Research Commentary

Working at the Boundaries of Mathematics Education and Statistics Education Communities of Practice

Randall E. Groth
Salisbury University

Statistics education has begun to mature as a discipline distinct from mathematics education, creating new perspectives on the teaching and learning of statistics. This commentary emphasizes the importance of coordinating perspectives from statistics education and mathematics education through boundary interactions between the two communities of practice. I argue that such interactions are particularly vital in shared problem spaces related to the teaching and learning of measurement, variability, and contextualized problems. Collaborative work within these shared problem spaces can contribute to the vitality of each discipline. Neglect of the shared problem spaces may contribute to insularity and have negative consequences for research and school curricula. Challenges of working at the boundaries are considered, and strategies for overcoming the challenges are proposed.

Key words: Boundary interactions; Communities of practice; Mathematics education; Statistics education

A great deal has been written about differences between mathematics and statistics. Although statistics makes extensive use of mathematics, the two disciplines have different origins, foundational questions, and standards (Moore, 1988). The general nature of discourse within each discipline differs. Much mathematical discourse is grounded in deductive reasoning and language of definitive proof, whereas statistical discourse is often characterized by inductive reasoning and qualified conclusions (delMas, 2004; Rossman, Chance, & Medina, 2006). Extensive focus on variability and the synthesis of statistical and contextual knowledge are hallmarks of statistical practice (Cobb & Moore, 1997; Wild & Pfannkuch, 1999). Such differences between statistics and mathematics have naturally led those in each discipline to establish their own academic departments, journals, conferences, and professional organizations.

In a similar vein, statistics education has begun to mature as a discipline distinct from mathematics education. This maturation has been accelerated by increased prominence of statistics in pre-K–12 curricula internationally (Shaughnessy, 2007). Jones and Tarr (2010) noted, “Over the past century, statistics and probability have moved from relative obscurity in the mathematics curriculum to important, fundamental topics that should be studied by all students at each grade level” (p. 73). Although statistics is customarily taught as

part of the mathematics curriculum, the emerging statistics education community does not consist solely of mathematics education researchers. Statistics education research has also been published by those specializing in the fields of psychology, science education, educational technology, and statistics (Garfield & Ben-Zvi, 2008). Such diversity of contribution often strengthens a discipline, but it can also be an impediment because research is reported in so many different venues. Garfield and Ben-Zvi (2008) observed, "For many people interested in reading this area of scholarship, statistics education can seem to be an invisible, fragmented discipline" (p. 21).

Significant "de-fragmentation" of those involved in statistics education research has occurred over the past two decades. The discipline of statistics education has developed the three dimensions of a community of practice described by Wenger (1998): creation of contexts for mutual engagement, negotiation of a joint enterprise, and accumulation of a shared repertoire of resources. Mutual engagement has been fostered through the creation of statistics education specific conferences (e.g., International Conference on Teaching Statistics; United States Conference on Teaching Statistics; International Collaboration for Research on Statistical Reasoning, Thinking, and Literacy), journals (e.g., *Journal of Statistics Education, Statistics Education Research Journal, Teaching Statistics, Technology Innovations in Statistics Education*), and professional organizations (e.g., International Association for Statistical Education and Consortium for the Advancement of Undergraduate Statistics Education). These forums for mutual engagement have helped foster the negotiation of joint enterprises such as the negotiation of theory specific to research on teaching and learning statistics (Ben-Zvi & Garfield, 2004), the formation of curriculum documents devoted exclusively to teaching statistics in schools (Franklin et al., 2007; Garfield et al., 2005), and the design of doctoral programs in statistics education that are not necessarily tracks within mathematics education (Garfield, Pantula, Pearl, & Utts, 2009). The products of these joint enterprises have become part of the shared repertoire of resources for statistics education and continue to help the discipline grow.

