Learn the value of having students create their own stories and pictures to represent number sentences as classroom assessments.

Kelly K. McCormick and N. Kathryn Essex
A third-grade student wrote the following repeated-addition story as part of an assessment given to 583 third graders near the end of the school year (see fig. 1).

Once a group of five little marbles were walking. They ran into five more marbles. Now the five is ten. Five more marbles came by. Now a group of fifteen marbles are walking, and then they all bought lollipops.

We had asked the children to “make up a story and a picture about marbles for this number sentence: $3 \times 5 = 15.$” Students in this study came from predominantly low- to average-income families living in three distinct geographical areas within the United States. We also collected work, which included a similar division task, from these students at the end of their fourth-grade year. In this article, we present findings describing the children's multiplication and division stories and discuss the value of having students create their own stories and pictures as classroom assessments.

We wanted to capture and examine the children’s understanding of multiplication and division. Research suggests that providing a foundational understanding of the meaning of an operation supports students’ competence in problem solving and computation (Fuson 2003). Correspondingly, understanding multiplication is a powerful tool; multiplication is a primary operation that can be properly defined so it is fundamental for representing and solving many different situations (Otto et al. 2011). When solving word problems, children—

frequently choose an operation without making sense of the choice. . . . Knowing why an operation is an appropriate choice for a solution strategy is an important part of establishing a robust understanding of mathematics. (Otto et al. 2011, p. 15)
The Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) clearly emphasizes the importance of understanding the meanings of multiplication and division; CCSSM states that developing an understanding of multiplication and division is one of four critical areas in third grade, when students are to—

develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; this includes understanding the meanings of whole number multiplication and division. (CCSSI 2010, p. 21).

Third graders are to—

interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$. (CCSSI 2010, p. 23)

These standards state that third graders should be able to—

interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$. (p. 23)

CCSSM extends this focus to fourth grade, when students should be able to “interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5” (p. 29) and to the fifth, sixth, and seventh grades, when students should learn to “apply and extend previous understandings of multiplication and division” to fractions and rational numbers” (pp. 34, 41, and 48).

Having children create their own multiplication and division stories and pictures provided us with rich information about their understanding of these operations. As educators, we could show students how to represent a problem situation, but we learn more about their understanding of the operation and quantities involved when they create their own stories (Otto et al. 2011). In addition, because we recognize the importance of children concurrently developing an understanding of multiplication and division and the relationship between the operations (CCSSI 2010; Fosnot and Dolk 2001; Mulligan and Mitchelmore 1997; Otto et al.
groups of objects in two ways: each group represents one thing at the same time it is a number of things. Before constructing the idea of unitizing, number is used to represent single units—six represents six marbles (Fosnot and Dolk 2001).

As Fosnot and Dolk (2001) note, children do not construct mathematical ideas in any set or ordered sequence. “They go off in many directions as they explore, struggle to understand, and make sense of the world mathematically” (p. 18). We saw evidence of this when some of the children’s stories contained elements of both additive and multiplicative thinking. For example, the story in figure 4 starts with one group of five marbles, then having another, and then a third. For example, in figure 2, the child writes a story for which the mathematical structure is $5 + 5 + 5 = 15$.

The multiplicative stories were about equal groups (see fig. 3): either three groups of five things or five groups of three things. The development from repeated addition to multiplication requires children to understand a higher-order treatment of number, unitizing, in which groups are counted as well as the objects in the group (Fosnot and Dolk 2001). Children must be able to think about the numbers involved with

![A third-grade student’s story has both multiplicative and additive aspects present as well as a picture for the number sentence $3 \times 5 = 15$.

**Story:**

Mark had a bag of 5 marbles. He found 2 more bags of marbles now he has 15 marbles.

**Picture of marbles:**

![For the number sentence $3 \times 5 = 15$, one third grader wrote a multiplicative-compare story and picture.

**Story:**

I have 3 marbles. My friend has 5 times of that amount. How many marbles does my friend have?

**Picture of marbles:**

Of the 583 students who were a part of this
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On Wednesday, September 13, 2017, 9:00 p.m. EDT, we will discuss “Capturing Children’s Multiplication and Division Stories” by Kelly K. McCormick and N. Kathryn Essex. Follow along using #TCMchat.

You can also follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.

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Third graders’ correct multiplication stories

![Graph showing the distribution of types of stories: 76% multiplicative, 13% additive, 8% additive and multiplicative, and 3% multiplicative-compare.]

- Multiplicative: 262 stories
- Additive: 28 stories
- Additive and multiplicative: 46 stories
- Multiplicative-compare: 9 stories

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Study, 345, or 59 percent, wrote a correct story; 312 children also drew a correct picture. Fifty-two children wrote an incorrect story but drew a correct picture. Figure 6 shows an example of an incorrect multiplication story and picture. Among the 345 correct stories, 262 were multiplicative stories, 28 were additive stories, 46 stories had both additive and multiplicative aspects, and 9 were comparison stories (see fig. 7). Thus, the majority of students who wrote correct stories exhibited the ability to think about equal groups; they had constructed the big idea of unitizing, which underlies the understanding of place value, multiplication, and division (Fosnot and Dolk 2001).

**Children’s division stories**

The fourth-grade division story task mirrored the multiplication task:

Make up a story and a picture about marbles for this number sentence: 18 ÷ 6 = 3.

Once again, we looked at not only the correctness of students’ responses but also the different types of stories composed. Division stories, like multiplication stories, are about equal-size groups.

