

A kindergarten teacher uses Gutiérrez's Four Dimensions of Equity to design and facilitate geometry instruction.

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Equitable instruction is reflected in how students are positioned in the classroom and how their identities evolve through purposeful interactions that value and recognize the intellectual capacity of each student (Gutiérrez 2013; Lemons-Smith 2008). These integral interactions occur when teachers and students exchange problem-solving strategies, discuss relations among various mathematical representations, and listen to the viewpoints of others (NCTM 2000; 2014).

**"SLIDING" INTO
EQUITY**



AN BLESSON

However, without explicitly and purposefully attending to *whose* voice is represented in these conversations or valuing the out-of-school knowledge that students bring, we are not providing students the support, confidence, or opportunities necessary to reach their highest levels of mathematical success (NCTM 2014). Equity in mathematics education requires meaningful experiences that embolden each student to achieve success not only through content objectives but also in disposition, perseverance, and application of mathematical concepts (Gutiérrez 2002; NCTM 2014). In fact, Gutiérrez argues that equity is not one dimensional, but rather involves four dimensions (*achievement, access, identity, and power*)—a powerful framework that teachers and administrators can use to guide instructional practices and decisions.

The dimensions of *achievement* and *access* are more readily accessible in the design and implementation of mathematical activities. *Achievement* pertains to students' participation and outcomes on standardized and non-standardized mathematical assessments. *Access* includes implementing mathematical tasks that relate to students' daily lives, local community, or shared experiences, as well as ensuring that mathematical tasks have multiple entry points. Embedding scaffolds, prompts, and extensions within the lesson provides appropriate challenges that promote access to content.

The remaining dimensions, *identity* and *power*, are integral elements that promote mathematical learning. *Identity* involves teachers drawing on students' cultural backgrounds and encouraging learners to view themselves as mathematicians capable of having an impact on broader society. *Power* is closely tied to *identity*. To provide students with *power*, teachers must be cognizant of whose voice is privileged when answering questions, solving tasks, or sharing mathematical strategies, and they must implement instructional decisions that recognize the contributions of a wide variety of students.

By intentionally integrating Gutiérrez's (2013) Four Dimensions of Equity into mathematical discussions, teachers value students' voices and provide an equitable learning experience for all. In this article, we describe how a kindergarten teacher, Bell, planned and

facilitated a geometry lesson that attended to the dimensions of equity. We highlight key actions and exchanges that demonstrate how he provided a voice to each student while engaging the class in discussions focused on identifying, describing, and comparing basic geometric solids.

Lesson preparation: Planning through frameworks

To promote productive discourse in the classroom, teachers purposefully incorporate opportunities in which students are encouraged to explore, discuss, hypothesize, and question ideas or viewpoints. This process is complex because during planning, teachers must decide (a) *how* to incorporate students' responses that contribute to mathematical understanding, and (b) *who* will speak to "advance the mathematical storyline of the lesson" (NCTM 2014, p. 30).

In this lesson, Bell planned to implement the Slide-Stack-Roll activity to address the mathematical content—analyze and compare three-dimensional (3D) solids. The Slide-Stack-Roll (SSR) investigation offers students the chance to tangibly explore the attributes of geometric solids and determine whether shapes would slide down an incline, would stack vertically, or would roll across a flat surface. During planning, Bell considered how to apply Gutiérrez's (2013) four dimensions when implementing the task with his students.

Achievement and access

In reflecting on the *achievement* dimension, Bell purposefully planned to use a recording sheet to document his students' thinking rather than drawing name sticks. Initially, he thought that randomly drawing name sticks would offer students equal chances to participate. However, he recognized that it actually hindered students' participation—and consequent *achievement*—because he realized that several of his students generally disengage before and after their stick is pulled.

When considering different tasks to explore geometric solids, Bell selected the SSR investigation because it was a tactile activity grounded in most students' block-play experiences. Additionally, the exploratory nature of the task allowed a diverse group of students *access* to

TABLE 1

This overview of kindergarten teacher Bell's lesson aligns with Gutiérrez's (2013) Four Dimensions of Equity.

