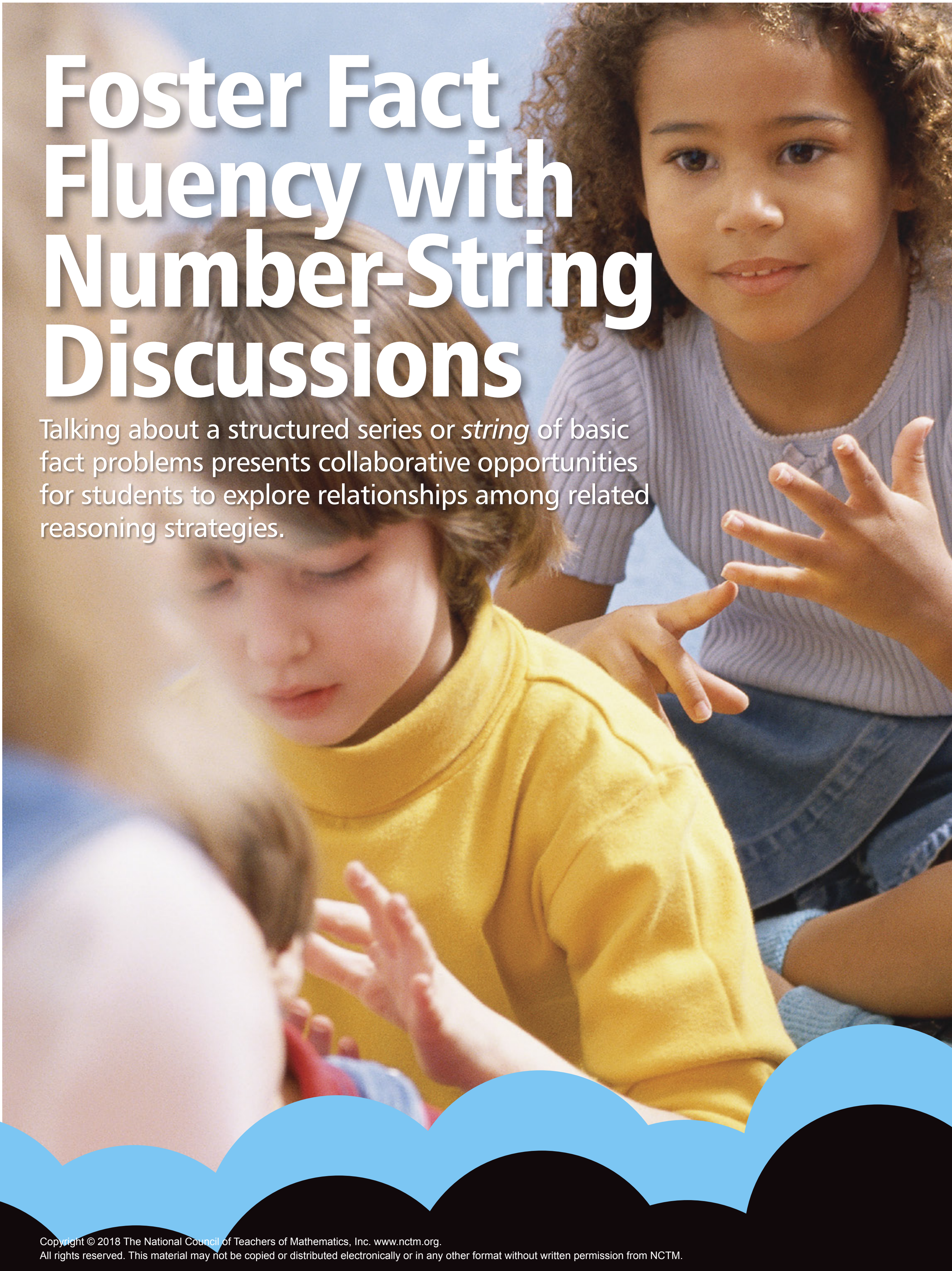
# Foster Fact Fluency with Number-String Discussions

Talking about a structured series or *string* of basic fact problems presents collaborative opportunities for students to explore relationships among related reasoning strategies.



