NATALYA TKACHENKO/THINKSTOCK

Jennifer M. Bay-Williams and Graham Fletcher

Walk into any primary classroom and odds are that you will see a hundred chart hanging on the wall. For years, the hundred chart has been a staple in K–grade 2 classrooms and has been continuously called on to help develop early numeracy concepts. The structure of the hundred chart makes it an effective tool for anchoring students' reasoning and understanding of foundational concepts, including place value, addition, and subtraction (Van de Walle, Karp, and Bay-Williams 2016).

A Bottom-Up Hundred Chart?

Putting a twist on a popular mathematical tool, this collection of activities shows how placing a number 1 in the bottom-left cell and a 100 in the top-right cell can better support student reasoning.

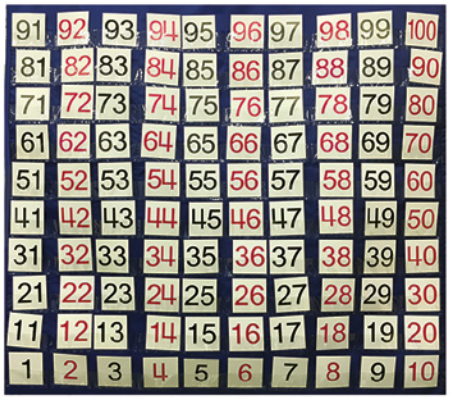
FIGURE 1

"I added twenty more and went down two rows."

5	6	7	8	9	10
15	16	17	18	19	20
25	26	27	28	29	30
35	36	37	38	39	40
45	46	47	48	49	50

FIGURE 2

First graders explored numbers within 100 using the bottom-up hundred chart.



The following features make the hundred chart a useful tool in the early grades:

- It is organized in a way that children see the tens as 10 ones (counting across rows) and as 1 ten (a full row) (i.e., unitizing).
- Children can see that 1 hundred is 10 tens (i.e., place value).
- The structure connects adding (or subtracting) to place value. Specifically, children notice that adding one changes the ones place and adding ten changes the tens place.
- It is a public number chart that serves as a reference as children reason and explain their reasoning to others.

The hundred chart certainly aligns with Tools and Technology, the third of NCTM's Essential Elements in *Principles to Actions: Ensuring Mathematical Success for All*, because the chart is a mathematical tool that helps "students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM 2014, p. 5). Despite its common use in classrooms, a dearth of research exists on the relationship of the hundred chart to student thinking. In fact, a review of NCTM publications identified only one other article, published more than forty years ago, connecting the hundred chart to student thinking. That article focused on supporting students with special needs and recommended a "new look" for the hundred chart—a bottom-up chart (Randolph and Jeffers 1974). The concern raised then, and that we raise now, is that as students *talk* about adding and subtracting (conceptual talk) and how this relates to moves on the hundred chart (directional talk), a problem arises. Listen to this child share her strategy for adding 16 + 23: "I found sixteen on the hundred chart. I added twenty more, so I went down two rows, then added three by going over three. The total is thirty-nine" (see **fig. 1**).

Notice that as the student connects her reasoning to the chart, she *increases* her quantity by twenty, but moves *down* the chart. In other words, her quantitative statement (add twenty) does not match her directional statement (move down). This apparently opposite language can cause a "directional conflict" for students, in particular, students with disabilities (Randolph and Jeffers 1974, p. 203).

The language issue has nagged at us for years because it seems to be counterintuitive. Children have asked, "Why do you go down [the chart] when you are going up [in quantity]?" Recently, through Twitter and other dialogue, we asked why the hundred chart is top-down, rather than bottom-up. The ideas we heard are that (1) children learn to read from left to right, top to bottom; (2) it looks like a calendar; and (3) countless resources and activities accompany the hundred chart. This list of reasons does not seem like enough to counter the negative aspects of the traditional hundred chart.