Although the evolution of a statistics education community of practice can be viewed as a positive development for research on the teaching and learning of statistics, there is also a danger that the disciplines of statistics education and mathematics education may become increasingly insular and noncommunicative with each other. Sustained boundary interactions are vital to preventing insularity from contributing to the stagnation of interrelated communities of practice (Wenger, 2000). When boundary interactions occur, borders between disciplines can become exciting sites for learning rather than prohibitive barriers (Wenger, 1998). One way to encourage productive boundary interactions is for those working in each discipline to recognize shared problem spaces (Akkerman & Bakker, 2011). The purpose of this commentary is to identify potential shared problem spaces for mathematics education and statistics education and to consider how interactions within these problem spaces can contribute to the vitality of each discipline.

Shared Problem Spaces for Mathematics Education and Statistics Education

This research commentary was written from my perspective as a mathematics educator who became intrigued with issues in statistics education while working within a number of shared problem spaces. Working within those problem spaces brought to light the complex nature of interacting with the two disciplines simultaneously. I believe it is important for mathematics educators to appreciate this complexity because the discipline of mathematics education still maintains a great deal of stewardship over the teaching and learning of statistics and over research in statistics education. Problem spaces that are shared among mathematics education, statistics education, and related disciplines provide ideal sites for envisioning how the disciplines can interact in mutually beneficial ways.

Many shared problem spaces for mathematics education and statistics education can be identified. My intent is not to be exhaustive in discussing all such potential spaces but to focus on two in particular that appear to be ripe for investigation: (a) the treatment of measurement and variability in the *Common Core State Standards* (CCSSM) in the United States (National Governors Association for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) and (b) the use of contextualized problems in mathematics and statistics curricula. My aim is to discuss these examples in enough detail to conceptualize the work needed in each case. In doing so, I also hope to catalyze readers' thinking in regard to identifying other high-priority shared problem spaces.

Measurement and Variablility in the Common Core

Measurement is an important concept in both mathematics and statistics. In mathematics classes, students are frequently taught to measure attributes such as length, area, volume, weight, and time. Although these types of measurements are frequently used in statistics as well, measurement in statistical practice also includes cases in which there is not widespread agreement about how an attribute should be measured (Rossman et al., 2006). Such cases arise early in children's everyday experiences. For example, they may make judgments about who in their class is best in mathematics on the basis of grades given by the teacher, even though these represent just one (often flawed) measure of mathematical ability. Curriculum documents represent conflicting views on the extent to which children should engage in critically examining measurement protocols. For instance, the Measurement and Data domain of the CCSSM mentions only situations having well-defined measurement protocols in the standards preceding high school. High school is when students are first to "encounter novel situations in which they themselves must conceive the attributes of interest" (NGA & CCSSO, 2010, p. 58), such as deciding how to measure the attribute of highway safety. In contrast, in the *Guidelines for Assessment and Instruction in Statistics Education* (*GAISE*) *Report*, Franklin et al. (2007) recommend that definitions of measures be examined in some of students' earliest learning experiences with statistics. For example,

if children are examining the lengths of words on a page, the GAISE report explicitly recommends that they discuss the definition of *word* (e.g., Is a number on the page considered a *word*?). Without such discussion, different notions of how to measure an attribute may go unnoticed and covertly influence the conclusions students draw from data.

The CCSSM and GAISE documents also represent two different views of variability in the K–12 curriculum. The GAISE report proposes a learning trajectory designed to help students develop understanding of variability. It suggests that children's first experiences with variability in the early years of school encompass measurement, natural, and induced variability. These experiences are to provide a basis for later developing understanding of sampling variability and then chance variability. In the CCSSM, variability is not explicitly mentioned as an object of study until Grade 6. In Grade 6, no explicit recommendations are made about which types of variability students should study. Instead, focus is placed on using the mathematical measures of interquartile range and mean absolute deviation to summarize variability with a single number. Hence, though CCSSM and GAISE both offer guidelines for the teaching of "variability," mathematical quantifications take precedence in the CCSSM, whereas understanding different types of variability one may encounter in various contexts is emphasized in GAISE.