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# MOVING BEYOND SHOW & TELL

Intentional Mathematical Discourse

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A consensus is growing that an important aspect of students attaining fluency with basic facts is developing facility with reasoning strategies that leverage knowledge of number relationships (Baroody 2006; Carpenter et. al 2015; Fosnot and Dolk 2001; Kling 2011). This stance is quite different from the traditional focus on memorizing isolated facts by rote through repetition and reinforcement. A reasoning approach subscribes to the view that fact fluency, "grows out of discovering the numerous patterns and relationships that interconnect the basic combinations" (Baroody 2006, p. 24). This approach demands that a major goal of instruction is to support students in exploring increasingly flexible, strategic, and efficient ways to solve basic fact problems (Kling and Bay-Williams 2014). In particular, as students start to know some facts with automaticity, instruction should encourage students to consider how they might use known facts to *derive*, or figure out, unknown facts

A particularly promising instructional strategy for promoting a reasoning approach to fact fluency is to engage students in a discussion of *number strings*. A number string is a structured series of computation problems that are selected and sequenced to provoke discussion of particular strategies or mathematical ideas (DiBrienza and Shevell 1998; Fosnot and Dolk 2001; Carpenter, Franke, and Levi 2003, Lambert, Imm, and Williams 2017). See examples in **table 1**. When carefully crafted in light of students' current strategies, fact-focused number-string discussions provide opportunities for students to reason together to identify and clarify relationships among facts and to explore related fact strategies. The instructional goal of such discussions is to stimulate consideration of *how* known facts can be used to derive unknown facts and to promote class understanding of *why* the strategies work. In this article, we will offer a detailed account of the design and implementation of an addition-fact number-string discussion and highlight key teacher moves employed to advance students' fact fluency.

**TABLE 1**

Number strings can be designed to emphasize a variety of mathematical ideas.

**Number string examples**

| A multidigit subtraction number string focused on jumps of ten | A multiplication-fact number string focused on developing halving and double strategies | A fraction-addition number string focused on the strategy of making a whole |
|---|---|---|
| 30 − 10 | 2 × 6 | 1/4 + 1/4 + 1/4 + 1/4 |
| 32 − 10 | 4 × 6 | 7/8 + 1/8 + 2/8 |
| 52 − 10 | 6 × 4 | 7/8 + 2/8 |
| 52 − 20 | 3 × 4 | 7/8 + 5/8 |
| 52 − 24 | 2 × 7 | 7/8 + 1/4 |
| 82 − 24 | 4 × 7 | 3/4 + 3/8 |
| 91 − 35 | 6 × 8 | |

## Designing an addition-fact number string

Reflecting NCTM's stance on the importance of teachers' eliciting and using evidence of student thinking (2014), we find that fact-focused number-string discussions are optimized when they are designed and carried out with understanding of students' current fact strategies at the forefront. The activities described in this article occurred on one day of a nine-day Cognitively Guided Instruction (CGI) professional-development program designed by Teachers Development Group (TDG) offered to teachers as part of the Foundation for Success in STEM project at Florida State University. The PD session followed the TDG CGI classroom-embedded protocol (Levi 2017). A group of teachers used data on second graders' current addition-fact strategies (gathered in one-on-one interviews) to design an addition-fact number-string discussion for the PD leader, Dr. Maldonado (the second author), to implement with their class. The teachers first classified the strategies that each student used in the interview using the progression of strategies described by Carpenter and his colleagues (2015) (see **fig. 1**). Interview data revealed that most students had achieved automaticity with certain easier addition facts (e.g., 5 + 5) and that counting on was the most prevalent strategy used to solve unknown facts. Five students

**TABLE 2**

When children approach addition facts by focusing on relationships, certain derived-fact strategies are commonly invented (Fosnot and Dolk 2001). Any one of these strategies can be used to focus an addition number string.

