


A smiling woman with short brown hair is holding a bright yellow book. She is wearing a black long-sleeved top and a grey skirt. The background is white with large, colorful, abstract shapes in shades of red, green, blue, and yellow. Overlaid on the image is the word "Cataly" in a large, light blue, cursive font.

Cataly

MIYAZAKI/GETTY IMAGES




A shared
responsibility for
teachers of all grades
can empower every
student as a capable
lifelong learner and
confident doer of
mathematics.

DeAnn Huinker

Envision for a moment that every student graduates from high school as a confident, capable mathematics learner. Furthermore, envision that students are not sorted into separate tracks and that all students graduate after successfully completing four years of rigorous mathematics instruction. Additionally, students are ready to make wise decisions in their personal lives and appreciate the beauty and usefulness of mathematics and statistics. These are some of the recommendations in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM 2018), which calls for fundamental changes in the way high school students experience mathematics.

Mathematics education at the high school level is not working for many students. Although we are seeing steady improvement in mathematics learning at the elementary and middle school levels across the United States, for the past decade, student achievement has remained essentially flat at the high school level (NCES 2015, 2016). One possible reason is that the shift at the elementary level to instructional pedagogies that build on student thinking and engage students in mathematical discourse have been slow to find their way into high school classrooms.

zing Change for Elementary School



We build the foundation in the elementary grades to launch students on their mathematical journeys. Therefore, it is important for us to keep an eye on the mathematical horizon for our students, be aware of future expectations, and work toward a consistent pre-K–12 vision of mathematics teaching and learning. The purposes of this article are to overview *Catalyzing Change* and examine implications for our professional work. I begin by listing the four major recommendations for high schools and then elaborate on key messages. These include a need for equitable instruction that fosters positive mathematical identities and agency as well as a need for equitable structures, such as elimination of tracking and creation of a common course pathway. I close with next steps for collaborative conversations and actions.

Four major recommendations

Catalyzing Change broadens the purpose of learning mathematics beyond college and career readiness and challenges us to create systems that open doors and mathematical opportunities for each and every student. Four specific recommendations frame the serious discussions necessary for strengthening high school mathematics.

1. To expand professional opportunities; be able to understand and critique the world; and experience the wonder, joy, and beauty of mathematics, each and every student should learn the Essential Concepts identified in *Catalyzing Change*.
2. High school mathematics should discontinue the practice of tracking students, as well as teachers, into qualitatively different or dead-end course pathways.
3. Mathematics instruction at the high school level should be consistent with research-informed and equitable teaching practices.
4. High schools should require four years of mathematics, including a common shared pathway for the initial two or three years, with all students studying the Essential Concepts.

These recommendations, although targeting high school, also have relevance for teaching, policies, and practices in elementary schools. For example, students in the early grades begin to develop beliefs about what it means to do mathematics and whether they have the capac-

ity to be successful, thus beginning to shape their own personal mathematical identities (Ashcraft 2002). We also know that the mathematical foundation built in the elementary grades is predictive of later success in mathematics (Duncan et al. 2007). Progress toward the recommendations must be viewed as a long-term process and must involve all stakeholders, including teachers and leaders at the elementary school level. As NCTM's former president Matt Larson stated in the book's preface, "We owe this effort not only to our students but also to ourselves as we work together to create and nurture the society we wish to inhabit" (NCTM 2018, p. xiii).

Creating equitable structures

Catalyzing Change draws attention to structures that produce opportunity gaps and lead to unacceptable disparities in learning outcomes for students. System structures are defined as school or district policies, practices, or conditions that support or impede student learning of mathematics. *Catalyzing Change* specifically calls for the dismantling of student and teacher tracking, the creation of a common initial course pathway for all students, and the need for students to study mathematics for all four years of high school.

Student tracking

Student tracking in high school mathematics is prevalent, yet much research shows that it does more harm than good (Oakes 1985) and that de-tracking leads to success for more students (Boaler and Staples 2008). Tracking occurs as students are sorted into fixed sequences of courses. Students in the "higher" track are viewed (and view themselves) as capable of being successful with cognitively demanding tasks and are often engaged in problem solving, reasoning, and deepening of conceptual understanding. In contrast, students in the "lower" track are viewed (and view themselves) as having less ability and tend to experience instruction that focuses on memorization and rote procedures. Students in different tracks receive qualitatively different experiences with mathematics, which heavily influences beliefs about their own mathematical abilities (Flores 2007). Some of the tracks may be terminal or dead-end pathways that do not prepare students for continued study of mathematics.