A classroom story

Co-author Graham Fletcher, a math specialist for his district, recently visited Mrs. G's first-grade classroom while her students were exploring a traditional hundred chart. Mrs. G recognized that her students were having a

difficult time navigating the hundred chart, in particular, how to describe moves up and down on the chart. After the lesson, she asked Fletcher, "Why does the hundred chart go down when it is increasing in quantity?"

With no sound answer, Fletcher asked if he could return the next day and share something with her students. The next day, Fletcher returned to Mrs. G's class after consulting his professional community via Twitter and deciding to create a bottom-up hundred chart (see **fig. 2**). Fletcher asked the children what they noticed about *this* hundred chart. As students examined what they called *the upside-down* hundred chart, two themes emerged:

1. The language made sense. Children noticed and were able to explain that when a number increases by ten it gets taller, bigger, and greater, which are all physically modeled when they moved "up" on the chart.
2. Connections to contexts were made. Children were able to think of real examples in their world where something became more as it went up. Elisa [a pseudonym] shared, for example, "It's like my carton of chocolate milk. When I drink some, there's less, and it goes down. But when I [pour] it back in the carton, there's more milk, and it goes up."

Applying Elisa's example to a traditional hundred chart just does not sound right: *When we put more milk in the carton, it goes down. When we take milk out, it goes up.* For this reason, we suggest that we (elementary school mathematics teachers) take a *numeracy* approach to the hundred chart and go *bottom-up*. Our experience has been that the alignment of quantitative and directional language as children talk about their reasoning is worth the time it takes initially for us to explicitly teach students to explore and read this chart.

Benefits of a bottom-up hundred chart

As we began to integrate and explore the bottom-up hundred chart, we identified some compelling reasons to use it in our K–grade 2 classrooms and beyond:

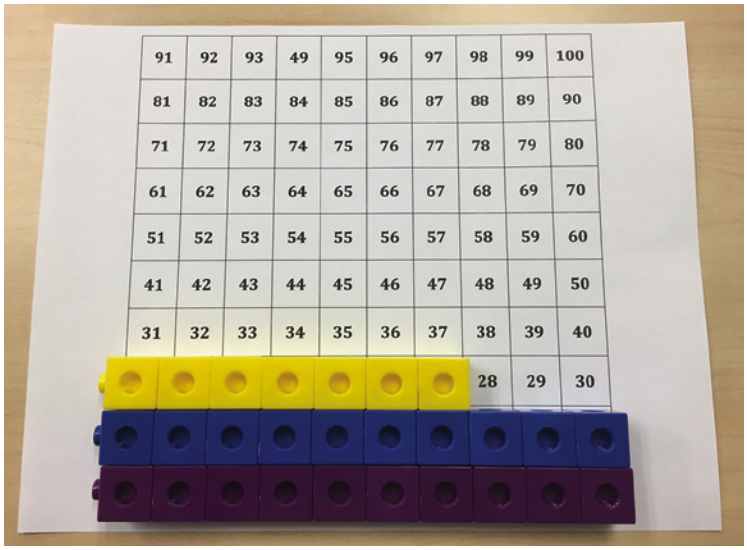
FIGURE 3

"I added twenty more, so I moved up two rows."

45	46	47	48	49	50
35	36	37	38	39	40
25	26	27	28	29	30
15	16	17	18	19	20
5	6	7	8	9	10

FIGURE 4

The stack vertically increases as the quantity increases.

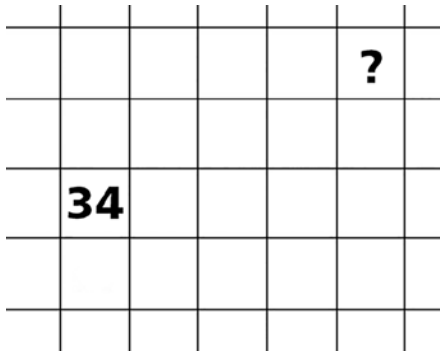


- Students' language while describing moves within the bottom-up hundred chart accurately captures what is happening to the size of the numbers (going up on the chart, the value of the number is also "going up"). This is a conceptual connection for all students but certainly can avoid confusion for English language learners (ELLs). In fact, strong evidence suggests that ELLs benefit from explicit attention to academic language, using oral language, and using charts and visuals (Baker et al. 2012).