Overview of Bell's lesson

Dimension	Launch	Explore	Summarize
Achievement —participation and outcome	Think-pair-share strategy Make predictions and justifications about shapes' attributes	Recording sheet for students' responses Prompts connecting shape properties to students' predictions	Multiple students contribute to whole-group discussion Justify results on the basis of shapes' attributes
Access —high-quality tasks, multiple entry points	Community connections	Hands-on SSR block activity Community connections	Open-ended questions
Identity —viewing students as mathematicians		Refer to students as <i>mathematicians</i>	Refer to students as <i>mathematicians</i>
Power —recognizing students' voices	Think-pair-share strategy Cooperative small groups predict results	Intentionally focus on students who do not regularly participate	Multiple students contribute to whole-group discussion

the mathematical content through the conversations that emerged from the investigation. Bell planned questions and challenges that focused the kindergartners' attention on properties of solids and anticipated their areas of difficulty. For instance, he anticipated that students would predict prisms would stack but the cone would not. Therefore, Bell planned additional scaffolds to provide students *access* to the mathematical content as they analyzed and compared five geometric solids (i.e., a rectangular prism, cube, cylinder, cone, and sphere). Bell also planned conversations that would connect geometry concepts to events in the students' community, such as the new retaining wall being built near the school. When facilitating these discussions, Bell would ask students to hypothesize why particular shapes were used to construct the wall.

Identity and power

Because *achievement* and *access* involve tangible outcomes, planning for these dimensions was more straightforward. Addressing *power* and *identity* through the SSR lesson was more complex, but attainable. Bell consciously planned to highlight *identity* by referring to his kindergartners as *mathematicians*. In addition, he prepared for a variety of interaction formats (e.g., think-pair-share, cooperative

small groups, whole-group discussions) to recognize and *empower* each student's voice in the classroom.

Implementing mathematical discussions that promote equity

The planning stage is crucial, but it is only part of the process and does not ensure that the learning experience is equitable or that each student's voice is represented during classroom discussions. In **table 1**, we provide an overview of Bell's actions during the implementation of the SSR lesson that align to Gutiérrez's (2013) four dimensions. Then we describe how Bell facilitated the SSR activity and how his pre-planned actions integrated equity throughout the launch, explore, and summarize phases.

Launch (5–10 minutes)

Bell began the lesson by showing pictures of slides from the school playground and from neighborhood parks that his students frequented with their families and friends. He asked his kindergartners to first put a "picture in their head" of a slide and then to turn and talk with their math buddy about what happens when they go down the slide. The think-pair-share strategy encouraged *achievement* and *power* by presenting opportunities for students to visualize the term *slide*, discuss vocabulary

with a partner, and share ideas with the whole group. During this time, Kourtne and Deklan used gestures to show how their bodies moved down the slide. Deklan excitedly shared with his partner, “It’s like rolling, but you don’t turn over! You just slide down all at the same time.”

Next Bell showed pictures of books and boxes stacked in cabinets around the classroom and school as well as photographs from local businesses—particularly restaurants and hotels—to demonstrate how chairs and dishes are stacked. He asked students what was happening in the pictures. After being shown a picture of chairs stacked at a local pizza place, Josef replied, “We can’t do that with chairs at our house. We tried to put them on top of each other when we got a new carpet, and they wouldn’t go.”

Bell used this opportunity to highlight the term *stack*: “Yes, sometimes the shape gets in the way of trying to stack things, like when your family tried to put the chairs on top of one another.”

After students presented their ideas on what it means to slide and stack, Bell shared a story about a former student whose soda cans kept rolling and sliding everywhere because his sister put the cans on the refrigerator shelf the “wrong way.” He held up a soda can and asked his students to predict why the cans were rolling and sliding.

Maurice and his partner discussed the different parts of the can: “See, this side is round, and it won’t stop rolling unless you block it good with something. But this side is more flatter, so you can just put the can like this—just don’t knock it over.”

These personal connections between the terms and the students’ funds of knowledge provided *access* and encouraged mathematical discourse.

As Bell transitioned to the exploration, he sorted the kindergartners into groups of four. He asked them to think about the shapes’ attributes and to predict whether the figures on the recording sheet (see **fig. 1**; a full-size recording sheet is available in the **online more4U** materials) would slide, stack, or roll. After a two-minute discussion, Bell selected someone from each group to share the group’s thinking as he recorded the predictions. EJ suggested that the cylinder “can stack and roll because it is the can shape, and we just talked about that one. The blocks do different things if you put them different ways.” Students’ predictions and justifications provided insight into their mathematical understanding.

