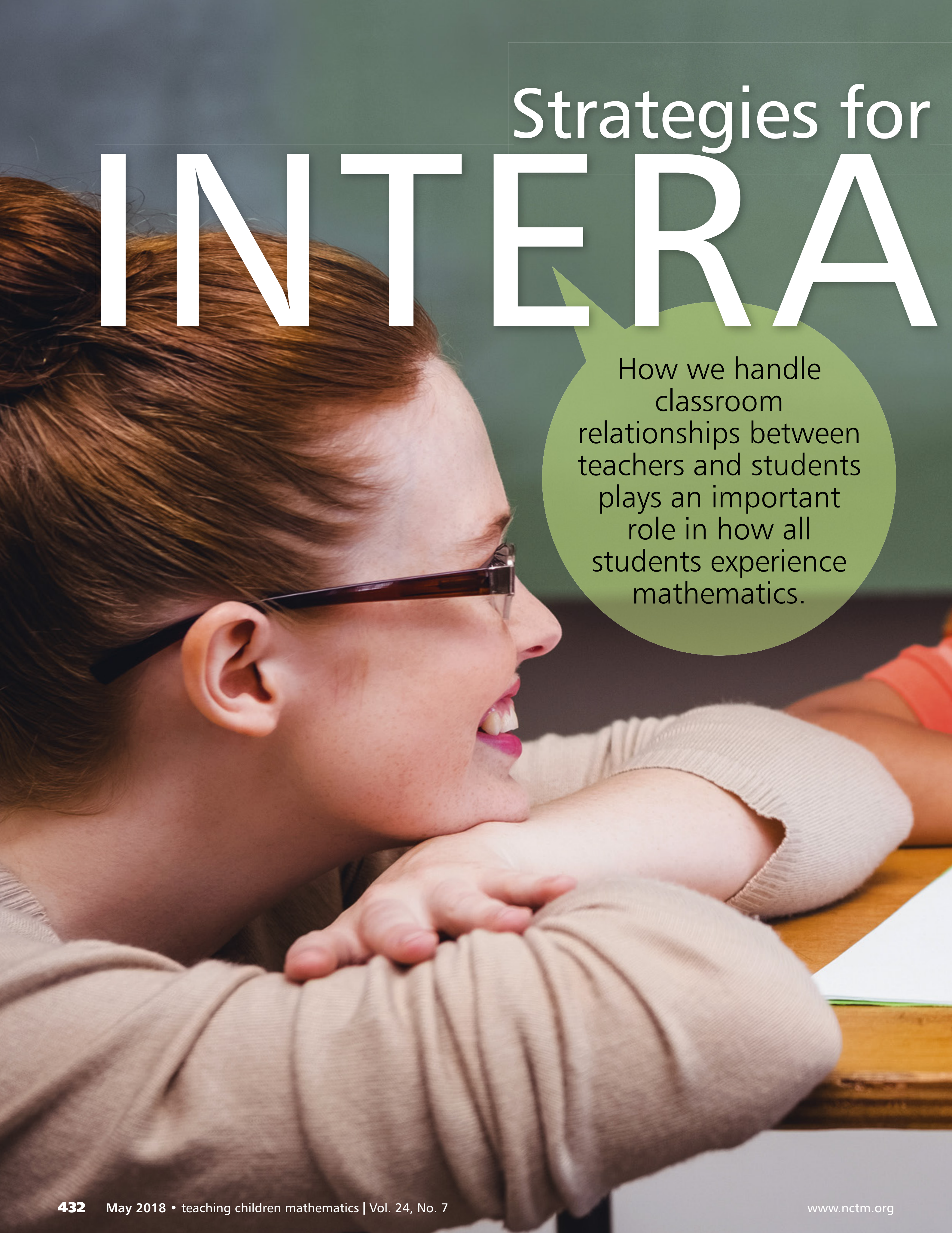