The differences between the CCSSM and GAISE approaches to measurement and variability appear to be partially motivated by a desire to shed the "mile-wide and inch-deep" label (Schmidt, McKnight, & Raizen, 1997) given to U.S. mathematics curricula. Limiting the number of statistics curriculum objectives in the earlier grades was one strategy employed in the CCSSM to attain this goal. The first page of the CCSSM cites Ginsburg and Leinwand's (2009) observation that mathematics curriculum standards in higher achieving countries include less emphasis on data analysis in the early grades in favor of more attention to number, measurement, and geometry. It would seem, then, that there is an impasse on the issue of how to deal with measurement and variability in the early grades. Adding more statistical curriculum objectives to the early grades would defeat the purpose of streamlining mathematics curricula, but omitting the objectives leaves concerns from the statistics education community unresolved.

When interrelated communities of practice reach an impasse, creating *boundary objects* (Star & Greismer, 1989) can be helpful. Boundary objects allow groups to operate collectively without consensus. In the case of variability, there is a lack of agreement on the nature and placement of curriculum objectives. It also seems unlikely that agreement will be reached anytime soon. However, it may be possible to create curriculum sequences that address parts of the GAISE guidelines about variability without adding more curriculum objectives to the CCSSM. For example, in one sequence of lessons (Groth, in press), I taught elementary school students how to create line plots (a CCSSM objective) by having them gather data from measurement, natural, and induced variability contexts (types of variability recommended for study by GAISE). Attending carefully to the contexts for data gathering did not take extra instructional time or require adding objectives to the

curriculum. This sort of lesson sequence can be considered a boundary object because it addresses concerns of the mathematics education community and the statistics education community without requiring consensus on objectives to be included in the CCSSM. Collaborations between mathematics education and statistics education communities of practice could result in the production of additional lesson sequences of this nature to embed in mathematics curricula.

The lack of consensus about how measurement should be taught in elementary school curricula may prove to be a more intractable problem for mathematics education and statistics education communities of practice to resolve. The GAISE guidelines recommend that nonmathematical aspects of measurement be addressed, potentially making collaborative work between mathematics education and statistics education more difficult. In such cases, mathematics educators and statistics educators might consider boundary interactions with other related communities of practice. In the case of measurement, there are natural connections with science education. Lehrer and Schauble (2007) described how a data modeling approach to science lessons for elementary students addressed nonmathematical issues of measurement while also meeting learning objectives from science. In one classroom they studied, fifth-grade students constructively argued about how to measure plant heights. Some favored measuring from the bottom of the pot to account for roots, while others wanted to measure from the top of the soil. As the plants grew, students had to decide on a fair way to measure bent stems. In essence, students were thinking deeply about measurement protocols rather than simply executing a protocol prescribed by the teacher. Hence, boundary interactions among mathematics education, statistics education, and other disciplines devoted to content-specific teaching and learning would seem to be a natural means for ensuring that expectations for younger students outlined in GAISE are met in meaningful ways, even if those expectations are not addressed in mathematics classes.

Contextualized Problems

It is recommended that students study contextualized problems in both mathematics education and statistics education curriculum documents (Franklin et al., 2007; National Council of Teachers of Mathematics, 2000). Because the importance of such problems is well established in both the mathematics education and statistics education communities of practice, the notion of a contextualized problem presents another opportunity for boundary interactions between the two communities. As noted earlier, the role that context plays in mathematics often differs from the role it plays in statistics (Cobb & Moore, 1997). Hence, in considering potential boundary interactions, it is important to unpack what mathematics educators and statistics educators may mean in recommending that contextualized problems be included in the curriculum. Toward this end, it can be helpful to compare the idea of mathematization (Freudenthal, 1991) to that of transnumeration (Wild & Pfannkuch, 1999). The former idea has been influential in mathematics education, and the latter has been influential in statistics education.