**Focusing an addition number string**

| Derived-fact strategies most commonly invented by children | Examples |
|---|---|
| Double plus or minus | 7 + 8 = 7 + 7 + 1 = 15 or 7 + 8 = 8 + 8 – 1 = 15 |
| Working with the structure of five | 7 + 8 = 5 + 2 + 5 + 3 = 10 + 5 = 15 |
| Making ten | 9 + 5 = 9 + 1 + 4 = 10 + 4 = 14 |
| Using compensation | 7 + 9 = 8 + 8 = 16 |
| Using known facts | 8 + 4 = 12, so 8 + 5 is 12 + 1 = 13 |

primarily employed direct modeling to determine unknown sums, and four students used derived-fact strategies at some point during the interview.

On the basis of these observations, it was affirmed that the second-grade class would benefit from a number-string discussion with the goal of increasing understanding of relationships among facts and related derived-fact strategies. Among the varied strategy types on which an addition-fact number-string might focus (see **table 2**), the teachers decided to design a string to draw out the making-ten strategy. They reasoned that this focus would stimulate noticing of relationships among combinations-of-ten facts (e.g., 6 + 4, 3 + 7) and would help students who had been observed as having automaticity with some combinations of ten to leverage that knowledge for unknown facts.

**Figure 2** presents the string drafted in the planning stage of the lesson. The entry problems 4 + 4 and 5 + 5 were known facts for most of the students and offered an access point for students who needed to direct model the problem with their fingers. Next 6 + 4 was selected for students to consider in relation to 5 + 5, with the intent to establish equality among different combinations that make ten. Then 6 + 5 was chosen for its potential to uncover a making-ten strategy (building on 6 + 4 and 5 + 5). Next 7 + 3 and 7 + 4 were selected to further develop the making-ten strategy. Finally 9 + 5 was chosen as a concluding challenge problem for students to engage in with less scaffolding.

FIGURE 1

Children use increasingly abstract strategies to solve addition problems within 20 (Carpenter et al. 2015).

**Direct modeling**
- With objects
- With pictures

→

**Counting on**
- From first
- From larger

→

**Number facts**
- Derived facts
- Known facts

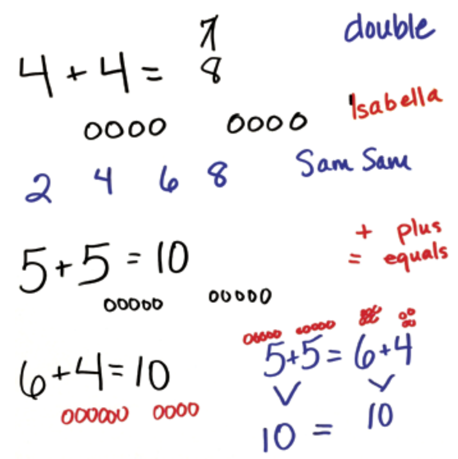
FIGURE 2

The planned string was designed to prompt discussion of the making-ten strategy.

| | | | |
|---|---|---|---|
| 4 + 4 | 6 + 4 | 7 + 3 | 9 + 5 |
| 5 + 5 | 6 + 5 | 7 + 4 | |

FIGURE 3

The number-string discussion opened with brief talk of facts that would set the stage for considering the making-ten strategy.




## In the classroom

To accomplish high-impact classroom discussions, teachers must simultaneously employ strategies to provoke and manage student engagement while also attending to the development of focal mathematics ideas and the varied profiles of the learners (NCTM 2014). To establish expectations, Maldonado explained to the second graders gathered on the carpet that they were going to work together to explore different ways to solve addition problems. She told students that she would put a problem on the board and give quiet time for everyone to think. She asserted, “We are not going to shout out answers,” and had students silently practice showing a thumbs-up sign to signal readiness to answer.

Maldonado wrote 4 + 4 and waited until all students showed their thumbs-up. She invited Isabella, a student observed to primarily use direct modeling with fingers, to share her answer. Isabella reported, “Seven.”