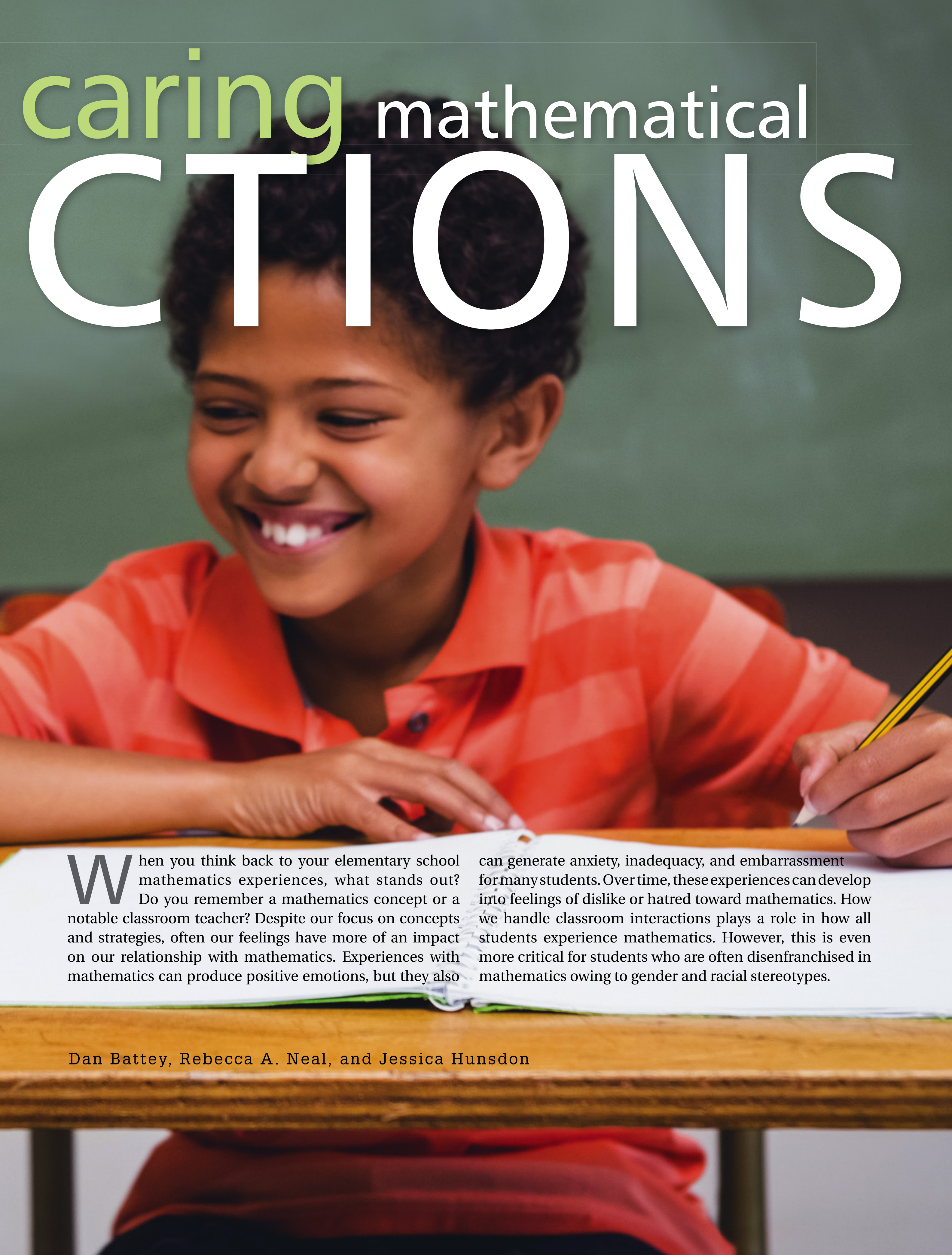