Two types of mathematization can be identified: horizontal and vertical (Freudenthal, 1991; Treffers, 1987). The two types have been characterized in the following terms:

> In the case of horizontal mathematizing, mathematical tools are brought forward and used to organize and solve a problem situated in daily life. Vertical mathematizing, on the contrary, stands for all kinds of re-organizations and operations done by the students within the mathematical system itself. (Van den Heuvel-Panhuizen, 2003, p. 12)

The *Mathematics in Context* curriculum provides an instructive example of how both types of mathematization may be encouraged among students. In one curriculum module, "Insights into Data" (Wijers, de Lange, Shafer, & Burrill, 1998), a great deal of horizontal mathematization is encouraged as students are asked to perform tasks such as measuring the typical income in a data set and interpreting the meanings of points on a scatter plot in terms of their context. Each lesson in the module ends with generalizations about the uses of mathematical and statistical tools introduced during the lesson. These generalizations are not meant to be bound to one specific context, but they are powerful enough to cut across a variety of similar contexts. Hence, they reflect vertical mathematization in the sense that students are asked to think beyond the specific contexts presented in the lesson to consider the general structures of the mathematical and statistical tools themselves.

The term *transnumeration* was coined to speak specifically about the role that context plays in empirical statistical inquiry. Transnumeration can be characterized as follows:

> It pervades all statistical data analysis, occurring every time we change our way of looking at the data in the hope that this will convey new meaning to us. We may look through many graphical representations to find several really informative ones. . . . We might try a variety of statistical models. And at the end of the process, transnumeration happens yet again when we discover data representations that help convey our new understandings about the real system to others. (Wild & Pfannkuch, 1999, p. 227)

Shaughnessy and Pfannkuch (2002) described a lesson designed to encourage students to engage in transnumeration-type activities. The main task for the lesson was to analyze data sets showing the numbers of minutes between eruptions of the Old Faithful geyser. Students were asked, "If we had some friends who were planning to visit Yellowstone National Park, how long should we tell them to expect to wait between eruptions of Old Faithful?" (Shaughnessy & Pfannkuch, 2002, p. 253). This question prompted students to produce a variety of graphs and statistics to make sense of the data. Some constructed box plots to show the typical wait time and the variation among wait times in the data. Some of the most revealing representations were plots of consecutive wait times, which showed an oscillating pattern not apparent in a box plot. As students shared their representations with one another, they noticed how different representations illuminated hidden patterns within the data.

At first glance, one might argue that the students in Shaughnessy and Pfannkuch's (2002) study were simply engaging in horizontal mathematization, making the distinction between mathematization and transnumeration trivial. Rather than entering into a lengthy debate about the technical meanings of the terms (though such a debate might also prove fruitful), I draw attention toward the difference between how the ideas of mathematization and transnumeration influenced the instructional goals in the examples. In the *Mathematics in Context* module, students engage in horizontal mathematization with the goal of progressing to vertical mathematization at the end of each lesson. Thus, the specific contexts in the module were ultimately intended to help reveal general characteristics of mathematical and statistical tools that could be used across a variety of similar contexts. In Shaughnessy and Pfannkuch's lesson, context was included in a fundamentally different way. An explicit goal of the lesson was to build knowledge of the Old Faithful context itself rather than to use the context as a launching pad for vertical mathematization. Shaughnessy and Pfannkuch's lesson was also intended to give students experience integrating contextual and statistical knowledge, again with the end goal of marshaling statistics in service of understanding the Old Faithful geyser (as opposed to using the Old Faithful context as a backdrop for introducing tools such as box plots and histograms).

Although the extent to which the *Mathematics in Context* and Old Faithful examples reflect the intended meanings of mathematization and transnumeration, respectively, is debatable, the two do exemplify how different communities of practice have appropriated the ideas. In the former example, the curriculum incorporates mathematization as a guiding framework, and the latter incorporates transnumeration. In the former, context is used in the service of providing a setting for introducing mathematical and statistical generalizations, and in the latter, context is introduced for the sake of using mathematical and statistical tools to make sense of the given context. The former resembles how mathematicians engaging in pure mathematics may view context in relation to their work, and the latter resembles how statisticians working in consulting roles may view context in relation to their work. Students are led toward different views of the role of context in each example.

