Assessing Students' Understanding of Fraction Multiplication

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The authors of *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) call on educators to use assessments as an ongoing teaching tool to help promote student understanding and to inform future lessons:

> An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement. (p. 89)

Similarly, Wiliam (2007) suggests that effective use of assessment for learning includes "engineering effective classroom discussions, questions, and learning tasks that elicit evidence of learning."

We integrated these elements into a project during which we unpacked fraction standards, created rigorous tasks and lesson plans, and developed formative and summative assessments to analyze students' thinking about fraction multiplication. The purpose of this article is to (1) illustrate a process that can be replicated by educators interested in using rigorous mathematical tasks and assessments to support and advance their students' mathematical thinking, and (2) share the artifacts and instructional products that educators can use to improve mathematics assessment practices.

Read about how the authors used many technological tools and platforms to engage a team of educators across the country in this collaborative project.

PAFFY69/THINKSTOCK (GIRL); ISURFACE_NIKA/THINKSTOCK (GEOMETRIC BAR)

Project goals and background

For this collaborative project—which was initiated by the Smarter Balanced Assessment Consortium, led by Illustrative Mathematics, and supported by the Teaching Channel—our goal was to follow the development of a mathematical concept and students' understanding of that concept through the creation of lesson plans and formative and summative assessment tasks, specifically in the area of fraction multiplication. We will share the collaborative process of unpacking a specific mathematical concept and developing lessons and rigorous formative and summative assessment tasks that inform teacher decisions.

Our team (which included university and college faculty, mathematics supervisors, and classroom teachers) chose to focus on multiplying fractions (5.NF.B.4–6) and investigate the major skills that led up to this topic. We designed tasks and lesson plans to highlight key aspects of the topic that would further students' mathematical understanding. **Figure 1** illustrates the phases of the project and also serves as an outline to the sections within this article.

Development of the research tasks and lessons

Because our team geographically spanned from northwestern Washington State to southern California to the East Coast and in between, we used Google Hangouts to meet virtually, and we used Google Drive to collaborate on documents (see **table 1 online** for a list of helpful resources and **table 2 online** for fifth-grade instructional tasks that correlate to CCSSM).

One of our activities, the Cornbread task (see **fig. 2**), was designed to include the multiplication of fractions, building from students' understanding of multiplication that is related to area. With a similar representation, students can use the rectangle to show multiplication with fractions. When using the area representation to multiply fractions, we are again looking for the area, but we need to remember to relate the "shaded" area to the whole unit. Students have multiple quantities to keep in mind as they multiply, and identifying the "whole" in the area representation is important when multiplying fractions.

FIGURE 1

Using the topic of fraction multiplication as an example, university and college faculty, mathematics supervisors, and classroom teachers collaborated to design a process of unpacking a mathematical concept, developing lessons, and reworking rigorous formative and summative assessment tasks to inform teacher decisions.

For the Cornbread task (see **fig. 2**), students consider a pan of cornbread that costs $12 and various scenarios where fractional parts of the cornbread are sold. The choice of the number 12 as the cost was deliberate, as 12 is a highly divisible number. Students were asked to represent situations and use those diagrams to answer questions, such as finding the cost when the pan is half full and someone wants to buy 5/6 of what remains. Students also considered how a customer's cost might change (scale) if the whole pan costs more or less than $12. Part A involved unit fractions only, and

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FIGURE 2

The choice of the number 12 as the cost in the Cornbread task was deliberate. Students drew diagrams to represent situations and used them to answer questions about fractional parts. Part A involves unit fractions only; part B includes nonunit fractions.

The Cornbread task after revisions

The fifth graders want to raise money for their overnight camping trip by selling cornbread during the school Chili Cook-Off contest. All the pans of cornbread are square. A pan of cornbread costs $12. The customers can buy any fractional part of a pan of cornbread and pay that fraction of $12. For example, $\frac{1}{2}$ of a full pan costs $\frac{1}{2}$ of $12.

