Measu Pen

PLESHKO74 (DOG); WOTERS (PAPER CLIPS)/GETTY IMAGES

Activities generated from a children's book can support

Mi Yeon Lee and
Dionne Cross Francis

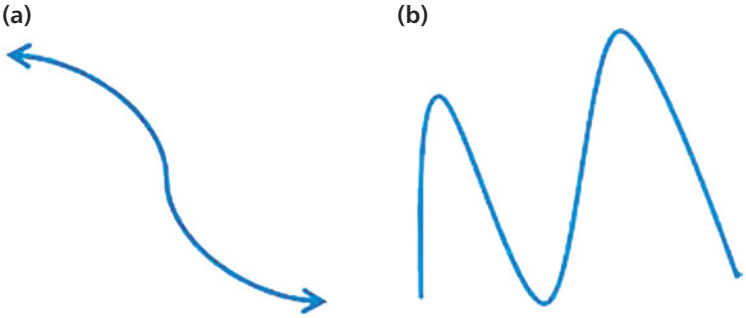
Children's literature can provide useful contexts for teaching mathematics. Books, stories, and poems can motivate students to think deeply about mathematical concepts by presenting rich contexts to explore these ideas in ways that connect with students' experiences (Cross et

youngsters in developing conceptions of measurement.

al. 2012; Clarke 2001). In this article, we discuss the use of the children's book *Measuring Penny* (Leedy 1997) to support Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) related to length measurement. We first describe three types of misconceptions about length that we observed in kindergarten students as they engaged in a measurement task. Then we describe activities generated from *Measuring Penny* that we used to support young children in developing measurement conceptions; specifically, that the length of an object can be found by laying multiple copies of shorter, equal-size objects end to end.

FIGURE 1

Here are two examples of nonstraight paths. (Adapted from Van de Walle, Karp, and Bay-Williams's (2013) Crooked Paths activity)



When asked to measure path B, which he thought looked "longer because it looks like an *M*," this kindergartner made an *M* with a piece of string.

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Students' difficulties in understanding conceptions of length

We engaged twenty-four kindergartners in a task-based interview prior to a five-day summer camp. Students were asked to compare the lengths of two nonstraight paths (see **fig. 1**) using paper clips, toothpicks, strings, or rulers. We observed three types of challenges in completing the task.

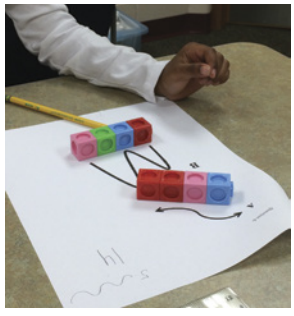
Visual comparison

First, students identified the longest path by casual visual comparison (Battista 2012) without seeing the need to measure the paths and compare them. Students' responses included these: "*A* looks longer because it goes from here to here [*putting their fingers on both ends of the shape*]" and "*B* is longer because it looks like an *M*." When asked to measure it, the latter student simply made an *M* with a piece of string next to path B. These examples show that the students seemed to be thinking of length as an arbitrary visual comparison relating to the object's shape rather than as a measureable attribute of objects. This challenge was noteworthy as CCSSM expects kindergarten students to be able to identify and describe length as a measurable attribute of an object and use this information to compare two objects, stating which is longer (K.MD.1 and K.MD.2).

FIGURE 2

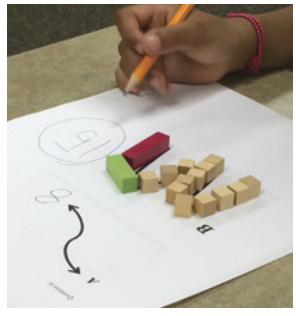
Students used nonstandard measurement tools in three inappropriate ways. They did not consider that—

(a) measuring the length of a path requires placing measurement tools on the path's actual curvature;



(b) it also requires iteratation of a same-size tool across the length of the path;

(c) and it requires calibration to add measurements from different units.