Student tracking is not unique to high school and often begins with ability grouping

in elementary school. Does your school sort students at a specific grade level on the basis of their perceived mathematical ability? Are students grouped for reading that defaults into ability grouping for mathematics? How do students' mathematical experiences vary across programs for general, special, and gifted education? Consider how grouping arrangements in your school might unintentionally impact mathematics learning opportunities, resulting in qualitatively different experiences and expectations, with some students viewed as more mathematically capable than others.

Another area to examine is math intervention programs. Interventions should focus on filling conceptual gaps in students' knowledge of mathematics, not on practicing memorized procedures. Students need deep understanding of mathematical ideas, structure, and connections as they move into the demands of high school mathematics. Mathematics instruction in intervention settings should be consistent with research-informed, equitable teaching practices and be in addition to the grade-level curriculum so students do not continue to fall further behind their peers.

Note that tracking differs from appropriate acceleration. A 2016 NCTM position statement is clear that acceleration should be for only a few students and that such opportunities must ensure that "no critical concepts are rushed or skipped." Mathematics learning is not a race, and students who speed through content are often the individuals who tend to drop out of mathematics when they have the chance (Seeley 2009).

Teacher tracking

We might recall from our own high school days that certain teachers, often the more experienced, were assigned to teach the "higher" tracks, whereas novice or the least experienced teachers were assigned to entry-level courses, usually ninth-grade algebra (Darling-Hammond 2007). *Catalyzing Change* calls for teaching assignments that are balanced to include both upper-level and entry-level mathematics courses. This approach deepens teachers' knowledge of the overall curriculum, lessens teacher isolation and burnout, and promotes a sense of collective responsibility for student learning (Strutchens, Quander, and Gutiérrez 2011). At the elementary level, how often do teachers in your school switch grade-level assignments? How might such shifts every few years strengthen one's own

teaching of mathematics and promote greater professional growth toward effective teaching and learning of mathematics?

Common initial course pathway

The recommendation is for all students to begin high school mathematics in a single, common pathway for two to three years. These courses would focus on the forty-one Essential Concepts identified in *Catalyzing Change*. The concepts represent the deep understandings that are most important for students to learn within the domains of number, algebra and functions, statistics and probability, and geometry and measurement. They are not another set of standards but rather serve as a refinement for focusing state, provincial, or district standards. Schools or districts will need to decide how best to distribute the Essential Concepts across the courses in the common pathway so that each student can achieve proficiency with all the concepts.

Four years of high school mathematics

The recommendation is for all students to study mathematics for four continuous years in high school. The initial common pathway creates an equitable structure for providing each student with a solid mathematical foundation. Students can then pursue additional courses, not on the basis of perceived ability but instead aligned with their goals, interests, and aspirations during the culminating portion of high school mathematics. Students with an interest in STEM (science-technology-engineering-mathematics) careers would likely study calculus; whereas students with interests in business, economics, humanities, or social sciences might study discrete mathematics, mathematical modeling, or statistical inference.

Mathematical identity and agency

Catalyzing Change repeatedly highlights the importance of engaging and empowering students as "doers of mathematics" (NCTM 2018, p. 25) because they derive mathematical identities from their experiences in learning mathematics. *Mathematical identity* is defined as—

the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives. (Aguirre, Mayfield-Ingram, and Martin 2013, p. 14)

TABLE 1

NCTM advocates for using the effective Mathematics Teaching Practices (NCTM 2014, p. 10) and equity-based practices (Aguirre, Mayfield-Ingram, and Martin 2013) across all grade levels.

Effective and equitable mathematics teaching practices

Effective teaching practices	Equity-based practices
1. Establish mathematics goals to focus learning.	Go deep with mathematics.
2. Implement tasks that promote reasoning and problem solving.	Leverage multiple mathematical competencies.
3. Use and connect mathematical representations.	Affirm mathematics learners' identities.
4. Facilitate meaningful mathematical discourse.	Challenge spaces of marginality.
5. Pose purposeful questions.	Draw on multiple resources of knowledge.
6. Build procedural fluency from conceptual understanding.	
7. Support productive struggle in learning mathematics.	
8. Elicit and use evidence of student thinking.	

We often think of mathematical agency as identity in action. Students with strong agency demonstrate confidence in their mathematical ability, believe they can make progress on challenging problems, and trust in their mathematical conclusions. These individuals have the capacity and willingness to engage mathematically.

One way to capture your students' emerging mathematical identities is to ask them to respond to the prompt, "What does it mean to be smart in mathematics?" The responses from students reveal the beginnings of both positive and negative identity formations. Their responses can provide insights for your work in strengthening or challenging current beliefs and for considering unintended impacts of instructional practices.

For example, some third-grade students with positive views wrote, "Being smart in math means you can solve math problems and have strategies to solve harder problems" and "Being smart in math means you know how to do many of the problems but not all of them, and you know how to defend your answer." Unfortunately, these encouraging types of responses are still too rare across all grade levels.