A. Mrs. Smith buys cornbread from a pan that is $\frac{1}{4}$ full. She buys $\frac{1}{3}$ of the remaining cornbread in the pan.

- What fraction of the whole pan of cornbread does she buy? Use objects or a diagram to show how much of the pan of cornbread she buys.
- What does she pay for the cornbread she bought? Use objects or a diagram to show how much pays.

B. The next customer is the school principal. He buys cornbread from a different pan that is $\frac{1}{2}$ full. He buys $\frac{5}{6}$ of the remaining cornbread in the pan.

- What fraction of the whole pan of cornbread does he buy? Use a diagram to show how much of the pan of cornbread he buys.
- What does he pay for the cornbread he bought? Use a diagram to show how much he will pay for his part of the pan.

C. What would be the cost of the cornbread the principal bought if the price of the entire pan changed to one of the following? (Use a diagram to explain each answer.)

- $24
- $60
- $18

We watched our proposed alternatives enacted in another classroom and observed the power of thoughtful, rigorous tasks that advance our assessment of students' mathematical thinking.

part B extended to include nonunit fractions. The numbers used for the whole in part C are all scaled versions of 12 (e.g., $12 \times 2 = 24$, $12 \times 5 = 60$).

Implementation

Initial teaching to students

Once the lesson including the Cornbread task was created, one of the teachers on the team, Kristin Gray, taught the lesson in her fifth-grade classroom in Delaware. The lesson started with a "Number Talk" (Parrish 2010) focusing on whole-number multiplication. The intention was to elicit students' prior understandings of multiplication before the lesson. We wanted students to connect the fraction multiplication within the task to concepts they already knew about whole-number multiplication. The discussion promoted rich strategies for multiplying whole numbers, but in the end, it seemed that students were unable to connect these whole-number multiplication strategies to the multiplication of fractions. A full description of the lesson can be found on Gray's blog: https://mathmindsblog.wordpress.com/2014/10/11/lesson-study-teaching-take-2/.

Gray uploaded the videos to a private space within the Teaching Channel website that we used for this project. This feature of the Teaching Channel allows for groups to share video, send messages, annotate video, engage in discussions, and share resources. Watching the video of Gray teaching the lesson, we used the Teaching Channel group in two ways: (1) to make refinements to the lesson before it was retaught in the second classroom, and (2) to use video clips and student work in a one-day professional development workshop.

Professional development workshop

After Gray taught the initial lesson and everyone had an opportunity to watch and annotate it on the Teaching Channel Teams site, we debriefed in person before a district professional development session organized by one of our team members that we conducted as a group. We had a range of participants in the workshop, including grades 3–5 teachers and mathematicians. During the afternoon of the professional development workshop, participants found solutions to the Cornbread task after they had spent time learning about the mathematics within the standards and reading the Fractions Progression document. They were asked to consider the possible mathematical goals of the task, its connections to the standards, and the prior knowledge students would need to engage in the task. The teachers anticipated students' responses and considered the misunderstandings that students might bring to the task. Because Gray had already taught the lesson, we could then bring in video clips and actual student work for the teachers to analyze. Even though we had revised the task numerous times, discussing it with the teachers in the workshop helped us refine the task further.

Refinement and reteaching

Some of the participants of the PD wondered about the wording of the problem, and so we worked on making it more precise. By first creating and refining the task, then teaching the lesson, sharing it with other educators, and collaboratively refining the task again, we were able to go through an in-depth, iterative revision process before teaching it again in another classroom.

One adjustment we made to the lesson was the exit task. Gray had her students complete a journal entry with the prompt, "Where did you see multiplication in your work today? What are some things you noticed?" We discovered that this prompt was too open-ended, given the various components of the task. Students in Gray's class commented on the multiplication

they used when finding the cost of the cornbread. Other students commented that they had used division to apportion the cornbread and mentioned that division is related to multiplication. Many responses were vague and did not connect back to a specific problem. This resulted in Gray being unable to fully assess her students' developing understanding of fraction multiplication. For the second teaching of the lesson in Alicia Farmer's classroom, we chose to have students solve a new Cornbread problem, 3/5 of 1/2, and then answer the open-ended prompt, "Where do you see multiplication in this problem? Explain your reasoning" (see **fig. 3**). Students connected their explanations to the "fraction of a fraction" problem.