Length as a countable attribute

The second challenge involved students perceiving the length of the paths as the number of segments each path comprised, so they conceptualized length as a "countable" attribute determined by counting the different segments that make up the path. Students' explanations for selecting a path included, "*A* has three lines, and *B* has four lines" (see **fig. 1**) and "*B* is longer because it has more zigzags." These responses show that students considered length as counting a whole number of segments of the given paths (Battista 2012) without attending to same-size length units that span the paths with no gaps or overlaps (compare 1.MD.2).

Inappropriate tool use

The third challenge involved the use of measurement tools. After guessing the longest path, students were asked to measure the paths to check their guess. For the students who thought that a tool would be useful (not all the students saw the need to use a tool), we observed two interesting behaviors. First, children used inappropriate tools. For example, to measure paths A and B, students placed measurement tools that were longer or shorter than the actual curvy length of paths A and B (see **fig. 2a**). Second, children used two different tools to measure the same path without calibrating the tools (e.g., unit cubes and paper clips or different sizes of rods). Then they tried to compare the measures from different measurement tools by counting the number of unit cubes, paper clips, or rods (see **fig. 2b** and **fig. 2c**). These responses showed that the students did not seem to consider (1) that to compare the lengths, each segment of the two paths should be equally partitioned (compare 1.MD.2); (2) that to compare the lengths, they had to iterate the same-size measurement tool (unit cubes, paper clips, toothpicks, etc.) across the length of the path (compare 2.MD.1); or (3) that calibration is required to add measurements from different units (compare 2.MD.2) (Sarama and Clements 2009).

We designed activities based on *Measuring Penny* to support students in considering that (1) the length of an object as a measureable attribute (K.MD.1), (2) obtaining the length of an object requires iterating same-size, nonstandard units across the object without gaps or overlaps (1.MD.2), (3) measuring length by selecting and using appropriate tools (2.MD.1), and (4) the size of the measurement tool influences the actual length measurement (2.MD.2). In the following sections, we describe how we used the children's book and five activities to support students in developing the ideas about length measurement described above.

Five activities before measuring penny

In *Measuring Penny*, Lisa completes her measurement homework by measuring different parts (e.g., leg, ear) of her dog, Penny. Lisa compares Penny's size to the sizes of other dogs, saying that "Penny is a Boston terrier and is bigger than a pug and smaller than a cocker spaniel. She's about the size of a Shetland sheepdog"

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Students initially made conjectures by looking at life-size cutouts of the dogs. Then they decided to measure all the dogs to determine if Lisa was correct.

(p. 8). Using the story as a launching pad for the activity, we asked the class, "Is Lisa right?" Students initially made conjectures (see **fig. 3a**) by looking at life-size cutouts of the dogs. Then we made a chart tallying everyone's conjectures (see **fig. 3b**). Through discussion, students decided that they could measure all the dogs to determine if Lisa was right.

By referring to students' misconceptions, we designed five activities to build knowledge of length and measurement (see **table 1** in the online **more4U materials**). We did not design activities focused on calibration because we believe the concept is too advanced for kindergartners, but we tried to encourage students to think about the relationship between measurements and the size of the nonstandard unit chosen as a necessary foundation for understanding calibration.

Activity 1: Comparing objects

The first activity involved comparing a set of objects: a paper clip and a toothpick, a toothpick and a straw, and a straw and a pencil. Students put the objects side by side to determine which was longer or shorter. To encourage the class to consider actually measuring objects when comparing them, we asked students to compare equal-length objects in different positions. Specifically, we used two equal-size straws, one straw farther away than the other, and asked students, "Which one is longer, or are they the same length?" Some students answered that the one farther away was longer; others answered that they were same. When asked how they could determine the answer, all students lined up the two straws and stated they were equal. Students were able to apply this idea of "measuring to compare" when asked to draw dots equally spaced on one of the equal-size straws (see **fig. 4**). After they drew the dots, the teacher asked whose straw was longer. Students lined up the straws, saw they were equal and noted that the dots did not matter in determining which was longer.

