

Classroom discussions become more productive when teachers design their inquiries to elicit and understand student thinking.

Refining Planning:
Questi



Delise R. Andrews
and Karla J. Bandemer

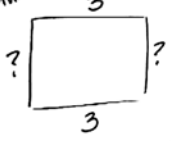


Have you ever witnessed a really powerful mathematics discussion and wondered, “How do I make that level of discourse the norm in my mathematics classroom?” More and more teachers are taking up the call to refine the way they teach mathematics. Whether or not your state has adopted the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), your mathematical practice standards likely include language that focuses on having students discuss their thinking and their work. Productive mathematical conversations are a significant component of developing students’ mathematical understanding and ability to effectively communicate their thinking, and the types of questions we ask can either hinder or advance that development. When we have asked teachers in our district who are adept orchestrators of student discussion how they got to where they are, their response always references the need for thoughtful planning. In short, rich mathematical discourse does not happen by accident.

Planning

with a Purpose

FIGURE 1

The teacher used a planning tool to anticipate student strategies for a particular problem, misconceptions the team believed that students might hold, and notes during the lesson about which students used which strategies and the order in which to discuss them.

Strategy	Who & What	Order
<p>active</p>  <p>$3 + 3 + ? + ? = 20$ $? = 7$</p>		
<p>Misconception</p> <p>$20 + 3 = 23$ or $20 - 3 = 17$</p>		
<p>manipulatives (error)</p> <p>20 square tiles</p>  <p>length = 9</p>		
<p>manipulatives (correct)</p>  <p>length = 7</p>		

A wealth of literature suggests that facilitating productive mathematical conversations is critical to developing students' mathematical understandings (Chapin, O'Connor, and Anderson 2009; NCTM 2014; NRC 2001; Seeley 2016; Smith and Stein 2011). The process is also complex and nuanced. We seek to support teachers in this work. In our professional roles, we support teams of elementary school classroom teachers as our district sharpens its focus on collaborative lesson planning for mathematics instruction. Teams of teachers who are accountable for instruction in many different subject areas often have limited time for collaborative planning of mathematics instruction. Thus, offering them strategies and supports has been crucial—those that are not only meaningful but also feasible given the significant demands on teachers' time. Toward that end, in recent years our district math department has focused professional development on the five practices for orchestrating mathematics discussions (Smith

and Stein 2011)—(1) anticipate; (2) monitor; (3) select; (4) sequence; and (5) connect—as well as the effective Mathematics Teaching Practices discussed in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014).

During the course of many collaborative planning sessions with teams, teachers expressed a need to make questioning more intentional as they engaged in the anticipating and monitoring phases of a task. According to Smith and Stein (2011),

Almost all good classroom discussions begin in the same way: by inviting a student to share how he or she solved a particular problem. After the initial student response, however, classroom discussions diverge—separating into the relatively rare fruitful ones and the much more frequent unproductive show-and-tells. (p. 69)

In most classrooms, teachers ask students to share their thinking, whether at random or by thoughtfully selecting and sequencing students' ideas. We have noticed that what separates the show-and-tell discussions from the productive mathematical discussions comes down to what teachers know about student thinking before the whole-class discussion even begins. That knowledge comes from observations made and questions asked during students' initial work on a task. Shifting the focus from checking students' answers to analyzing students' thinking—by asking purposeful questions—can elicit critical information. Seeking to understand students' thinking while monitoring their initial work on a task better equips the teacher to purposefully select and sequence students' ideas. This, in turn, can better support meaningful mathematical connections among students' work, which provides for a more productive discussion.

The following vignettes are not representative of any one teacher or team. Rather, they are a compilation of common themes that we have observed in our work. These themes are evidence to us of the need to focus on thoughtful planning for quality questioning in mathematics instruction.

Collaborative planning vignette 1: We have anticipated; now what?

A team of teachers collaboratively plans for discussion about the following problem:

A rectangular pool has a perimeter of 20 yards. The width of the pool is 3 yards. What is the length of the pool?