In pointing out differences between the mathematization and transnumeration examples, I do not mean to imply that one is superior to the other. Instead, I posit that it is essential for students to have opportunities to reason in the ways reflected in each example. Such opportunities can broaden students' perspectives on the types of thinking done by mathematicians and statisticians. Furthermore, it seems unlikely that either type of activity, by itself, could be adequate for a school curriculum. If students do not have opportunities to make generalizations about statistical tools that cut across various contexts (i.e., vertical mathematization), it seems unlikely that they would have an adequate repertoire of statistical tools with which to draw insightful observations about a given context. Additionally, if students do not have opportunities to use statistics primarily for the sake of building knowledge of a context (i.e., transnumeration), it seems unlikely that they

would appreciate how statistics can be used for the sake of making sense of the world around them and that statistical tools are not always just studied for their own sakes. Hence, mathematization-type activities and transnumeration-type activities appear to have an interdependent relationship in school curricula, as depicted in Figure 1.

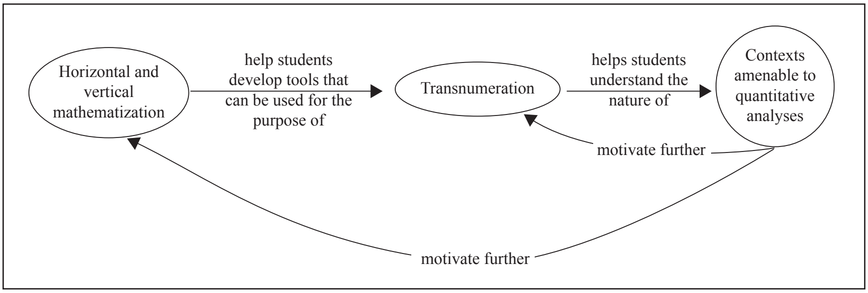

Figure 1. Hypothesized relationship between mathematization and transnumeration. To navigate the figure, begin by reading any node, and then follow the connecting arrow to the next node. The text in the first node is the beginning of a sentence, the text on the connecting arrow is the middle of a sentence, and the text on the second node is the end of a sentence.

The relationship between mathematization and transnumeration provides a space for discourse between mathematics educators and statistics educators about a common concern: the teaching and learning of modeling. Modeling can be described in the following terms:

> Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. (NGA & CCSSO, 2010, p. 72)

Modeling is valuable as "a means of giving children a sense of agency through recognizing the potential of mathematics as a critical tool for analysis of issues important in their lives" (Greer, Verschaffel, & Mukhopadyay, 2007, p. 89). Authentic modeling problems usually have multiple viable solution strategies and solutions. Mathematization is essential for helping students develop an understanding of the range of tools available for this enterprise and the mathematical structure of each tool. Transnumerative thinking is essential in judging the relative usefulness of different tools in constructing an overall model to understand phenomena. Coordinating the perspectives of mathematization and transnumeration, therefore, has the potential to help in the production of curricula that resonate with ways in which modeling is employed in practice.

Neglecting shared problem spaces related to mathematization and transnumeration comes with risk. I argue that some parts of the CCSSM illustrate this