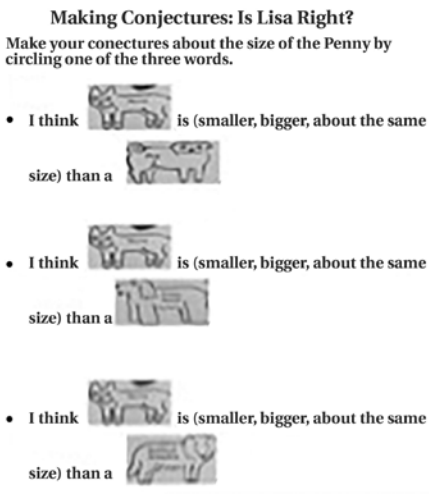
Activity 2: Ordering by length

The second activity involved ordering ten Cuisenaire® rods. Through trial and error (see **fig. 5a**) students were able to order the rods correctly, and some pairs of students combined their work so that it looked like a square (see **fig. 5b**). When asked, they described their approach as "lining them up" side by side from shortest to longest. To support students in comparing the rods, the teacher started

FIGURE 3

In *Measuring Penny*, Lisa completes her homework by measuring different parts of her dog, Penny, and comparing Penny's size to sizes of other dogs.

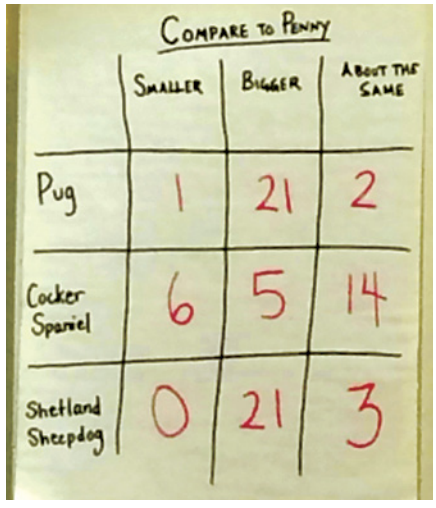
(a) On an activity sheet, students recorded their conjectures about Lisa's comparisons.


Making Conjectures: Is Lisa Right?

Make your conectures about the size of the Penny by circling one of the three words.

- I think is (smaller, bigger, about the same size) than a
- I think is (smaller, bigger, about the same size) than a
- I think is (smaller, bigger, about the same size) than a

(b) Then the class compiled a chart that tallied all the conjectures.



asking general questions (e.g., Can you tell me the relationship between the different rods?) and then specific questions (e.g., How many little cubes do we need to get to the next rod? How many little ones do we need to make the orange rod? Can you find other sets that have the same difference like orange and brown rods?). When asked these questions, some students aligned unit cubes between two rods to measure them (see **fig. 6a**). Others used unit cubes to measure each rod (see **fig. 6b**). Answering these questions seemed to help students think of using unit cubes for measurement.

Activity 3: Direct comparison

For the third activity, the Scavenger Hunt (Lee and Cross Francis 2016), the teacher gave students a straw and a toothpick as referent units and a piece of paper for recording their findings. Then the teacher asked them to directly compare the straw with objects they found and describe their findings using a comparison sentence. For example, object *x* is shorter than the straw (see Lee and Cross Francis 2016 for a full

FIGURE 4

The first activity was to compare sets of objects as well as equal-length objects in different positions. Students marked two straws of equal length with equally spaced dots and then aligned and compared them.