Strategies for INTERA

How we handle classroom relationships between teachers and students plays an important role in how all students experience mathematics.



caring mathematical CTIONS

When you think back to your elementary school mathematics experiences, what stands out? Do you remember a mathematics concept or a notable classroom teacher? Despite our focus on concepts and strategies, often our feelings have more of an impact on our relationship with mathematics. Experiences with mathematics can produce positive emotions, but they also can generate anxiety, inadequacy, and embarrassment for many students. Over time, these experiences can develop into feelings of dislike or hatred toward mathematics. How we handle classroom interactions plays a role in how all students experience mathematics. However, this is even more critical for students who are often disenfranchised in mathematics owing to gender and racial stereotypes.

Dan Battey, Rebecca A. Neal, and Jessica Hunsdon

All too often, teachers' relationships with African American and Latina students have been found to be overly conflictual in mathematics. Conflicts can play out in unconscious ways in terms of focusing on misbehavior, missing students' mathematical contributions, and discussing "low" students (Battey 2013). Caring for students involves taking on students' perspectives and challenging stereotypes about who is mathematically competent (Bartell 2011).

Classroom relational interactions (RIs)

The Common Core (CCSSI 2010) Standards for Mathematical Practice (SMP) emphasize the importance of quality instruction through problem solving, mathematical discussion, and student explanations—providing a foundation for a conceptually focused mathematics classroom. However, relational dimensions of instruction, central to developing a caring environment, are often overlooked (Neal and Battey 2014). Through our work in urban elementary school classrooms, we have captured different ways that teachers of various ethnicities relate to students in mathematics. We have chosen to name the ethnicity of the teachers, in addition to the students, to challenge the common notion that when ethnicity is not stated, the teacher is assumed to be European American (Lewis 2004). By categorizing interactions that contribute to mathematical learning, we have developed a framework of five dimensions of relational interactions (RIs):

Wow, David, you are thinking like a **mathematician**.

Relational dimensions of instruction are central to developing a caring environment in mathematics classrooms.

1. Address behavior.
2. Frame mathematics ability.
3. Acknowledge student contributions.
4. Attend to culture and language.
5. Set the emotional tone (Battey et al. 2016).

Each dimension can be used for either caring or disconnecting from students (see **table 1**).

Through classroom examples, we highlight three dimensions that are most applicable to mathematics and predictive of increased learning: framing mathematics ability, acknowledging student contributions; and setting the emotional tone. These three dimensions specifically support SMP 1: *Make sense of problems and persevere in solving them* (CCSSI 2010, p. 6) and SMP 3: *Construct viable arguments and critique the reasoning of others* (pp. 6–7). Additional examples can be found in **table 1**. Note that because negative RIs with students of color often lead to disengagement, misbehavior, or school dropout (Solórzano, Allen, and Carroll 2002), implementing positive strategies for attending to behavior is crucial. Furthermore, additional strategies for successfully incorporating culture and language into mathematics teaching have been carefully documented elsewhere (Civil and Khan 2001; Drake et al. 2015).

Positively framing mathematics ability

Framing mathematics ability as innate or intimately connected to someone's intelligence is common. Have you ever heard a parent or colleague say something akin to, "Jonathan can really write, but I don't know what happened in math!"? Unfortunately, this framing not only narrowly describes how mathematics learning occurs but also raises the stakes for how students participate in classrooms. Students may worry about judgments from peers and teachers related to their mathematics ability, or they may feel vulnerable to labels that might include "nonmath person" or "slow." The importance of countering this narrative in authentic, meaningful ways cannot be overemphasized (Boaler and Staples 2008).

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TABLE 1

Teacher-student interactions that contribute to mathematical learning can be organized by five dimensions.

Relational interactions (RIs): Dimensions, definitions, and examples

Dimension	Description	Positive examples of interactions
Addressing behavior	Responses to student behavior. Includes escalating behavioral issues or managing them privately.	Noting positive models of behavior Handling misbehavior discretely Avoiding a hyperfocus on misbehavior
Framing mathematics ability	Comments that frame students' ability to do mathematics. Considers the ways in which competence is framed as innate or incremental.	Framing students as competent mathematically Relating student thinking to more complex mathematical ideas Explicitly challenging societal stereotypes about success in mathematics
Acknowledging student contributions	Responses to students' mathematical contributions. Entails ways in which teacher values or devalues or praises or disparages students' thinking.	Recognizing correct aspects of student thinking with incorrect answers Pressing for complete explanations to show care about the details of student thinking for correct and incorrect answers Revoicing student explanations rather than focusing simply on answers
Attending to language and culture	References to cultural or linguistic contributions of students. Includes inclusion and omission of everyday experiences of students in mathematics instruction as well as varied uses of language and body movement.	Drawing on students' everyday experiences in designing problem contexts Allowing students to solve problems in language or dialect of preference before explaining their thinking Revoicing student explanations across languages
Setting the emotional tone	Communicating expectations of acceptable behaviors and emotions. Consists of preempting behavior (as opposed to a direct response) and creating emotional space for students.	Relating to students through personal stories and being emotive Validating student emotions during mathematics Supporting perseverance and effort in mathematics