Let's revisit our example of 16 + 23, using the bottom-up hundred chart (see **fig. 3**). A student now solves this addition problem by saying, "I was at sixteen, and I needed to add twenty more, so I moved up two rows of ten to thirty-six and then moved over three more to get me to thirty-nine."

FIGURE 5

What's the mystery number on the bottom-up hundred chart?



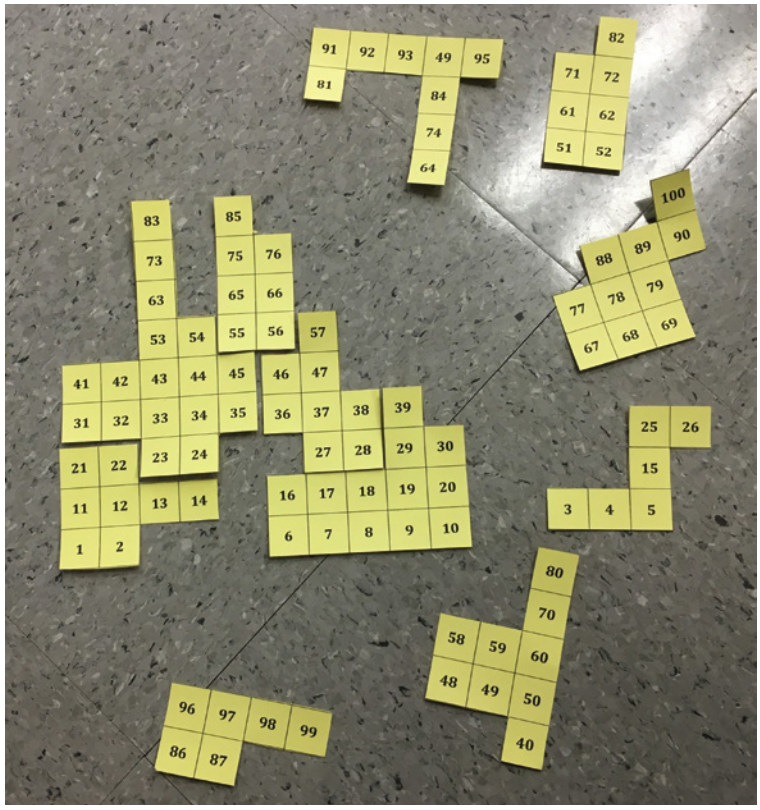
- Using the bottom-up hundred chart, students are able to model the chart with physical tools, stacked vertically, to physically see quantitative increases as more cubes are stacked (see **fig. 4**). This is a powerful conceptual connection for students because as they see the number increase by ten, they visually experience the quantity getting taller, bigger, and greater, which are all modeled as stacks of ten are added. Conversely, as cubes are removed, students see the quantity physically get shorter, smaller, and less as cubes are removed. The connection between concrete physical models and the semiconcrete hundred chart representation makes mental computation more accessible and easier to articulate.

- The moves on the hundred chart accurately reflect the way students describe their number moves on a number line. Moving to the right on a horizontal number line is an increase, and moving up on a vertical number line is an increase. This empowers students because their language, model, and reasoning all mirror one another, which has proven successful when engaging students in such activities as What's the Missing Number? (see **fig. 5**).

- The bottom-up hundred chart also is a better match to the coordinate axis. The coordinate axis is the coordinating of a horizontal number line and a vertical number line, so this is a natural extension of the previous bullet point. Notice that with a bottom-up hundred chart, you begin as you would in a quadrant I coordinate axis. In this case, the origin is in the lower-left corner of the coordinate axis, and locations are found by going to the right and up.

