Abbe H. Herzig

Goals for Achieving Diversity in Mathematics Classrooms

Commenting on the film *Good Will Hunting*, a mathematician noted, "We would do well to remember, in our efforts to include members of underrepresented groups in mathematics, that there can be as much resistance to our efforts from the students we work with as from the system we work in" (Saul 1998, p. 501). In other words, try as we may to include people in mathematics—in *our version* of mathematics—they might not be interested. Alternatively, some individuals may reject mathematics not out of a sense of choice but because they feel that mathematics has rejected them. Students' reactions to mathematics are affected both by their interests, abilities, and goals and by the particular ways that mathematics is conceived and taught within the mathematics classroom. It is possible that the people who succeed in mathematics are those who are able or willing to adapt themselves to the existing structure of mathematics education in schools. Individuals whose talents, values, skills, or interests make it difficult or undesirable for them to adapt to that structure may not be able to negotiate successfully the educational systems in a way that allows them to succeed in mathematics. In this argument I turn attention away from features of students—for example, their preparation or their ability—and toward our assumptions about mathematics education itself, presenting a unique challenge for us to build a context for mathematics education that is truly accessible and inviting to a broad range of students.

In this article I consider the access of all students—regardless of race, ethnicity, gender, socioeconomic status, sexuality, disability status, national origin, or English language ability—to the learning of meaningful mathematics. In thus considering the diversity of students, I first discuss the many ways in which such diversity is important. I then turn to a discussion of what it takes for students to succeed in mathematics, based on sociocultural theories of learning. Finally, I explore obstacles that some groups of students have to those paths to success and offer some thoughts to help teachers minimize the effects of those obstacles.

This department consists of articles that bring research insights and findings to an audience of teachers and other mathematics educators. Articles must make explicit connections between research and teaching practice. Our conception of research is a broad one; it includes research on student learning, on teacher thinking, on language in the mathematics classroom, on policy and practice in mathematics education, on technology in the classroom, on international comparative work, and more. The articles in this department focus on important ideas and include vivid writing that makes research findings come to life for teachers. Our goal is to publish articles that are appropriate for reflective discussions at department meetings or any other gatherings of high school mathematics teachers. For further information, contact the department editor.

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WHY DOES DIVERSITY MATTER?

Although this might seem like a rhetorical question, we need to be clear about our goals and beliefs about educating diverse students in mathematics. As we work toward educating diverse students equitably in the mathematics classroom, our beliefs and goals for diversity have an important impact on our efforts. By making those beliefs and goals explicit, we are in a better position to make meaningful progress. Building opportunities for all students to achieve in mathematics is important both for the individuals currently excluded from such opportunities and also for mathematics classrooms and for society at large.

Diversity Is a Question of Justice

Mathematics in education operates as a powerful filter, serving as an important prerequisite to many fields of study. Without a solid mathematical foundation and a record of achievement in mathematics, students' choices about higher education are severely limited. Beyond that, mathematics serves as an important gateway to many careers in our economy that are sites of affluence, influence, and power. Despite disparate outcomes for students of different races, ethnicities, genders, and social classes, the validity and the reliability of this filter have not been adequately addressed. A premise of life in a democratic society is that all members of that society have equitable opportunities. If more people participate in mathematics, those people will have increased opportunities to avail themselves of those benefits (Secada 1989).

> The learning of all students would be enriched by an expanded range of perspectives and approaches

Furthermore, mathematics has high prestige as a set of tools that help us understand, model, and manipulate the natural and social worlds. Concentrating this prestige in the hands of an elite few confers substantial power on that minority, which also allows them to maintain hegemony over mathematical development. This power also acts to inhibit equitable access to mathematics and to limit the types of mathematical development that are valued and pursued. Alternatively, if more individuals were to be included as full participants in mathematics, then the groups that those individuals are seen to represent would share in that power, and the equity implications might reach beyond the domain of mathematics. Finally, with growing national concerns about the lack of a quantitatively literate populace, precollege students might benefit from having a broader range of appropriate role models to emulate.

Diversity Enhances the Intellectual Climate of the Classroom

The strategy of using diversity to ensure long-term vitality has served well in a variety of natural, social, and economic systems; systems that fail to diversify are often unstable and vulnerable (Wilson 1992). In education, this observation implies that by including all students in meaningful interaction within the mathematics classroom, the diversity of perspectives and problem-solving approaches that ensues is critical for the intellectual health of both the classroom and for the discipline of study as a whole. "New entrants bring questions, fresh ideas, new and different perspectives on old problems, new energies, and new skills. They are not blinded by the familiar. The experience they bring enlarges the repertoire of strategies that can be employed" (Wilson 1992, p. 4). By broadening mathematical discourse in the classroom to include a more diverse range of thinkers, the learning of all students would be enriched by an expanded range of perspectives and approaches.