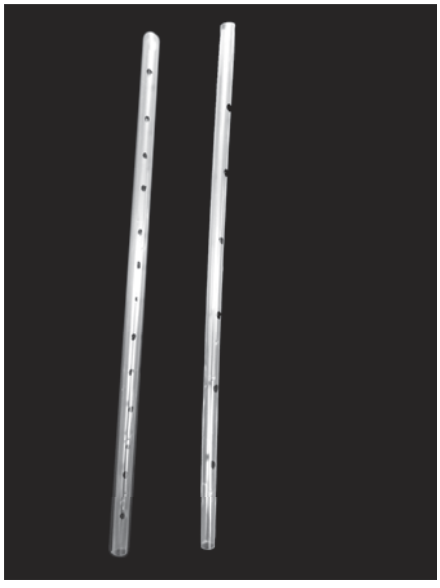


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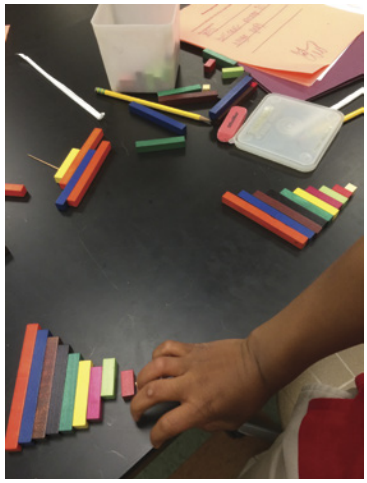
FIGURE 5

The second activity, ordering ten Cuisenaire rods, involved trial and error.

(a) Individuals ordered the rods correctly by "lining them up" side by side from shortest to longest.



(b) Some student pairs combined their work to form a square.

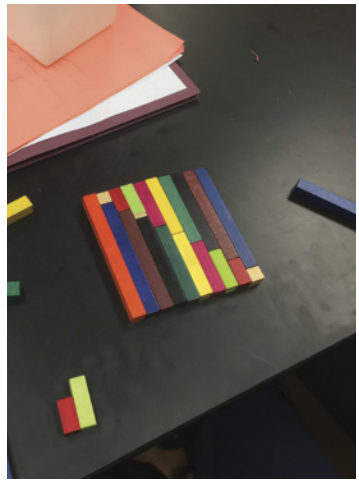
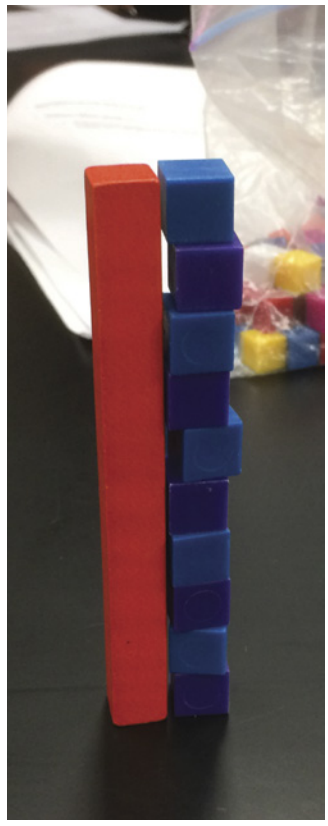


FIGURE 6

Answering the teacher's general and specific questions helped students consider how to use unit cubes for measurement.

(a) Some students measured two rods by placing small cubes between them.

(b) Other kindergartners used unit cubes to measure each rod separately.



description). Students were able to find a range of items in their classroom to compare with the straw or toothpick. As an extension, we asked students how much taller the straw was than the objects they found. Although this question was challenging for kindergartners to answer, it encouraged them to find ways of making more specific, quantifiable comparisons, thereby laying the foundation for developing measurement skills. Most students struggled to quantify their comparison, stating, "It's more bigger" or "It's a little bit bigger." One student stated, "My pencil is two toothpicks tall," explaining, "I borrow a toothpick and put two toothpicks next to my pencils." To estimate the length of his pencil, this student iterated the toothpicks, although the concept of iteration had not been explicitly taught. We encourage teachers to engage students with a range of comparison activities before working on measuring tasks.

Activity 4: Using nonstandard measurement tools

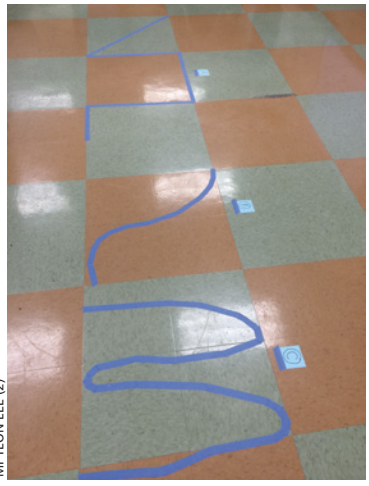
For the fourth activity, students worked in groups to measure large objects (e.g., a work table) in the classroom using two nonstandard units (e.g., straws, markers, strings, toothpicks, big paper clips). In this activity, by assigning large objects that were impossible to measure by direct comparison (unlike the previous activities), the teacher encouraged students to work on measurement using two nonstandard units as referent units. Before students measured, the teacher engaged them in considering how they might go about measuring the large object. In this whole-class discussion, they explored their different ideas and were able to discuss incorrect ways of measuring length (e.g., measuring with gaps or overlaps, measuring using two different units at once, measuring in a nonstraight manner).