point. Statistics is included in the standards, but a large part of its role, particularly in the elementary grades, is to help students develop knowledge of mathematics rather than to use statistics to generate knowledge about a context of interest. In kindergarten through Grade 5, the study of statistics is folded into the domain of Measurement and Data. Within the domain for these grade levels, students are to develop mathematical skills such as describing and comparing measurable attributes, classifying objects, and counting the number of objects in each category. Learning standards are narrowly focused on producing specific types of graphs to solve a few specific types of mathematical problems, as in the third-grade standard: "Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph" (NGA & CCSSO, 2010, p. 20). This learning objective does not address transnumeration because it, along with other learning expectations for the elementary grades, does not explicitly mention encouraging students to use various graphs to make sense of contexts of interest. The decision not to emphasize this sort of transnumerative thinking is questionable, given that researchers have provided cases of how such thinking might be accomplished in the elementary grades (Konold, 2002; Paparistodemou & Meletiou-Mavrotheris, 2008; Watson, 2006). A risk of delaying transnumerative-type thinking until the later grades is that students may develop an entrenched view, early on, of statistics as a set of tools with little use for generating knowledge about contexts of interest.

Several strategies might be used to deal with the impasse over transnumeration-type learning experiences. As discussed earlier, boundary interactions between mathematics educators and statistics educators that focus on coordinating transnumeration and mathematization perspectives could be helpful. Such interactions might focus on integrating transnumeration-type lesson sequences into mathematics curricula in a manner that does not require the addition of extra learning objectives or instructional time. Boundary interactions among mathematics educators, statistics educators, and those within other disciplines devoted to content area teaching and learning could also prove to be fruitful. For example, social studies educators have an interest in using data to make sense of phenomena. The National Council for the Social Studies (2002) wrote,

> Teachers of the middle grades can help learners relate their personal experiences to happenings in other environmental contexts. They can provide learning experiences which encourage increasingly abstract thought as learners use data and apply skills in analyzing human behavior in relation to its physical and cultural environments. (p. 23)

Similarly, the importance of using data to make sense of scientific phenomena, beginning in kindergarten, is expressed in the *Next Generation Science Standards* (Next Generation Science Standards Lead States, 2013). Hence, as in the case of measurement and variability, boundary interactions between mathematics education and statistics education can be supplemented with boundary interactions among other disciplines devoted to content area teaching and learning.

Potential Challenges of Working at the Boundaries

We are currently at an exciting and pivotal moment in the relationship between mathematics education and statistics education. Boundary interactions between the two disciplines have the potential to be invigorating, whereas neglecting critical shared problem spaces has negative consequences. In cases where boundary interactions between mathematics education and statistics education do not successfully address impasses, supplementary boundary interactions with other related disciplines are desirable. In suggesting boundary interactions within shared problem spaces, however, I do not mean to imply that such interactions constitute a panacea. On the contrary, boundary interactions themselves are fraught with challenges. To conclude, I attempt to anticipate some of those challenges. Just as anticipating students' difficulties when planning a mathematics lesson can prepare teachers to plan for potential trouble spots, anticipating challenges during boundary interactions may help those working within each discipline to do the same.

Stimulation and support of meaningful discourse about boundary objects will require involvement from individuals Wenger (1998) referred to as "brokers." Brokers are those who participate in each community of practice. This article represents an attempt at brokering by encouraging those involved in each community of practice to discuss the roles of measurement, variability, and contextualized problems within school curricula and research. Brokering, however, can be a difficult enterprise. Wenger noted that being a broker entails staying at the boundaries of a practice rather than moving to its core. Because of this, brokers may constantly feel uprooted. The value of their contributions may also be difficult to assess because they maintain a degree of distance from the core of each community of practice. Wenger (1998) recommended that brokers "recognize one another, seek companionship, and perhaps develop shared practices around the enterprise of brokering" (p. 110). Some of this interaction among brokers can occur within forums for statistics education that already exist within mathematics education conferences and journals. However, to more fully recognize brokering as an enterprise that occurs on the boundaries, this type of interaction could also be encouraged by establishing and expanding forums specifically for mutual engagement among brokers, such as the American Statistical Association and National Council of Teachers of Mathematics Joint Committee on Curriculum in Statistics and Probability. Such forums could increase the profile of brokering and emphasize it as a valuable contribution to multiple communities of practice.