Figure 1 shows the initial notes that one teacher took during the team's planning session. The figure reflects the team members' focus on anticipating strategies for one particular problem in the lesson and misconceptions they believe that students may have about finding the perimeter of a rectangle. The planning tool used by the teacher, based on the work of Smith and Stein, details anticipated strategies and allows space for the teacher to take notes during the lesson about which students used which strategies and the order in which to discuss the strategies during the whole-class conversation.

On the day of the lesson, students use several of the anticipated strategies as they work on the task. Many errors are also observed that reveal potentially significant misconceptions. Eight of the eleven pairs of students in the class have incorrect answers (see fig. 2). Although the teacher and the team have anticipated some of the strategies and misconceptions, they have not planned any specific prompts or questions to elicit students' reasoning about their work. Seeing so many incorrect answers, the teacher feels somewhat overwhelmed. The source of students' confusion is unclear, so the teacher resorts to a funneling pattern of questioning (Herbel-Eisenmann and Breyfogle 2005; Wood 1998) to get students to the right answer. Students who have incorrect answers are asked a series of low-level, closed questions such as the following:


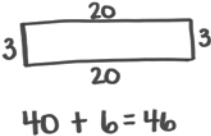
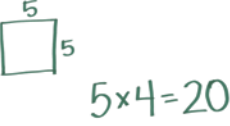
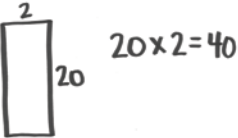

- What is the width of the pool?
- So, what is the width on the opposite side?
- Remember, the perimeter is the distance around the edge of the pool, so if we already have six yards, what must these other two sides add up to?

Because the goal of the questions is to get students to the correct answer, the teacher does not feel able to walk away until the pair comes up with an answer of seven yards. Students who have correct answers are not asked questions at all but are directed to help classmates who have wrong answers. The teacher does not have enough time to touch base with every student pair.

As a result of asking mostly low-level questions, the teacher is unable to determine the nature of students' confusion (i.e., whether students do not understand the problem, are unsure of how to find the perimeter of a rectangle when given only two side lengths, are

FIGURE 2

Although some students used anticipated strategies, many did not. The teacher resorted to questions that funneled students toward the correct answer but did not allow time to elicit and discuss students' reasoning or misconceptions.

$\begin{array}{r} 20 \\ + 3 \\ \hline 23 \\ \times 2 \\ \hline 46 \end{array}$	$20 \times 3 = 60$ 60 yards 
	
$\begin{array}{r} 20 \\ + 20 \\ \hline 40 \\ + 3 \\ \hline 43 \\ + 3 \\ \hline 46 \end{array}$	
$P = 20$ 	$20 \times 3 = 60$ 60 yards

confused by the difference between area and perimeter, or have some other misconception). To begin the classroom discussion, the teacher selects a student who has a correct solution to share her work with the whole class. Students who also have the correct answer agree, and students who still do not have the correct answer erase their boards or change their work. After the lesson, the teacher expresses concern that the resulting conversation was not truly a mathematical discussion that connected strategies and deepened students' understanding about perimeter; instead it funneled students toward a correct answer. Consequently, the teacher did not address any of the misconceptions that appeared prevalent in student work. The teacher also wonders if true evidence of understanding was elicited from the students who had independently arrived at the correct answer. Is it

Instead of looking for correct answers and correcting students who are struggling during the monitoring phase of the task, team members wanted to elicit and use students' thinking as a foundation for the whole-class discussion.