Explore (10–15 minutes)

Bell gave each group a box containing the five geometric solids. As students explored the materials and determined whether the solids would slide, stack, or roll, Bell walked around the classroom and asked such guiding questions as, “How are the sphere and the cylinder similar? Do you think they will have the same results? Why?” These questions stimulated the kindergartners’ thinking about the properties and attributes of the solids and how these connected to their predictions. Bell also prompted his students to make connections to the real world as they explored the activity. For example, when one group was trying to determine if the solids would stack, he asked students to think about how different items like cereal boxes or cans of soup are placed on grocery store shelves. This example prompted students to consider the orientation of the solid and how that affects the results (e.g., cylinders positioned horizontally will roll, but when positioned vertically, they will stack). Bell helped students *access* the mathematical content by encouraging






FIGURE 1

A full-size Slide-Stack-Roll recording sheet is available as an online **more4U** student activity sheet.

→ “Sliding” into an Equitable Lesson” activity sheet

Name _____

Recording Sheet

		Will the shape SLIDE?	Will the shape STACK?	Will the shape ROLL?
Rectangular prism				
Cube				
Cone				
Cylinder				
Sphere				

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TABLE 2

Bell used the information in his recording sheet to decide which group would share during a whole-group discussion.

Example of Bell's recording sheet

Student's name	Student's strategy	Select to share (yes or no)?
James	Angle of ramp differences—when does it no longer roll or slide?	If time—extension
Jasmine	Threw the cube in the air to see if it rolled down the ramp when it landed	Yes
Casey	Placed the rectangular prism on floor and pushed it to see if it would slide or roll	No
Marshall	Held the cone in his hand and stacked the circular base on top of the one in his hand	Yes
Mapani	Placed the rectangular prism on the floor and pushed it to see if it would slide or roll	Yes

connections between students' experiences and the geometric solids.

As Bell monitored students' explorations, he overheard Rosie's group discussing whether or not a cone could stack. Rosie claimed that if you "put the flat circles of the cone together with one upside down," you can stack two cones. But she was unsure whether this would work for more than two cones. Bell encouraged group members to try it and see, and then he went over to James's group, which was investigating whether the angle of a ramp affects whether a cube will slide.

James: Hey, guys! Let's move the ramp higher and see what happens! [*Students moved the ramp and dropped the cube.*]

Robbie: It rolls!

De'Cretia: No, it didn't roll. It flipped over on its side.

As Bell circulated around the room, he recorded students' ideas. He used this information to decide which group would share during a whole-group discussion (see **table 2**). Bell briefly conferenced with the groups to identify who would report their discoveries. For example, he asked Marshall to share his group's insistence that each shape could be stacked if they balanced it properly, and he asked Mapani to discuss how his group got the prism to roll down the ramp by angling the ramp higher and pushing the block. Bell intentionally focused on students like Mapani, who does not normally raise his hand or offer suggestions in class, to give him a *powerful* voice in the mathematical discussion. Encouraging students to share their

mathematical discoveries with one another also nurtured their *identities* as mathematicians.

Summarize (15–25 minutes)

The students picked up materials and returned to the carpet for the summary, or math huddle. Bell strategically sequenced students' strategies to reflect a developmental direction—moving from concrete to more abstract and complex ideas. He asked Mapani's group to share their strategy of the rectangular prism sliding down the ramp, and then Bell asked the class to compare Mapani's group with Jamie's group, whose members had tossed the cube up to see whether it would slide down the ramp. Following each group's sharing, Bell posed questions and reiterated the focus on comparing the attributes of the solids and how those properties would make the shapes slide, stack, or roll. For example, Bell first had students compare their predictions and actual results of the rectangular prism and the cube. The kindergartners compared the shapes and made connections as to why the results were similar by discussing the solids' common attributes. For instance, Henry explained, "Both solids are almost exactly the same, but this one [a rectangular prism] is a lot longer, and they both have flat sides all over the block."