As brokers work to produce school curricula within shared problem spaces, they need to be mindful of the distinction among written, intended, and enacted curriculum (Stein, Remillard, & Smith, 2007). The written curriculum exists in print, the intended curriculum exists in teachers' instructional plans, and the enacted curriculum is what students actually experience in the classroom. Teachers tend to implement curricula with varying levels of fidelity to the intentions of the authors of the materials (Remillard, 2005). This phenomenon may be especially pronounced in the case of curricula produced at the boundaries of

mathematics and statistics education because teachers are not always familiar with distinctions between the two disciplines (Burrill & Biehler, 2011). In many cases, mathematics teachers exhibit the same types of problematic statistical reasoning patterns as their students (Groth, 2007). Hence, it seems imperative that curriculum development not end with the production of written curriculum. Providing support for teachers as they form the intended curriculum and enact it could help ensure that the intended spirit of the curriculum materials produced during boundary interactions is not lost. Additionally, as curriculum writers interact with teachers, they may find that some adaptations teachers make to the written curriculum help to improve it (Morris, 2012).

Finally, it is important for communities of practice to maintain their distinctiveness as boundary interactions occur (Akkerman & Bakker, 2011; Wenger, 2000). The goal is not to attain homogeneity between mathematics education and statistics education. On the contrary, the potential boundary learning opportunities I have identified depend on interactions between distinct disciplinary perspectives. The fact that these interactions may at times be contentious only adds to the potential richness of boundary learning. Disagreements can be seen in a positive light insofar as they prompt those within a given discipline to reflect on their professional beliefs and practices and refine them. In the case of mathematics and statistics education, conflict may also lead to the development of new and stronger connections with related disciplines such as science education and social studies education. Ultimately, sustained boundary interactions between mathematics education and statistics education may contribute not only to the vitality of the two disciplines but also to the strength of the constellation of disciplines devoted to teaching and learning within which they are situated.

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169. doi:10.3102/0034654311404435

Ben-Zvi, D., & Garfield, J. (2004). Statistical literacy, reasoning, and thinking: Goals, definitions, and challenges. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 3–15). Dordrecht, the Netherlands: Kluwer.

Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education* (pp. 57–70). Dordrecht, the Netherlands: Springer.

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *American Mathematical Monthly, 104*(9), 801–823. doi:10.2307/2975286

delMas, R. C. (2004). A comparison of mathematical and statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 79–95). Dordrecht, the Netherlands: Kluwer.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007, August). *Guidelines for assessment and instruction in statistics education* (*GAISE*) *report*. Alexandria, VA: American Statistical Association. Retrieved from http://www.amstat.org/education/gaise/GAISEPreK-12_Full.pdf

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht, the Netherlands: Kluwer.

Garfield, J., Aliaga, M., Cobb, G., Cuff, C., Gould, R., Lock, R., . . . Witmer, J. (2005). *Guidelines for assessment and instruction in statistics education: College report*. Alexandria, VA: American Statistical Association.

Garfield, J. B., & Ben-Zvi, D. (2008). Research on teaching and learning statistics. In J. B. Garfield & D. Ben-Zvi (Eds.), *Developing students' statistical reasoning: Connecting research and practice* (pp. 21–43). New York, NY: Springer.

Garfield, J., Pantula, S., Pearl, D., & Utts, J. (2009, March). *Statistics education graduate programs: Report on a workshop funded by an ASA member initiative grant*. Retrieved from http://www.causeweb.org/research/programs/statedgradprogs.pdf

Ginsburg, A., & Leinwand, S. (with Decker, K.) (2009, December). *Informing grades 1–6 mathematics standards development: What can be learned from high-performing Hong Kong, Korea, and Singapore?* Washington, DC: American Institutes for Research. Retrieved from http://www.air.org/sites/default/files/downloads/report/MathStandards_0.pdf

Greer, B., Verschaffel, L., & Mukhopadhyay, S. (2007). Modelling for life: Mathematics and children's experience. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI Study* (pp. 89–98). New York, NY: Springer.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education, 38*(5), 427–437.