Strategy	Questions/Prompts	Who/What	Order
<p>Ⓐ</p> $\begin{array}{r} 12 \\ 202 \\ -198 \\ \hline 4 \end{array}$ <p>4 more cans</p>	<ul style="list-style-type: none"> • tell me about your ungrouping. • why did you subtract? • does 4 make sense? why? • partner student with someone who used a mental math strategy - discuss connections 		
<p>Ⓑ</p> $198 + 2 = 200$ $200 + 2 = 202$ <p>↳ 4 cans</p>	<ul style="list-style-type: none"> • some kids subtracted... why did you decide to add? • does 4 make sense? why? • partner with strategy C or misconception D to discuss. 		
<p>Ⓒ</p> <p>4 cans</p> $2 + 2 = 4$	<ul style="list-style-type: none"> • can you explain how you thought about this? • does 4 make sense? why? • if counted up, ask "would it work to count back from 202?" or vice-versa 		
<p>Ⓓ</p> <p>misconception</p> $\begin{array}{r} 202 \\ +198 \\ \hline 400 \end{array}$	<ul style="list-style-type: none"> • tell me how you thought about this problem. • does 400 make sense? why? • what does 202 (or 198) represent in this problem? • partner w/ A or B to discuss 		
<p>Ⓔ</p> <p>error</p> $\begin{array}{r} 202 \\ -198 \\ \hline 196 \end{array}$ <p>196 cans</p>	<ul style="list-style-type: none"> • how did you think about this problem? • does 196 make sense? why? • would estimation help you? • could you draw a model? • partner w/ A to discuss. 		
<p>Ⓕ</p> <p>error</p> $\begin{array}{r} 12 \\ 202 \\ -198 \\ \hline 114 \end{array}$ <p>114 cans</p>	<ul style="list-style-type: none"> • explain how you thought about this problem • does 114 make sense? • tell me about your ungrouping. • partner w/ A and/or E to discuss. 		

Collaborative planning vignette 2: Eliciting student thinking

In subsequent collaborative planning sessions, we refined our process and began to incorporate a greater focus on eliciting students' thinking. Given limited planning time and the shifts in practice that we were asking teachers to make, we initially focused on reorienting teacher prompts (from getting answers to seeking information) during the monitoring phase of the lesson. In the following vignette, a teaching team plans for discussion about the following problem:

The local food bank needs 202 cans of beans for the food drive. So far, they have collected 198 cans of beans. How many cans does the food bank still need?

As the team plans, they anticipate possible strategies and misconceptions (see **fig. 3**). Instead of simply giving corrections to students who are struggling and looking for correct answers during the monitoring phase of the task, team members want to elicit and use students' thinking as a foundation for the whole-class discussion. As they talk together about how they can shift from answer-getting strategies to information-seeking methods, they plan some general open-ended prompts they will use to elicit students' thinking during this phase. Prompts written for successful strategies are designed to press students' reasoning, uncover potential confusions, and extend their thinking.

Informal conversations about previous lessons had uncovered a common theme of students neglecting to consider the reasonableness of their answers. As a result, one of the goals for this class discussion is to have students talk about whether different answers make sense. With that in mind, the team plans to listen carefully to how students support their thinking in response to planned prompts (e.g., "Does 400 make sense? Why?"). The team decides to select and sequence student work with a goal of engaging the whole class in a discussion about reasonableness. That is, the teachers plan to look for students to share one correct solution and one or two examples of unreasonable results during the class discussion.

In one classroom on the day of the lesson, the teacher uses the planned prompts as a guide for interacting with students who are working on the task. She takes notes after students share their thinking (see **fig. 4**); student

possible that some of them had stumbled on the correct answer but still had hidden misconceptions about perimeter and area?

Although the teaching team in vignette 1 had discussed the task in advance, thinking carefully about potential student conceptions and misconceptions, the teachers had not considered how to elicit student thinking about the strategies they used. The work was valuable in that it helped the teacher feel prepared for what students might do with the task. However,

the need to generate all the questions/prompts "in the moment" during instruction cost the teacher valuable time and cognitive energy and ultimately limited the ability to effectively engage with all students as they worked on the task.



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initials indicate which pairs of students used which strategy. The teacher notices that several students used a counting-on strategy to find the correct answer. Two of the three anticipated incorrect answers are also discovered in student work. The teacher writes the three most common answers on the board: 4 more cans, 196 more cans, and 114 more cans. Students are asked to consider the reasonableness of the three answers within their small groups before the class comes back together for a whole-class discussion.

Prepared with prompts that the team collaborated on for the monitoring phase of the lesson, the teacher was able to devote more energy during that time to thinking about students' thinking. These prompts also served to hold the teacher accountable to listen to students' ideas instead of simply focusing on whether they had the right answer. The teacher felt that with the shift in the purpose of the discussion, more students could meaningfully engage in the conversation, so she challenged the class to determine which answer made the most sense and then pushed students to support their own thinking and consider the thinking of others.