Although the "nonmath" narrative may have an impact on all students, considering the ways that underserved students are often positioned as deficient or incapable in relation to mathematics, challenging this narrative by explicitly framing underserved students as mathematically able is especially important for teachers. Compounding this, students who are in urban schools comprising African American and Hispanic students often receive low-quality mathematics instruction (Lubienski 2002). Countering these impoverished practices, teachers can frame students' mathematics ability positively by focusing on three strategies:

1. Eliciting a complete explanation from students.
2. Revoice student thinking.
3. Highligh students' mathematical competence.

These strategies cannot be implemented when norms of mathematical classroom discourse require only an answer (Forman and Ansell 2001). In contrast, these strategies require an environment in which teachers use follow-up questions to elicit complete explanations (Franke et al. 2009), which is consistent with the last of the eight Effective Mathematics Teaching Practices, to *elicit and use evidence of student thinking* that is found in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014, p. 10). Revoicing—repeating, highlighting, or rephrasing—students' ideas is a way of recognizing the mathematical importance of specific students' thinking. Additionally, explicitly noting the mathematical competence

This is what my **junior high teacher** *taught me*

Junior high teacher . . . ?

Uh, huh. And this is second grade. That's **amazing**.

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of students, on the basis of evidence of student thinking, serves to counter ideas of accuracy and speed as the only mathematics abilities. The more that teachers highlight all the different mathematics competencies (e.g., problem solving, spatial thinking, logic), the more that students will view themselves as mathematically smart in various ways (Featherstone et al. 2011).

The following vignette illustrates how Ms. May, an Asian American second-grade teacher, enacts the three strategies in her classroom in an effort to promote perseverance (SMP 1). The class is solving the problem $a + a = 15 + 1$, and May emphasizes the mathematical thinking of two female students, Grace and Amy, who are African American and Hispanic, respectively, as she presses them for another solution (bold text indicates emphasis).

May: You made a new problem out of it. OK, is there another way to solve it?

Amy: Eight plus eight equals sixteen; sixteen take away fifteen equals one.

May: Sixteen—can I see this? So, you're telling me that eight plus eight—this turned into a what? A sixteen? [*She rewrites it as* $16 = 15 + 1$, *eliciting a complete explanation.*]

Amy: Uh, huh.

May: And then you put minus fifteen on this side? And so it was one, and you know this way it equals one? [*She rewrites it as* $16 - 15 = 1$, *revoicing the student's thinking.*]

Grace: We did it backwards.

May: You did it backwards, so you got eight plus eight right here, and you made that to a sixteen, like **that**? [*May writes* $15 + 1 - 15 = 8 + 8 - 15$, *again revoicing the student's thinking.*]

Grace: Uh, huh.

May: Like **that**? And then you minused [*i.e., subtracted*] fifteen and got one? That's **interesting**. You know, the way I just wrote it like this? This is what my junior high teacher taught me [*highlighting mathematical competence*].

Amy: Junior high teacher . . . ?

May: Uh, huh. And this is second grade. That's **amazing**.

One important aspect of framing students as mathematically competent is reflecting on how we are recognizing students' *multiple* competencies in mathematics classrooms. Are we noting specific instances of student competence related to mathematical reasoning, efficiency, communicating ideas, and connecting ideas? This can be as simple as stating, "Wow, I love David's questions to his partner about how she solved the problem." Discovering the variety of mathematical competencies, especially for those who struggle, can significantly change students' relationship with mathematics.

Acknowledging student contributions

Students' views of mathematical abilities are also constructed in smaller moments through how teachers choose to acknowledge student contributions. The subtle ways that competence is constructed mathematically are often related to classroom patterns in responding to students' answers and posing purposeful

Discovering students' mathematical competencies and acknowledging their contributions during class discussions can significantly improve their relationship with math.

questions as stated in NCTM'S effective mathematics teaching practices (2014). Do we question students only when their answers are incorrect, or do we reserve questions for students with correct answers? Classroom patterns may communicate that questions are only for students whose mathematical thinking is considered valid or, in contrast, that questions always mean a solution is incorrect.