Although each object could have different kinds of lengths to measure (e.g., vertical, horizontal, and diagonal length; perimeter), the class agreed to measure the vertical length of each object by using a consistent unit and putting the unit end to end, straight, and without any gaps or overlaps. Each group measured its assigned object twice using different nonstandard units. At the end of the activity, the class discussed how the two measurements related to the size of the nonstandard unit chosen. For example, when asked about the relationship between different units and their measurements, a student answered, "So many toothpicks [are] need[ed] because the toothpick is too small," and another student replied in a more general way, "If we have smaller units, we will have larger numbers [of measurement]."

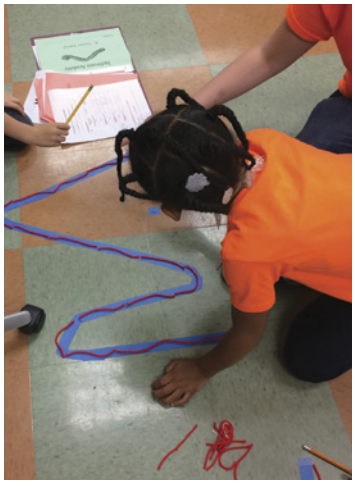
FIGURE 7

The fifth activity gave students a chance to consider how to measure paths that were not straight.

(a) Three paths were created with blue tape.



(b) Students measured path C with strings.



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Activity 5: Measuring nonstraight paths

The fifth activity was implemented by measuring three paths on the floor, which were marked with blue tape (Lee and Cross Francis 2016). This activity presented the opportunity for students to consider how to measure paths that were not straight. Before measuring the three paths (see **fig. 7a**), students were asked to estimate which path was the longest and why they thought so. Similar to the task-based interview, students guessed on the basis of vague visual comparison or by focusing on the countable parts that path A, which had five segments, would be longest. Unlike the interview, however, students concluded—on the basis of their experiences in previous activities—that they could determine the longest path by measuring and that they should use only one unit to measure. However, students had difficulty identifying an appropriate measurement tool that could be put end to end to

easily follow the curvy lines without going outside of them. When the teacher reminded students of how they indirectly compared straight objects using unit cubes or nonstandard units, students answered that they needed to line up the unit cubes along the objects. When the teacher asked which measurement tool would be good to put along the path, students suggested that straws would not work to measure paths B and C, but string or paper clips would be good because they can be used easily to follow the path.

After selecting the measurement tool, each group of students measured the three paths. They struggled with measuring the corners in path A or curved points in paths B and C. They also struggled with keeping the units end to end without gaps or overlaps because of the long extension of the paths. Despite the struggle, the activity served to help students consider useful ways of comparing lengths and the appropriateness of different measuring tools. However, we recommend using shorter curvy paths with early childhood students. Student groups were then tasked to visit other groups to check that units were used correctly to measure the paths (see **fig. 7b**).