In this second vignette, the teacher also worked with a collaborative team to plan the task in advance, thinking carefully about potential student conceptions and misconceptions. The team developed purposeful questions and prompts as an added part of their planning process. Being intentional about planning questions that connected to the anticipated student strategies minimized the tendency to fall into a funneling pattern of questions. Instead, the teacher was prepared both for the solutions that students offered and how to respond in a way that helped elicit more information about students' thinking. This, in turn, provided the substance for a whole-group discussion (Smith and Stein 2011).

The teachers and teams we have had the opportunity to work with have inspired us to think deeply about the role of planning for questioning in the development of a powerful mathematics discussion. The work of anticipating students' strategies, conceptions, and misconceptions, and the planning of questions has helped teachers move away from a show-and-tell discussion and toward intentional discourse. NCTM's 2017 Taking Action series (e.g., Huinker and Bill 2017) includes planning templates that can support teachers who are seeking to incorporate a purposeful focus on questioning as they plan for mathematics instruction and student discourse.

FIGURE 4

This teacher used the planned prompts as a guide when interacting with students, took notes after students shared their thinking, discovered two of the three anticipated incorrect answers, and asked the class to consider the answers' reasonableness within small groups before engaging in a whole-class discussion.

Strategy	Questions/Prompts	Who/What	Order
A $\begin{array}{r} 100 \\ 12 \\ 202 \\ -198 \\ \hline 4 \end{array}$ 4 more cans	<ul style="list-style-type: none"> • tell me about your ungrouping. • why did you subtract? • does 4 make sense? why? • partner student with someone who used a mental math strategy 	K/S F/S P/C C/B - discuss connections	
B $\begin{array}{r} 198 + 2 = 200 \\ 200 + 2 = 202 \\ \hline 4 \text{ cans} \end{array}$	<ul style="list-style-type: none"> • some kids subtracted... why did you decide to add? • does 4 make sense? why? • partner with strategy C or misconception D to discuss. 	K/A - "faster to count up" R/Z O/D	3
C 4 cans $\begin{array}{c} 2 + 2 = 4 \\ \leftarrow \quad \quad \quad \rightarrow \\ .198 \quad 200 \quad 202 \end{array}$	<ul style="list-style-type: none"> • can you explain how you thought about this? • does 4 make sense? why? • if counted up, ask "would it work to count back from 202?" or vice-versa 	K/B R/M - counted on in head "4 makes sense b/c 198 is close to 202."	2
D misconception $\begin{array}{r} 202 \\ + 198 \\ \hline 400 \end{array}$ none	<ul style="list-style-type: none"> • tell me how you thought about this problem. • does 400 make sense? why? • what does 202 (or 198) represent in this problem? • partner w/ A or B to discuss 		
E error $\begin{array}{r} 202 \\ -198 \\ \hline 196 \end{array}$ 196 cans	<ul style="list-style-type: none"> • how did you think about this problem? • does 196 make sense? why? • would estimation help you? • could you draw a model? • partner w/ A to discuss. 	S/F - "8-2 is 6, 9-0 is 9, 2-1 is 1" K/T E/B - "196 makes sense b/c they need a lot of cans!"	
F error $\begin{array}{r} 100 \\ 12 \\ 202 \\ -198 \\ \hline 114 \end{array}$ 114 cans	<ul style="list-style-type: none"> • explain how you thought about this problem • does 114 make sense? • tell me about your ungrouping. • partner w/ A order E to discuss. 	F/V - noticed & corrected regrouping error when explaining N/L - partnered w R/M to compare approaches	1

A call to action: Examining beliefs

Just as the teachers we work with have inspired us to refine our planning practices, we hope that this article encourages readers to reflect on how they prepare for meaningful mathematical discussions. Wood and Hackett (2017) remind us that "the substantial learning outcomes for everyone in the classroom (teachers and students alike) make it worth the time investment of using purposeful questions" (p. 58). We challenge readers to examine their own beliefs about the planning process, its purpose, and the potential impact it can have on student learning. Reflecting honestly on their beliefs about students' capacity to contribute to a mathematics discussion is also important for teachers (and instructional leaders). These beliefs play a key role in either supporting or impeding the work of planning for mathematics discourse. See **table 1** for a summary of some productive and

TABLE 1

The authors and team members they collaborated with have encountered some productive and unproductive beliefs in their work, including in their own thinking.