In contrast, other third graders wrote, "Being smart in math means that you know a lot of stuff that other people do not know" and "Being smart in math means you can answer problems quickly." Sadly, these students are forming beliefs that speed, rather than thoughtful sense making, is valued in mathematics classrooms. The mathematical

beliefs and identities that students form in the elementary grades follow them into high school. *Catalyzing Change* acknowledges that all teachers are identity makers and must proactively foster positive mathematical identities and agency through effective and equitable mathematics teaching.

Implementing equitable instruction

Across all grades, NCTM advocates for the use of the effective Mathematics Teaching Practices articulated in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) and the equity-based practices described by Aguirre, Mayfield-Ingram, and Martin (2013), listed in **table 1**. Together these practices form the basis for connecting research-informed teaching and the development of mathematical identity, agency, and competence. This connection is discussed further in the NCTM Taking Action series (Huinker and Bill 2017).

Catalyzing Change recommends mathematics instruction that actively engages students in solving cognitively demanding tasks by working collaboratively and using multiple representations, including technological tools to access, model, and solve problems. Students would have many opportunities to share and clarify ideas as well as make their mathematical reasoning and strategies visible for consideration and critique by other students. Teachers in such classrooms allow students to grapple with mathematical ideas, and they use purposeful

TABLE 2

The vision of equitable structures and teaching unites teachers from elementary school through high school. Here are some ideas to begin having productive conversations with your colleagues about *Catalyzing Change*.

Questions and actions for next steps

Reflection and discussion questions	Getting started
<p>How are we building positive mathematical identities in our students?</p> <p>Are some students receiving messages that they are not as capable in mathematics as other students?</p> <p>What can we do to ensure each and every student develops a positive disposition toward mathematics?</p>	<p>Do you want to know more about mathematical identity and agency?</p> <p>Start a book study on <i>The Impact of Identity in K–8 Mathematics</i>.</p> <p>Ask your students to reflect on what it means to be smart in mathematics and then hold an anonymous online chat.</p>
<p>Are we grouping or tracking students in ways that limit their mathematics options as they move forward in their education?</p> <p>What policies, procedures, or practices might we need to change?</p>	<p>Do you want to examine grouping practices?</p> <p>Watch a video on “Ability Grouping” by Jo Boaler (tinyurl.com/JoBoalerGrouping).</p> <p>Examine grouping practices in your classroom, grade level, or school; note differences in students’ mathematical experiences and expectations.</p>
<p>To what extent are teachers consistently using effective and equitable teaching practices?</p> <p>How can we support and hold each other accountable so that equitable teaching is the norm in our school?</p>	<p>Do you want to strengthen your instruction?</p> <p>Create a self-reflection tool from the teacher and student action lists for each teaching practice in <i>Principles to Actions</i>.</p> <p>Select a specific teaching practice and work with a peer-coach to strengthen that aspect of your teaching.</p>

questioning to assess students’ emerging thinking and to advance their understanding. Thus, students come to realize that their approaches and thinking serve an important role in learning mathematics. This vision of equitable and effective teaching is in stark contrast to mathematics classrooms in which students are passive recipients of information, with few opportunities to engage in discourse.

Furthermore, equitable teaching continually emphasizes the essential role of mathematical reasoning and sense making in developing deep and connected understanding of mathematical concepts and procedures. Teachers view students as mathematically capable and frequently leverage the strengths that students bring to the learning environment. This might involve tapping into students’ expertise and experiences, including prior mathematical understanding, culture, language, peers, family, or community. Throughout instruction, teachers seek ways to affirm and nurture positive mathematical identities and strong mathematical agency within their students.

Next steps

Catalyzing Change identifies issues worthy of consideration at all levels. The vision of equitable structures and teaching unites teachers from elementary school through high school. It provides us with a common agenda to come together with the aim to create consistent mathematical learning experiences for students. I encourage you to read *Catalyzing Change* and to have critical conversations with your colleagues about its recommendations, messages, and vision. An initial set of questions and actions are listed in **table 2**. You might also want to watch and discuss the recorded webinar by Matt Larson on *Catalyzing Change* (www.nctm.org/webinars/authortalks). It is our collective responsibility to ensure that each and every student has experiences throughout one’s prekindergarten through grade 12 education to develop deep mathematical understandings, a positive mathematical identity, strong agency, and a sense of competence and pride in one’s mathematical abilities.



Let's chat

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and readers about an important topic in our field.

On March 13, 2019, at 9:00 p.m. EDT, we will discuss "*Catalyzing Change for Elementary School*," by DeAnn Huinker. Follow along using #TCMchat.

Unable to participate in the live chat? Follow us on Twitter@TCM_at_NCTM and watch for a link to the recap.

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