Groth, R. E. (in press). Royalty, racing, rolling pigs, and statistical variability. *Teaching Children Mathematics*.

Jones, D., & Tarr, J. E. (2010). Recommendations for statistics and probability in school mathematics over the past century. In B. J. Reys & R. E. Reys (Eds.), *Mathematics curriculum issues, trends, and future directions: 72nd yearbook* (pp. 65–75). Reston, VA: National Council of Teachers of Mathematics.

Konold, C. (2002). Teaching concepts rather than conventions. *New England Journal of Mathematics, 34*(2), 69–81.

Lehrer, R., & Schauble, L. (2007). Contrasting emerging conceptions of distribution in contexts of error and natural variation. In M. C. Lovett & P. Shah (Eds.), *Thinking with data* (pp. 149–176). Mahwah, NJ: Lawrence Erlbaum Associates.

Moore, D. S. (1988). Should mathematicians teach statistics? *College Mathematics Journal*, *19*(1), 3–7.

Morris, A. K. (2012). Using "lack of fidelity" to improve teaching. *Mathematics Teacher Educator, 1*(1), 71–101. Retrieved from http://www.nctm.org/publications/article.aspx?id=33989

National Council for the Social Studies. (2002). *National standards for social studies teachers*. Silver Spring, MD: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Next Generation Science Standards Lead States. (2013). *Next generation science standards: For states, by states*. Washington, DC: National Academies Press.

Paparistodemou, E., & Meletiou-Mavrotheris, M. (2008). Developing young students' informal inference skills in data analysis. *Statistics Education Research Journal, 7*(2), 83–106. Retrieved from https://www.stat.auckland.ac.nz/~iase/serj/SERJ7%282%29_Paparistodemou.pdf

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246. doi:10.3102/00346543075002211

Rossman, A., Chance, B., & Medina, E. (2006). Some important comparisons between statistics and mathematics and why teachers should care. In G. F. Burrill & P. C. Elliot (Eds.), *Thinking and reasoning with data and chance: 68th yearbook* (pp. 323–333). Reston, VA: National Council of Teachers of Mathematics.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision: An investigation of U.S. science and mathematics education*. Boston, MA: Kluwer Academic Press.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 957–1009). Charlotte, NC: Information Age Publishing.

Shaughnessy, J. M., & Pfannkuch, M. (2002). How faithful is Old Faithful? Statistical thinking: A story of variation and prediction. *Mathematics Teacher, 95*(4), 252–259.

Star, S. L., & Greismer, J. R. (1989). Institutional ecology, “translations,” and boundary objects: Amateurs and professionals in Berkeley’s Museum of Vertebrate Zoology, 1907–39. *Social Studies of Science, 19*(3), 387–420. doi:10.1177/030631289019003001

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Charlotte, NC: Information Age.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction—The Wiskobas Project*. Dordrecht, the Netherlands: Reidel Publishing.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9–35. doi:10.1023/B:EDUC.0000005212.03219.dc

Watson, J. M. (2006). *Statistical literacy at school: Growth and goals*. Mahwah, NJ: Lawrence Erlbaum.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, United Kingdom: Cambridge University Press.

Wenger, E. (2000). Communities of practice and social learning systems. *Organization, 7*(2), 225–246. doi:10.1177/135050840072002

Wijers, M., de Lange, J., Shafer, M., & Burrill, G. (1998). Insights into data. In National Center for Research in Mathematical Sciences Education & Freudenthal Institute (Eds.), *Mathematics in context: A connected curriculum for grades 5–8*. Chicago, IL: Encyclopaedia Brittanica Educational Corporation.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223–248. doi:10.1111/j.1751-5823.1999.tb00442.x

Author

Randall E. Groth, Department of Education Specialties, Salisbury University, 1101 Camden Avenue, Salisbury, MD 21801; regroth@salisbury.edu

Submitted September 9, 2013

Accepted June 3, 2014