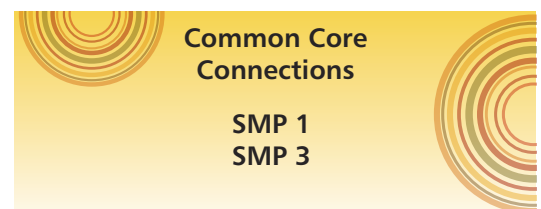
Authors' and team members' beliefs

Unproductive	Productive
Planning questions in advance is not a good use of time; we must wait and see what students do and hear what students say.	Anticipating what students will do reduces a teacher's cognitive load during instruction, allowing the teacher the freedom to be open to and curious about different strategies students might use (Smith and Stein 2011).
You are either a good questioner or you are not. Good questioners do not need to plan prompts; it will just happen. It is an organic process.	Organic conversations can still happen and in fact are better facilitated if the teacher has anticipated student thinking. Participating regularly in this process helps us get in tune with students' ideas. Teachers will become better at predicting possible strategies and crafting questions that respond to those strategies.
We have a prescribed curriculum, so we do not need to plan together. All the questions are in the book.	Just as we cannot anticipate every strategy or write every question, neither can a textbook. We have a professional responsibility to use what we know about our students and connect their conceptions and misconceptions to the mathematical objectives set out in the curriculum.
We put discussion prompts and sentence frames on the wall, but some students just cannot or will not ever participate in a discussion about mathematics.	Teachers must model effective, authentic questioning and provide structured, intentional opportunities for students to practice questioning one another.
If students with the incorrect answer share their work, other students might learn it incorrectly. It will just confuse everyone.	Sometimes misconceptions are more useful for conversation than correct answers. Errors reveal important information about students' understanding and are often connected to some correct reasoning. Quality questions help teachers find strengths in students' ideas.

unproductive beliefs we have encountered in our work (and sometimes needed to overcome in our own thinking).

We believe that it is possible to make productive mathematics discussions the norm in every classroom. Student thinking is the foundation for meaningful conversations that advance learning. Gaining the necessary insight into students' thinking can be done by asking purposeful questions during the monitoring phase of a lesson. In our work with teachers, we have focused on shifting the purpose of teachers' questions away from getting students to the correct answers and toward understanding students' conceptions and misconceptions about the mathematics within a task. This shift better equips teachers to orchestrate powerful mathematics discussions. By gaining an understanding of students' thinking before a discussion, teachers can select work to be shared and

discussed not on the basis of how correct the answer is, but rather because of how the reasoning behind it can be used to build students' understanding about the concept. Students are empowered to defend their reasoning about a task and position themselves as mathematicians in their own right.



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“Refining Planning: Questioning with a Purpose”

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions related to “Refining Planning: Questioning with a Purpose,” by Delise R. Andrews and Karla J. Bandemer, are suggested prompts to aid you in reflecting on the article and on how the authors’ ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

Reflecting on and challenging unproductive beliefs about planning and discourse is a powerful way to move toward productive mathematical conversations in your classroom. Wherever you are in your journey toward such discourse, we encourage you to take time to analyze the types of questions asked and the way they are used in your classroom. Here are some questions for reflection:

- Does my team plan together?
- Should we include other teachers who could provide valuable input?
- Are team planning sessions focused on student thinking?
- Do we plan questions and prompts in advance?
- Do our plans support a focusing pattern of questions (as opposed to funneling)?

We invite you to tell us how you used Reflect and Discuss as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to Teaching Children Mathematics at tcm@nctm.org. Please include Readers Exchange in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgment. Letters are also edited for style and content.

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Delise R. Andrews, dandrews@lps.org, is the grades 3–5 math coordinator for Lincoln Public Schools in Lincoln, Nebraska. She is interested in supporting high-yield instructional practices that engage students with mathematics in meaningful ways. Karla J. Bandemer, kengh@lps.org, is the grades 3–5 math teacher leader for Lincoln Public Schools in Lincoln, Nebraska. She is interested in strengthening student learning by supporting teachers’ instructional planning and professional growth.