We coded stories about sharing items equally across a given number of groups as fair-sharing or partitive-division stories. The three examples of the fourth graders’ stories (see fig. 8) demonstrate that these children have a strong understanding that 18 ÷ 6 = 3 means that eighteen objects are divided equally into six groups and that there are three objects in each of the groups. Their stories show they understand that the groups must be equal, and their questions focus on the number of objects in each group.

Repeated subtraction or measurement division stories had contexts in which a given number of marbles were repeatedly subtracted from the whole group. We found fewer examples of these stories in fourth graders’ work. Three examples of fourth graders’ stories (see fig. 9) demonstrate that these students have a strong understanding that 18 ÷ 6 = 3 means that eighteen objects are divided into groups of six and that three groups of six objects are in eighteen. With a repeated subtraction problem (see fig. 9a), the question is, “How many sixes are in eighteen?” However, because we had...
asked the children to “make up a story [not a problem] and a picture about marbles for this number sentence: 18 ÷ 6 = 3,” the story shown in figure 9b also demonstrates a correct understanding of division. Even though the child does not directly pose the question, “How many sixes are in eighteen?” she explains how to determine the number of sixes that are in eighteen by repeatedly subtracting six from eighteen, twelve, and then six; and as the child then states, after that, “there were no marbles left, so the answer was 3.” If this were a classroom assessment, a follow-up question would be to ask this student what the three in her story means to ensure that she understands that the three is three groups of six.

Some children wrote correct stories for this task that we coded as multiplication stories (see fig. 10). In other words, the action in the story was about finding out how many marbles were in three groups of six objects or six groups of three. This suggested to us that these children understood that multiplication and division may be used to represent the same situation, that is, situations involving a given number of equal-size groups. Mulligan and Mitchelmore (1997) also found that children naturally relate these two operations and that when they do, they do not necessarily find one more difficult than the other, again emphasizing the importance of providing children with opportunities to link the operations of multiplication and division more closely.

In the fourth grade, 356, or 61 percent of the 583 students, wrote a correct division story, and 309, or 53 percent of all the children, also drew a correct picture. An additional 42 students drew a correct picture but did not write a correct story. Among the correct stories, 280, or 79 percent, were stories with fair-sharing contexts. Another 46 students, or 13 percent, wrote stories with repeated-subtraction contexts. Eleven students wrote equal-group division stories that had neither a fair-sharing nor a repeated-subtraction context. One child wrote a comparison story. Eighteen students wrote multiplication stories.

In the classroom
Research highlights students’ difficulty solving story problems; they often guess at which operation to use to solve a problem if they do not understand what the operations mean (Verschaffel et al. 2007). For students to develop adaptive expertise in interpreting problems and carrying out appropriate computation to solve them, instruction—including assessment—must emphasize students’ understanding of the action and the meanings of the operations in context (Russell 2010). Understanding the multiple meanings of operations and the relationship between the meanings and operations is a critical part of establishing
Three fourth graders’ stories demonstrate that these students have a strong understanding that $18 \div 6 = 3$ means that eighteen objects are divided into groups of six and that three groups of six objects are in eighteen.

(a) With a repeated subtraction problem, the question is, “How many sixes are in eighteen?”

Story: **One day a girl wondered how many marbles of 6s are in 18. 6ths are in 18?**

Picture of marbles:

(b) Although this student never directly posed the problem, her example shows a correct understanding of division.

Story: **After finding out that he had 18 marbles, Joe decided to do a division problem: $18 \div 6$. He took 6 away. That’s 11. He had 12 left. He took 6 away. That’s 2! He had 6 left. He took them away. That’s 2!**

Picture of marbles:

(c) **Story:**

*No has 18 marbles he wants to give 6 to each of his friends how many friends does he have?*

Picture of marbles:

Using problem writing as an assessment reveals students’ understandings and misunderstandings of operations in a manner in which traditional assessments cannot (Drake and Barlow 2007–2008). The most powerful part of the learning experience described previously is that the stories come from the children. As students write and discuss their problems, they generate stories or story problems for a given equation or expression is a powerful way for classroom teachers to assess students’ knowledge of the action and meaning of the operations (Drake and Barlow 2007; Van De Walle et al. 2013). It requires a higher level of thinking than simply solving a variety of story problems, which is how teachers typically assess students’ understanding of the meaning of the operations.

The stories and diagrams that children create offer a multitude of opportunities for teachers to facilitate rich classroom discussions about the different meanings of the operations and how the meanings are related. For example, a teacher might frame a discussion around having students compare a child’s repeated-addition multiplication story to another child’s equal-groups multiplication story. Similarly, another discussion could be built around having students compare a child’s fair-sharing division story to another child’s repeated-subtraction problem.

When exploring part-whole relationships, asking children, “What did you know?” and “What were you trying to find out in your problem?” is a powerful tool (Fosnot and Dolk 2001). Through the previous discussion, students should come to realize that with both types of division stories, the total number of marbles is known. However, in one story, they are trying to figure out how many marbles are in each group; and in the other, they know how many marbles are in each group and are trying to figure out how many groups. Building on the previous discussion, having the children determine which interpretation of division is involved (how many groups or how many in each group) in other children’s stories would deepen their operation sense. Other follow-up discussions might focus on the following questions: “Some of the how-many-groups stories say that the groups need to be equal or the marbles need to be shared evenly [see fig. 8a, b, and c]; is it important that the groups are equal and that the marbles were shared evenly? Why did you include that in your story?”
reveal their mathematical thinking: the value of student discussions, such as those previously described, is quite evident from the NCTM (2000) Standards and the Common Core’s (2010) Standards for Mathematical Practice.

### REFERENCES


**Ed. note:** For more on this topic, consult *Multiplication and Division in Grades 3–5* in NCTM’s Essential Understanding series.

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