The students had a similar discussion about the cylinder and cone. When Marshall shared his group's cone-stacking success, Rosie excitedly described her group's similar experience and new hypothesis, "I can do four at most, but only if I hold two cones in each hand stacked up."

Because the sphere did not share common attributes with the other solids, Bell concluded the discussion by having students share their



By attending to the Four Dimensions of Equity (Achievement, Access, Identity, and Power) in designing and implementing mathematical activities, teachers create structures to support the success of each student.

thinking about the sphere's unique properties. He highlighted Victoria's question, "Does this ball shape roll or slide? I think it's both, but my group says no." Bell asked the kindergartners what they thought.

Bell recognized the importance of having a variety of students (e.g., race, gender, knowledge) contribute to the discussion because it helps students see themselves as mathematicians. Moreover, as students shared, Bell asked questions related to students' reasoning for why the solid would slide, stack, or roll:

- What do you think about [a student's] idea?
- How does [a student's] idea connect to your idea?
- How is [a student's] idea similar to [a different student's] idea?
- How are they different?

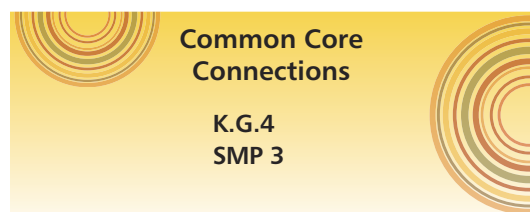
Bell concluded the lesson by asking his students to explore: "How could people in your family think about shapes when they organize boxes and cans of food in the kitchen?" This

open-ended question presents an opportunity for students with various experiences to continue mathematical conversations at home. Furthermore, these prompts stimulated critical thinking regarding the properties of geometric solids and the ways in which shapes are used in various contexts.

Summary in context of equity

Teachers encounter tensions when implementing equitable mathematics lessons like Slide, Stack, and Roll because they are neither simple nor innate (Gutiérrez 2009a; 2009b). For example, one tension that teachers have to embrace when teaching from an equity stance is "being in charge of the classroom and not being in charge of the classroom" (Gutiérrez 2009b, p. 12). Teachers must do what is necessary to encourage participation and give students a voice. Yet, students decide whether they will participate or let their voice be heard. The process of creating integrated experiences that provide opportunities for exploration, critical thinking, connections, and applications while also promoting the *identity* and *power* of diverse individuals is

a complex, long-term, multifaceted approach. It requires teachers to think holistically and purposefully as they plan and implement key tasks, activities, and questions intended to promote dialogic exchanges that benefit learners' mathematical reasoning and development in early mathematics. By explicitly considering how to incorporate Gutiérrez's (2013) Four Dimensions of Equity into lesson planning and implementation, teachers can create experiences that give all students opportunities to voice their thinking and contribute to the mathematical conversations occurring in the classroom.



REFERENCES

- Gutiérrez, Rochelle. 2002. "Enabling the Practice of Mathematics Teachers in Context: Toward a New Equity Research Agenda." *Mathematical Thinking and Learning* 4, no. 2–3 (July): 145–87.
- . 2009a. "Framing Equity: Helping Students 'Play the Game' and 'Change the Game.'" *Teaching for Excellence in Equity in Mathematics* 1, no. 1 (Fall): 4–8.
- . 2009b. "Embracing the Inherent Tensions in Teaching Mathematics from an Equity Stance." *Democracy and Education* 18, no. 3 (January): 9–16.
- . 2013. "The Sociopolitical Turn in Mathematics Education." *Journal for Research in Mathematics Education* 44, no. 1 (January): 37–68.
- Lemons-Smith, Shonda. 2008. "Dr. Asa G. Hilliard III: Trumpeter for the Academic and Cultural Excellence of African American Children." *Review of Educational Research* 78, no. 4 (December): 908–20.
- National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- . 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.



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On **January 9, 2019**, at **9:00 p.m. ET**, we will discuss "'Sliding' into an Equitable Lesson," by Kelley Buchheister, Christa Jackson, and Cynthia E. Taylor. Follow along using **#TCMchat**.

Unable to participate in the live chat? Follow us on Twitter **@TCM_at_NCTM** and watch for a link to the recap.



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