more carefully at student work samples. By sorting and classifying student work according to common strategies, rather than by right or wrong answers, the teacher can form instructional groups on the basis of how students are thinking about a particular mathematical concept. Furthermore, distinguishing representations of mathematical ideas from strategies used to understand or model the mathematics keeps the focus on student thinking. Representations are important, but they are only symbols of the thinking that lies underneath.

Adopting a “What’s right?” lens on student work changes the assessment process. Because I am seeking evidence of the accomplishments that students have already made, I am more likely to find positive evidence of student progress in the details of their work. The “What’s right?” lens also better reveals the next steps for instruction.

Using Kling and Bay-Williams’ (2014) four-component framework for fluency as a guide, I have shown that accuracy is not the sole determinant of fluency in comparing fractions. Equally important features of student work are flexibility, efficiency, and appropriateness of strategy. The four components of fluency are also important for areas of mathematics outside of basic facts. The student work samples shown here reveal that even skills with comparisons of fractions go beyond simple accuracy. A detailed examination of a small set of student work samples also revealed the importance of fluency, particularly the flexibility component, in comparing fractions.

**Common Core Connections**

**3.NF.A.3.A, B, D**
**4.NF.A.1**

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**Kimberly Morrow-Leong, morrowmath@gmail.com**, is a doctoral candidate at George Mason University and adjunct instructor at George Mason University in Fairfax, Virginia, and a consultant for Math Solutions. She is interested in how students think and learn mathematics and enjoys facilitating professional development, particularly when we put our pencils down and get messy with manipulatives!


## Let's chat about "Evidence-Centered Assessment"

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and *TCM* readers about a topic important in our field. You are invited to participate in the fun.

This month, we will discuss "Evidence-Centered Assessment" by Kimberly Morrow-Leong on **Wednesday, September 14,** at **9:00 p.m. EDT**. Follow along using #TCMchat.

Unable to participate in the live chat? Follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.

## Suggestions for conducting an evidence-centered assessment

- Select a rigorous task that is accessible by all students. Strongly encourage students to give their answers using as many representations as possible. Also encourage them to solve the problem in more than one way.
- After writing the problem, solve it yourself. Do the math! Make a record of as many ways to solve it as you can think of.
- Anticipate and record what students might do to solve the task.
- As you pick up each work sample, ask "What is right?"
- Sort the work samples by student strategies. Define the strategies.
- Initially, cite evidence. Do not evaluate until the end. As you find yourself evaluating the student work samples, hold yourself to the highest standard of evidence.

(Adapted from Smith and Stein 2011)