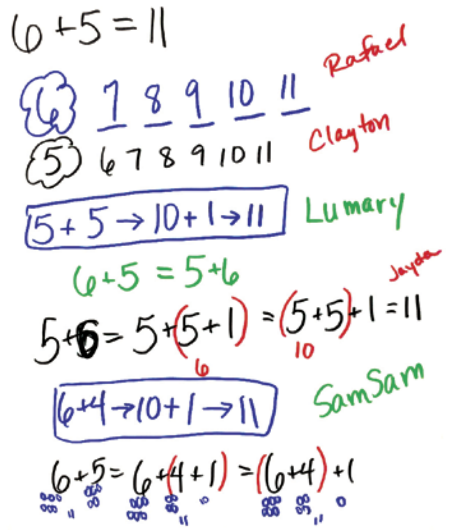
Maldonado recorded Isabella’s answer and asked if anyone had a different answer. Another child reported, “Eight.”

In an effort to promote class engagement in the strategies of peers and also to highlight the mathematical validity of Isabella’s strategy, Maldonado shared that she would have Isabella explain her strategy and then would call on someone to repeat what Isabella had done. Isabella explained with gestures and words, “I put four fingers up and four fingers and counted them together.”

After two students had repeated Isabella’s strategy, Maldonado recorded it by drawing two sets of four circles (see **fig. 3**) and guided the class to count the circles chorally. Next

FIGURE 4

Through class discussion of 6 + 5, Dr. Maldonado introduced two ways to notate derived-fact strategies: arrow notation and use of equations with parentheses.




Maldonado checked back with Isabella, allowing her an opportunity to revise her answer, before eliciting other strategies for 4 + 4.

Using the same think-signal-discuss procedure, Maldonado facilitated a brief discussion of 5 + 5 and 6 + 4 (see **fig. 3**). Recognizing that many students knew these facts with automaticity, minimal time was spent probing strategies. Instead, discussion was focused on establishing 5 + 5 and 6 + 4 as two different ways to "break apart" the number 10 while also prompting discussion of students' conceptions related to the meaning of the equal sign (see Carpenter, Franke, and Levi 2003). Maldonado wrote 5 + 5 = 6 + 4 on the board and said, "Talk with a neighbor about whether this equation is *true* or *false*."

After thirty seconds of partner discussion, Windlyn shared that she thought it was false because 5 + 5 is not 6. Raheem argued that the equation was true, because 5 + 5 and 6 + 4 were both 10. At this point, Dr. Maldonado wrote 10 = 10 on the board and stimulated consideration of whether that was *true* or *false*. Then she used Isabella's picture strategy to establish the validity of the equation 5 + 5 = 6 + 4.

Next Maldonado wrote 6 + 5 on the board. After every student had signaled thumbs-up, she invited Rafael, a student who had consistently used counting on in the interview, to share his strategy. Rafael said, "I got six in my head and counted five more."

To reinforce the importance of engaging with ideas shared by peers, Maldonado had two students repeat Rafael's strategy description. Then she elicited elaboration from Rafael, "What numbers did you say in your head?"

Rafael reenacted his strategy while Maldonado made a record on the board (see **fig. 4**). The next student also described a counting-on strategy in which he counted on from five (the first number) rather than six (the second number). At this point, Maldonado opted not to prompt comparison of the two counting-on strategies to reserve time for the intended lesson focus—derived fact strategies.

Next Lumary described her approach to 6 + 5: "I started with five, and I added five more to get ten. Then I added one, to get eleven."

Careful to avoid modeling incorrect use of the equal sign, Maldonado recorded the flow of Lumary's strategy with arrow notation (see **fig. 4**). Then, recognizing this as an opportunity for students to engage with a making-ten strategy, Maldonado directed students to turn and talk with a neighbor about why Lumary added 5 + 5 as the first step in her strategy for the fact 6 + 5. After one minute of partner discussion, students shared ideas with the class. Mily asserted that Lumary had used 5 + 5 first because 6 + 5 = 5 + 6. Around the carpet, students murmured agreement. Jayda elaborated on Mily's idea, "It's like five plus six is the same as five plus five plus one."

After recording 5 + 6 = 5 + 5 + 1 on the board, Maldonado elicited explanations of the equation from two students. Then, using a red marker, she sought to clarify the strategy by imposing parentheses notation and extending the equation (see **fig. 4**). She asked students to identify the 6 in 5 + 5 + 1 and then to identify the 10 in the equivalent expression.