The Measuring Penny activity

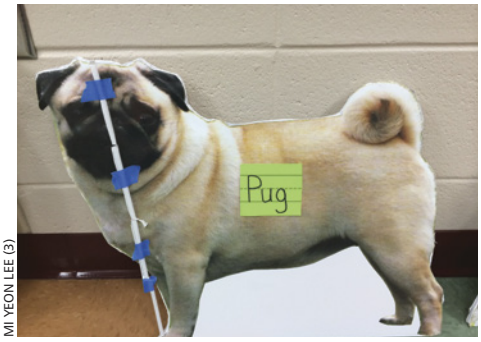
After implementing the five activities, the teacher revisited *Measuring Penny* and the question previously asked: "Is Penny the largest dog?" This question provides a good context for the learning activity by having students consider different kinds of measurements (e.g., weight, height, breadth, perimeter, etc.) that could help them answer the question. In everyday life, the weight of a dog is often used as an indicator of size (e.g., allowable size for airline travel, apartment pet restrictions). However, in this activity, students decided to measure the four dogs mentioned in the book to test their conjectures. To do so, they first discussed which part of the four dogs they wanted to measure, including "head to feet," (length up, or height), "foot to foot" (length across, or width) and "around the dog" (perimeter). From their experiences with the other activities, students were able to determine that dogs have different lengths that can be measured. For example, the suggestions of measuring the dogs "foot to foot" (horizontally) or "around the dog" were countered by most students, who suggested measuring the dogs "head to feet" (vertically). Advocates of "head-to-feet" measurement argued that the height of the dog is a good measure to determine size because "we [human beings] are measured by height." For a valuable, deeper discussion, provide cases of oddly shaped dogs (such as a dachshund, for example) and have students consider whether the height of the dog is the best measure for size. One way to stimulate such thinking would be to include a dog like a dachshund in the story—long body and short legs—to guide students in considering alternative measurements to height as an indicator of size.

Considering the affordances of the measurement tools these kindergartners had access to (e.g., smaller tools take more time), the students selected straws to measure the dogs because the length of a straw looked to be a medium size among all the measurement tools they had used. Each group was assigned to measure a dog, and the groups used different approaches. One group affixed the straws to the dog at a slant, using blue tape (see **fig. 8a**). Another group affixed the straws to the dog vertically, but the length of the straw was over the head (see **fig. 8b**). One group tried to follow the contours of the dog to measure

FIGURE 8

Some examples of ways that the kindergartners used straws to measure dogs included—

(a) using blue tape to affix straws to the cutout of the dog at a slant,



(b) affixing straws to the dog vertically but having the length of the straws exceed the dog's height,

(c) affixing straws following the dog's contours to measure from head to feet.

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length from head to feet (see **fig. 8c**). Teachers can support students in expanding these ideas in three key ways:

1. Introduce the concept of "more of"/"less of" (K.MD.2) to help students compare lengths of the dogs.
2. Allow students to deepen their thinking about their selection of the measurement tool, investigating what happens to the length of the object when they select a smaller or larger measurement tool to measure the same dog (2.MD.2).
3. Encourage students to consider what would be the most appropriate tools for measuring the perimeter of a photograph of a dog. That is, teachers can revisit the conversation they had with students about measuring curvy paths and draw connections to the bodies of the dogs. Because the dogs' perimenters are more curvy than straight, string is a more appropriate measurement tool. To determine its perimeter, young children can iterate short, same-size strings around the dog (see **fig. 7b**). Older students can use tape to place a long string along the contour of the dog and then measure the length of the string with a ruler.

A foundation for measurement skills

For young children, grasping the concept of length and developing measurement skills are important but difficult because they require having a sound understanding of attributes, units, and unit iteration as a foundation. In this article, we have demonstrated (1) how a children's story was used to create a meaningful context to engage students in learning about measurement concepts, and (2) what kinds of activities could be useful to promote students' understanding of length measurement. By implementing such activities, classroom teachers will be able to motivate young children to participate in inquiry experiences through which they learn about length.

Common Core Connections

1.MD.2
K.MD.1 2.MD.1
K.MD.2 2.MD.2

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Mi Yeon Lee, mlee115@asu.edu, is an assistant professor at Arizona State University in Tempe. Her research interests include K–grade 8 students' geometric and informal algebraic thinking as well as preservice and in-service teachers' understanding of students' reasoning. **Dionne Cross Francis, dicross@indiana.edu**, teaches elementary mathematics methods courses at Indiana University. She is interested in the development of teachers' mathematical content knowledge and the psychological constructs that influence teachers' classroom practices.

more4U

Go to **http://www.nctm.org/tcm** to find a table of five activities. Access to this online material is a members-only benefit.