It is important to avoid positioning students' contributions as having little mathematical merit even when misunderstandings arise. In these situations, teachers can acknowledge student thinking through a process:

- Recognize the correct aspects of the student's thinking.
- Note the problem that the student solved correctly.
- Pose that revised problem to the class.
- Return to the original problem as a check for understanding.

Using questioning to deconstruct incorrect answers helps teachers identify the correct aspects of a student's thinking. Bringing the part of the problem that the student has solved correctly before the class highlights the difference between the problem that the student solved and the original one, which also engages students in SMP 3. Posing this adapted problem recognizes that the student solved a problem of mathematical value, just not the intended one. Returning to the original problem extends the student's correct thinking to the material to be learned. This process stands in stark contrast to a teacher choosing not to question a student's thinking about an incorrect answer, embarrass them, or merely provide the correct answer.

In the following example, second-grade European American teacher Ms. Nolan both modeled and provided scaffolding support for SMP 3 when she pressed for an explanation as Eileen, an Hispanic student, provided an incorrect answer, using the previously mentioned process for acknowledging student contributions:

[*Eileen shares her solution for* 5 + 5 – 5 = 2 + ___ *by putting the number 8 in the blank*.]

Nolan: OK, Eileen did two interesting things. The first one is [that] she took this right here, two plus eight? And she moved it where? [*recognizing correct aspects of the student's thinking*]

Allison: To the front [2 + 8 = 5 + 5]

Nolan: To the front, and she put eight. And she moved this part to the what?

Students: [*in unison*] To the back

Nolan: She flipped it. She made it a little more interesting. Now, Eileen said that two plus eight is the same as ten. Would you guys agree with that? [*noting the problem that the student solved correctly*]

[*Students call out yes and no*.]

Nolan: Would you agree that two plus eight equals ten? [*posing the revised problem to the class to solve*]

Mandy: I'm talking about the three [5 + 5 – 5].

Nolan: No, I'm talking about two plus eight. Would you agree with that? Yes or no?

Mandy: Yes, because eight, nine, ten.

Nolan: Eight, nine, ten. OK. And then she said, "Five plus five is the same as ten." Would you agree with that?

Students: [*in unison*] Yes.

Nolan: OK, so if we say that five plus five equals ten, can I do this? If I take this out and put a ten here [*replacing* 5 + 5 *with a* 10 *on the white-board*], can I do that?

Students: [*in unison*] Yes.

Nolan: I can switch it, right? You just said five plus five equals ten. So now, let me ask you, Eileen, would you keep this eight or would you change it? Take a second to look at this. Look at this new problem that I put on the board here [*writing* 2 + ___ = 10 – 5, *returning to the original problem*].

Knowing how to highlight students' thinking when their answer is incorrect can be difficult, but it can create a jumping-off point for extending learning. Nolan began by establishing that Eileen's thinking is correct for the problem she solved, 2 + ___ = 5 + 5, before asking her to revisit the original problem with the added support of adding the two fives. Looking for the patterns that exist in our own classrooms around answers and explanations makes explicit the ways in which mathematical competence is constructed in our classrooms.

Creating emotional space

From anxiety or fear of failure, to excitement at getting the correct answer, we sometimes forget the range of emotions that mathematics can evoke. Instructional practices that teachers develop may serve to create or limit emotional space. The emotional tone of a classroom that is focused on timed tests, rote learning, and algorithms can feel very different than one that attends to the effective mathematics teaching practice of *facilitat*[*ing*] *meaningful mathematical discourse* that encourages student justification, problem solving, and classroom conversations (NCTM 2014, p. 10). Additionally, social factors influence emotional tone and may include the role that productive struggle plays in mathematical learning, whether doing mathematics is seen as an individual or social endeavor, and the extent to which emotions are considered part of the mathematics experience. Comments and stories as well as the types of work that teachers ask students to do can convey messages about the role of emotions within mathematics classrooms. Teachers can create more emotional space in mathematics classrooms by—

- sharing their own mathematics stories and being more emotive during instruction;
- validating student emotions while doing mathematics; and
- supporting perseverance and effort

—the last of which supports SMP 1, to *make sense of problems and persevere in solving them*. We are in no way advocating for a disingenuous display of emotions; stories must be authentic to teachers. This may include communicating our own mathematical struggles or possibly how we excelled in mathematics but noticed that some types of instruction did not meet the needs of certain students. Modeling our own emotional engagement with mathematics and validating students' emotions and experiences encourage an approach in which students bring their full selves to mathematics. Acknowledging struggle and supporting students' efforts also serve to challenge the notion that mathematical intelligence is innate and instead highlight how persevering through mathematics is a rewarding experience that builds competence and confidence.