Next Sam proposed a different way to leverage a known combination of ten. He said, "We already figured out 6 + 4 = 10, but [in 6 + 5] the one next to the six is not a four; it's a five. So, I needed to add another one."

Maldonado used arrow notation to record the flow of Sam's strategy. Then she prompted the class to explain Sam's strategy and help her record it as an equation using parentheses. Finally, wanting to emphasize the equality of the three expressions in the equation, Maldonado led the class in using Isabella's picture strategy to draw and count circles under each number to double-check the equality of the expressions.

**TABLE 3**

Students discussed a range of strategies for 6 + 7, including three derived-fact strategies.

**Strategies for 6 + 7**

| Students' strategies | Teacher's notation and questions |
|---|---|
| "I counted six fingers and then seven fingers to get thirteen." (*Direct modeling*) | Teacher drew six dots and seven dots and asked students to count all. $6+7=13$ 000000 0000000 |
| "I put six in my head, and counted seven, eight, nine, ten, eleven, twelve, thirteen." (*Counting on*) | Teacher notated 6 with a circle to show it was a starting number, and then wrote the rest of the numbers that had been counted. 6 7 8 9 10 11 12 13 |
| "I got thirteen by doing five plus five plus three. I know five plus five equals ten. And three more is thirteen." (*Derived fact*) | Teacher asked, "Is there a five inside of this six [in 6 + 7]?" and, "Where does the other five [in 5 + 5] come from?" On the basis of students' responses, the teacher wrote the following: $5+5 \rightarrow 10+3 \rightarrow 13$ $6=5+1$ $7=2+5$ |
| "I knew six plus six equals twelve. Then I added one more because it is six plus seven." (*Derived fact*) | Teacher asked, "Where is the seven [from 6 + 7] in this strategy?" On the basis of students' responses, she boxed the six and one and wrote this: $6+\boxed{6} \rightarrow 12+\boxed{1} \rightarrow 13$ $6+1=7$ |
| "First I did seven plus three to get ten. Then I did ten plus three to get thirteen." (*Derived fact*) | Teacher asked, "Where is the six from six plus seven?" Based on student responses, she boxed the three and the three and wrote the following: $7+\boxed{3} \rightarrow 10+\boxed{3} \rightarrow 13$ $3+3=6$ |

It was almost time to end class, so Maldonado modified the planned number string and presented 6 + 7 as the final problem. The class generated multiple strategies for 6 + 7, including three derived-fact strategies. For each derived-fact strategy, Maldonado used probing questions to help students make sense of how numbers were "broken up and put back together" to make 6 + 7 easier to solve. **Table 3** presents the strategies that students shared, Maldonado's notation, and key questions. After the class had unpacked all three derived-fact strategies, Maldonado enthusiastically exclaimed, "You all are just breaking up numbers all over the place. Kiss your brains!"

## Implementing discussions to advance fact fluency

This thirty-five-minute number-string discussion provoked noticing of relationships among addition facts that in turn stimulated student reasoning and consideration of derived-fact strategies. Close and careful attention to student thinking and the use of strategies to orient students to one another and the mathematical ideas undergirded Maldonado's ability to engage the second graders in the mathematics of the lesson through discussion.

### Attention to student thinking

The number string in this lesson was designed in response to careful consideration of students' current strategies for a range of addition facts. By attending to the class profile of student thinking, the string was calibrated to encourage flexible use of known facts. At the same time, care was taken to ensure entry points and engagement opportunities for students whose next step along the learning progression was to move from direct modeling to counting on (rather than from counting on to derived facts). In particular, Maldonado continually invited and valued a range of student strategies, and she continually leveraged Isabella's direct-modeling strategy to help students make sense of strategies that are more sophisticated.

Because Maldonado was aware of the strategies that particular students were likely to use for solving particular facts, she was able to purposefully call on students likely to share specific types of strategies. She also targeted questions to individual students to engage them in their personal next step along the learning progression. For example, when counting-on strategies arose during discussion, Maldonado intentionally targeted questions promoting understanding of that strategy to students who were currently direct modeling.