FIGURE 6

Students worked to reassemble a "broken" hundred chart.



Adapting existing hundred chart activities

As noted earlier, one reason that the hundred chart has remained unchanged is that many activities use it as it was originally designed. Luckily, with technology, many activities can be readily adapted to the bottom-up hundred chart. A bottom-up hundred chart appendix accompanies this article so you can adapt your favorite hundred chart activity. The appendix is sized to be used with Unifix® cubes. We offer two examples of adapted activities here.

Example 1: A hundred chart puzzle

Cut up a bottom-up hundred chart into puzzle pieces and have students put the chart back together (see **fig. 6**). This helps students become familiar with the bottom-up approach of the hundred chart. As they share how they worked on the puzzle, they can begin to use the

language that reflects this hundred chart, for example, explaining that they knew the piece with the teen values was below the twenties because teens are less than twenties.

Example 2: Finding neighbors

Start with a blank bottom-up hundred chart, with only the 1 and 100 labeled (see **fig. 7**). Player 1 rolls two ten-sided dice, creates a two-digit number, and records that value on the hundred chart (double zero can be defined as "roll again"). For example, if player 1 rolls an eight (8) and a nine (9), he or she can form the number 89 or 98, whichever will lead to the most points. Player 1 then records that number's neighbors of ten more, ten less, one more, and one less in the appropriate boxes (see **fig. 7**). For each number recorded, the score increases one point. If a selected number is close to an already-completed number, one of the values may already be "taken" (as shown in **fig. 7** in red with the number 56 [55 and 66 are already taken]). Player 2 takes a turn. After ten rolls, the player with the most numbers on the board wins.

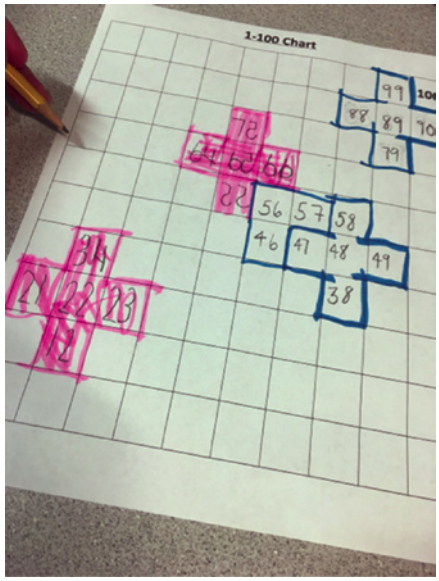
Many of the common ways that hundred charts are used can make much more sense using the bottom-up hundred chart, supporting student reasoning about quantities. Take, for example, arrow-moves activities. In such activities, children are given a starting value (e.g., 67) and a series of arrows (e.g., ➔↑↑↑). They make the moves (e.g., right one space to 68 and "up 30" to 37). *What*?! With a bottom-up hundred chart, "up 30" lands on 98, as it should. Arrow moves are often the bridge to adding and subtracting, helping students to connect the operations to place-value concepts. As you can see by this one example, discussing sums and differences will make more sense quantitatively with a bottom-up hundred chart.

Share your flipped-chart stories!

The idea of a bottom-up chart is not new; it was originally suggested forty-two years ago in *Arithmetic Teacher* (the predecessor of *Teaching Children Mathematics*). Yet mathematics teachers have not tackled our students' directional conflict when using the hundred chart. We think it is time to flip our hundred chart, making the upside-down right-side-up. Additionally, such a commonly used tool needs to be studied in terms of its impact on student thinking. Through research and professional discourse relating instruction to student learning, we can continue to improve as teachers and thereby better support students' reasoning and sense making. For us, the idea of a bottom-up hundred chart emerged from listening to students in classrooms and noticing their directional conflict. This led us to engaging in face-to-face conversations and Twitter chats about the potential risks and benefits of a bottom-up chart.