Mr. Gray, an African American fifth-grade teacher, showed vulnerability and encouraged perseverance in mathematics, SMP 1, by communicating his own mathematical struggles:

> Is there anybody that doesn't understand this problem? Don't be ashamed. You know you can't be ashamed in this class because I tell you all the time, there's a lot of stuff I don't understand in math. And math, I'm going to make it one of my—it *is* one of my—goals. If you're working on that too, it's OK.

Gray's example opened up multiple ways for students to experience mathematics. Immediately after sharing this, a student volunteered that she did not understand, and Gray responded by having students generate another strategy. This spoke to his students' comfort with him and the safety they felt in his response to their mathematical struggles.

In another example, May builds student confidence as students, in pairs, solve the true-or-false number sentence $4 + 9 = 5 \times 3 - 2$. Resisting the desire to tell students the answer or praise a correct answer, May challenges Abram to have more confidence and mathematical authority:

May: So, is it true or false?

Abram: It's true.

May: You bet your class money that it's true? [*Abram shakes his head no.*] Would you bet your class money that it's true? [*Serena shakes her head no.*] Have confidence in yourself!

Abram: I know I don't, 'cause sometimes I get stuff wrong.

May: But sometimes you get stuff **right**. [*The teacher leaves.*]

Abram: So, it's true.

Serena: Maybe it's wrong. You don't know. Maybe the teacher is lying.

Abram: It's right!

Serena: [*Giggling*] It's wrong.

Abram: Fifteen, fourteen, thirteen [*counting back on fingers*]. Come on, thirteen. I bet my class money. I bet you.

I know I don't, 'cause sometimes I get stuff wrong.

But sometimes you get stuff **right.**

Instructional practices that allow for both teacher and student vulnerability will open up more emotional space for students to experience mathematics in multiple ways.

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Teaching mathematics is clearly more than working with abstract numbers; it requires seeing the whole student, including his or her stories, emotions, and struggles. We can connect to our own emotional experiences during mathematics learning, and we can encourage students to do the same. How are we finding ways to authentically express a range of emotions in the classroom? Are there ways we can use our experiences to better connect to the mathematical struggles our students face? Opening up more emotional space for students to have multiple ways to experience mathematics is crucial, especially for those so often positioned as unlikely to succeed mathematically.

Reflecting on caring mathematical relationships

You can start implementing these strategies by monitoring and attending to your own relational interactions with students in your classroom. Use the framework as a tool of reflection and practice. Focus on one dimension to begin with, noting your own classroom patterns. Identify where changes are necessary and put them into practice, continuing to monitor interactions. For example, since constructing mathematics abilities on the basis of answers is common, how do you respond to answers? Reflect on the degree to which you support students by noting an aspect of their competence, highlighting correct parts of their thinking, and creating more emotional space. We posed questions in each section to guide reflection; however, reflecting on the ways in which our interactions relay specific messages to students is most critical. Purposively making sure that the messages we convey and our interactions with African American and Hispanic students do not align with—and instead challenge—negative stereotypes adds another dimension to this work. Remaining watchful can help us notice the ways that students interpret our interactions. We can transform mathematics classrooms into more caring environments.

Common Core Connections

SMP 1	1.OA.7
SMP 3	1.OA.8
3.OA.4	

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Let's chat!

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and readers about an important topic in our field. In May, we discuss this feature article, "Strategies for Caring Mathematical Interactions," by Dan Battey, Rebecca A. Neal, and Jessica Hunsdon on **May 9, 2018,** at **9:00 p.m. EDT**. Follow along using #TCMchat.

Unable to participate in the live chat? Follow us on Twitter @TCM_at_NCTM and watch for a link to the recap.

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