### Orienting students to one another and the mathematics

Another key to optimizing mathematical discussions is employing teacher moves that facilitate student engagement with and understanding of mathematical ideas that surface

(Carpenter et al. 2015; Kazemi and Hintz 2014). A significant hurdle that teachers face is bringing *all* students into the discussion and compelling them to value the ideas of their peers. Maldonado led by example as she intentionally invited and consistently valued the ideas of *all* students, not just those who had correct answers or used sophisticated fact strategies. For example, rather than glossing over Isabella's incorrect answer to 4 + 4, Maldonado had Isabella share the details of her valid direct-modeling strategy and directed the class to listen carefully and be ready to explain Isabella's thinking. Then, throughout the lesson, Maldonado strategically highlighted the usefulness of Isabella's picture strategy in making sense of other strategies—thus shaping the way the class viewed Isabella's contribution.

In addition to encouraging student attention to one another's contributions by prompting them to explain others' ideas, Maldonado sought to promote whole-class engagement at mathematically pivotal points in the discussion by having students turn and talk to a neighbor. For example, early in the discussion, Maldonado used a turn-and-talk to promote consideration of whether the equation 5 + 5 = 6 + 4 was true or false. Maldonado viewed this moment in the discussion as particularly critical because she knew that many students might hold flawed or limited conceptions about the meaning of the equal sign and valid equation structures. Establishing the equal sign as meaning *the same as* was foundational to students understanding the equation notation that Maldonado would use to unpack students' derived-fact strategies later in the discussion. When Lumary described how she solved 6 + 5 using 5 + 5, Maldonado recognized another pivotal moment. She prompted students to turn and talk about the basis of Lumary's strategy because she wanted to ensure that students were engaging in the idea of decomposing and recomposing numbers to make ten.

Also crucial to facilitating student access to others' mathematical ideas were the varied ways Maldonado recorded the strategies reported verbally and the ways she encouraged the rest of the class to interact with those notations. When Lumary and Sam described derived-fact strategies for 6 + 5, Maldonado captured the flow of the strategies with arrow notation (e.g., 5 + 5 → 10 + 1 → 11). Then, after using questions to prompt the class to justify each strategy, Maldonado recorded the justification provided by peers in equation notation. Also, as she pressed for deeper explanation through questioning, Maldonado added red parentheses to the equations to clarify the decomposition and recomposition of numbers inherent in the strategies (see **fig. 4**).

## Let's chat

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and readers about an important topic in our field.

On **October 10, 2018**, at **9:00 p.m. EDT**, we will discuss "Foster Fact Fluency with Number-String Discussions." Follow along using #TCMchat.

Unable to participate in the live chat? Follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.

## Final thoughts

A single fact-focused number-string discussion is unlikely to result in all students adopting more flexible and efficient fact strategies; however, we have found that repeated engagement in such discussions over time does promote student reasoning about relationships, which in turn leads to increased fact fluency. By cultivating a reasoning approach, we find that students are more likely to not only achieve automaticity with basic facts (Henry and Brown 2008; Kamii 1999) but also establish a strong foundation for using number relationships to devise multidigit computation strategies.

A number string is like a path that is designed to lead students toward consideration of a target strategy or mathematical idea. Although the teacher should certainly aim to facilitate a number-string discussion such that the target idea is illuminated, honoring students' ideas that deviate from the preconceived direction of the path is equally important. By keeping sight of the larger goal of encouraging and supporting student reasoning, we can use number-string discussions to leverage the collective thinking of the group and move an individual student's thinking forward.

For additional examples of lessons that use discussion of number strings to foster fact fluency, check out the "What's Next?" stories on our project website at https://www.teachingisproblemsolving.org/whats-next-stories/.

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**Common Core Connections**

**1.OA.B.3 1.OA.C.6**
**1.OA.D.7 2.OA.B.2**

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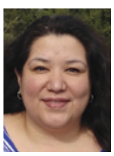

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