FIGURE 7

Two children played Finding Neighbors on an empty bottom-up hundred chart.



Let's chat!

On the second Wednesday of each month, TCM hosts a lively discussion with TCM authors and readers about a topic important in our field. You are invited to participate in the fun.

On **Wednesday, December 13, 2017**, at **9:00 p.m. EST**, we will discuss "A Bottom-Up Hundred Chart?" by Jennifer M. Bay-Williams and Graham Fletcher.

Follow us later on twitter@tcm_at_nctm, and watch for a link to the recap.

So, bottoms up! Try out the bottom-up hundred chart and share your stories and reflections. Conduct an action research project in your classroom or school. Share what you learn. Here is how to do so:

- Comment on the *TCM* Blog post related to this article: **http://www.nctm.org/tcm-blog/**
- Tweet about it, using #flipped100schart.
- Submit a letter to *Teaching Children Mathematics* by emailing **tcm@nctm.org** with Readers Exchange in the subject line.

Together we can continue to improve on the tools we use and the way in which we use those tools to support student reasoning in our classrooms.

Common Core Connections

1.NBT.B.3 1.NBT.C.6
1.NBT.C.4 2.NBT.B.5
1.NBT.C.5 2.NBT.B.9

REFERENCES

Baker, Scott, Nonie Lesaux, Madhavi Jayanthi, Joseph Dimino, C. Patrick Proctor, Joan Morris, Russell Gersten, Kelly Haymond, Micheal J. Kieffer, Sylvia Linan-Thompson, and Rebecca Newman-Gonchar. 2014. *Teaching Academic Content and Literacy to English Learners in Elementary and Middle School* (NCEE 2014-4012). Washington, DC: What Works Clearinghouse, National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, US Department of Education. https://ies.ed.gov/ncee/wwc/PracticeGuide/19

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Randolph, Winifred, and Verne G. Jeffers. 1974. "A New Look for the Hundred Chart." *Arithmetic Teacher* 21, no. 3 (March): 203–8. doi:http://www.jstor.org/stable/41188490

Van de Walle, John A., Karen S. Karp, and Jennifer M. Bay-Williams. 2016. *Elementary and Middle School Mathematics: Teaching Developmentally*. 9th ed. New York: Allyn and Bacon.

⇢ reflect and discuss

"A Bottom-Up Hundred Chart?"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions related to "A Bottom-Up Hundred Chart?" by Jennifer M. Bay-Williams and Graham Fletcher are suggested prompts to aid you in reflecting on the article and on how the authors' ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- Formal and informal language play a crucial role in the conceptual development of key mathematical ideas. What role (if any), do they have with the implementation of the bottom-up hundred chart?
- How might we, as teachers, emphasize the connections between the verbal and the physical movement to support students' understanding of the operations of addition and subtraction?
- Aside from the bottom-up hundred chart, what other tools or areas of elementary school mathematics might be seen as counterintuitive to how students naturally see and build number concepts?
- How might teachers incorporate the bottom-up hundred chart to supplement their current use of the traditional hundred chart and avoid the concern of confusing students?

We invite you to tell us how you used Reflect and Discuss as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to Teaching Children Mathematics *at* ***tcm@nctm.org****. Please include Readers Exchange in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgment. Letters are also edited for style and content.*

Jennifer M. Bay-Williams is a professor at the University of Louisville in Kentucky. **Graham Fletcher** is a district math specialist for Griffin-Spalding Schools, which is located in the Atlanta, Georgia, metropolitan area. Both authors are committed to seeking ways to support students' conceptual understanding and procedural fluency. They believe that through collaboration and purposeful discourse, teachers can empower themselves to identify and address content misconceptions and keep students' thinking moving forward.

Name_____

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

From the November/December 2017 issue of teaching children mathematics