As we analyzed the student work, we categorized the responses and discussed observations that emerged that would help us further the students' thinking. Some students did not see connections to multiplication: "I am not really sure because I don't know how to do that" and "I couldn't find the multiplication." A few of these students who represented and solved the problem, however, did not yet connect their thinking to an operation. Other students were beginning to notice patterns with the numbers, and a few students made the connection with the area model and the pan of cornbread. We have organized samples of student work into themes and described them (see **table 3 online**).

Our refinements to the task were effective in making student thinking visible. Although we continued to have questions about student thinking, we were positioned to pose some of those questions to the students and plan experiences to further address these ideas during instruction. The second lesson iteration and debriefing session were professionally videorecorded. They are available online through the Teaching Channel at the following links:

- Teaching Channel video of second teaching, in Farmer's classroom: https://www.teachingchannel.org/videos/fraction-multiplication-intro-sbac/?utm_source=newsletter20150516/
- Teaching Channel video of debriefing: https://www.teachingchannel.org/videos/elementary-school-math-teacher-collaboration-sbac

FIGURE 3

Realizing that Gray's inability to assess her students' understanding was because the exit task prompt was too open-ended, the team chose a new Cornbread problem for Farmer's class.

The Cornbread task

A. When Mr. Farmer walks up to buy cornbread, the pan is $\frac{1}{2}$ full. He wants to buy $\frac{3}{5}$ of the remaining cornbread. What fraction of the whole pan does he want to buy?

B. Where do you see multiplication in this problem? Explain your reasoning.

Meeting the challenges

Through engaging, ongoing collaboration, we created rigorous mathematical tasks to advance our students' understanding of multiplying fractions, and we incorporated the formative assessments to inform teacher decisions within and between mathematics lessons. Collaboration allowed us to see a lesson enacted multiple times and gave us the opportunity to deeply discuss the student thinking. After teaching a lesson and proposing alternatives, we watched those alternatives enacted in another classroom and observed the impact on student learning. We could see the power of thoughtful and rigorous tasks that advanced our assessment of students' mathematical thinking, which helped to inform teacher decisions as we explicitly moved through this process as a team.

This process was invaluable in our learning as teachers, but it did not come without challenges and opportunities for improvement. One of the biggest challenges we faced was the length of time between the first and second lesson. We wanted students in both classrooms to be in the same place with regard to the fraction work they experienced, so that engaging in the task would reflect similar learning experiences. Because we were in classrooms across the country, coordinating was a challenge. To resolve this, the teachers were able to rearrange the units within the subject of mathematics to accommodate this need. Even though this could be a challenge, it did not diminish the benefits gained through the opportunity to study a specific mathematical concept and develop tasks to help build students' conceptual understanding.

Despite possible challenges, opportunities exist for educators to find, join, and create communities with others interested in this type of work—perhaps a colleague down the hall, someone from a local school that teaches similar content, or people beyond your local area in larger communities, such as Illustrative Mathematics. We suggest that educators consider technological environments and open educational resources, such as classroom lessons and tools at http://achievethecore.org, videos available on the Teaching Channel, or resources you might have access to through your state or assessment organization. Google Hangouts and Google Drive enabled us to meet in real time, discuss student understanding and instructional moves, and reflect on the process. We hope our process of collaboration in a dynamic way with colleagues across the country will be used and replicated by educators to create rigorous mathematical tasks and assessments that support and advance their students' mathematical thinking.

Common Core Connections

5.NF.B.4
5.NF.B.5
5.NF.B.6

REFERENCES

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