Participating in a classroom with a diverse population of students exposes students to others with perspectives, skills, and experiences different from their own. In view of recent reform efforts emphasizing students' development of a broad range of problem-solving strategies, and the increasing diversity of our educational systems, workplaces, and society at large, this exposure has important educational value. Teaching all students mathematics within diverse settings enhances the mathematics they learn, as well as their skills at listening to, valuing, analyzing, evaluating, and incorporating a broad range of views.

Diversity Is an Economic Necessity

At the same time as society's needs for scientific leadership and a scientifically literate workforce are increasing, the new entrants to the workforce will be predominantly women and minorities. White males make up a shrinking proportion of our population; if we fail to educate large proportions of our increasingly diverse society to participate in our increasingly technological and quantitative economy, we will face the twin economic risks of a large pool of unemployable people and an undersupply of trained workers; this is an argument for student diversity that Secada (1989) calls "enlightened self-interest." In addition, if large numbers of people feel outcast from mathematics and science, they are less likely to support critical societal investments in mathematical and scientific development. As our need for mathematically literate workers and citizens increases, our economic security will depend on how well we have educated other individuals and incorporated them into full participation in the economy of the twenty-first century.

WHAT DOES IT TAKE TO SUCCEED IN MATHEMATICS?

Theories of situated learning posit that learning happens through participation in social practices and that learning is inseparable from that participation (Boaler 2002; Lave and Wenger 1991; Rogoff 1994; Wenger 1998). The idea is that students learn from what they do. The activities in which students engage determine the particular things they learn, and each set of activities—for example, listening to teacher explanations, completing various types of homework and classwork exercises, taking notes, working with other students to solve problems, reading texts, studying for and taking exams—constitutes a different type of learning opportunity. Focusing on learning in this context draws attention to how individuals change through their involvement with the activities of their classrooms and other educational experiences. "This is a process of becoming, rather than acquisition" (Rogoff 1995, p. 143).

According to this view of learning, students and teachers together constitute a "community of practice" and students learn through their participation in that community (Lave and Wenger 1991; Rogoff, Matusov, and White 1996). Etienne Wenger (1998) describes three dimensions that define a community of practice: a joint enterprise, a shared repertoire, and mutual engagement. The joint enterprise comprises the activities in which the members of the community engage together. In high school mathematics, the joint enterprise generally has to do with learning to produce solutions to a specific range of problems in algebra, geometry, trigonometry, and analytic geometry and with taking various types of standardized assessments, as the students interact with each other and with their teachers to acquire mathematical knowledge. A shared repertoire includes "tacit knowledge," the unspoken norms and practices by which classrooms and schools operate. For high school mathematics, the shared repertoire includes practices such as studying for courses, learning acceptable norms for answering particular types of questions, completing homework assignments, and taking tests. Both the joint enterprise and the shared repertoire are constructed and negotiated by participants as they mutually engage in the activities of their classroom communities. These three dimensions of the community of practice of school mathematics education entail students' appropriation of mathematical knowledge (entering and constructing the joint enterprise), practices (entering and constructing the shared repertoire), and sense of belonging within the discipline (engaging in mutual ways with the other community members) (Boaler 2002; Boaler, Wiliam, and Zevenbergen 2000; Herzig 2004). Each of these dimensions is affected by students' interactions with other members of the community—the students and the teachers. These three dimensions of students' learning of mathematics—acquiring knowledge, practices, and a sense of belonging—are all critical components of students' mathematical learning.

Extensive research on differences among students based on gender, race, and ethnicity has failed to turn up any evidence that students of different demographic groups differ in their abilities to acquire mathematical knowledge. Important differences exist, however, in students' access to the other two dimensions of learning. How do students learn the practices of the mathematics classroom? Jean Lave and Etienne Wenger (1991) argue that students initially participate in authentic ways at the periphery of mathematical practice, and as their skill level increases, they move to more central participation. As an example, they describe the work of apprentice tailors, who begin their apprenticeship by cutting fabric for the master tailor. This activity is an authentic part of the work of a tailor, and yet the task is relatively simple, with errors having only minor consequences—hence the label "peripheral." As the apprentice's skill and knowledge improves, the apprentice assumes responsibility for more central tasks, and this cycle continues until the apprentice's activities are at the very center of the work of the craft. This process highlights the importance of experienced members of the community modeling the appropriate practices and behaviors of the craft; the activity of the community provides a "curriculum" for students, who learn through their participation in that activity. If students have limited access to the activities and practices of the community—say, because of being socially excluded, not understanding the practices and activities, or not being able to participate in them—then they will be limited in their opportunities to learn mathematics.

> Building students' sense of *belongingness* in mathematics is critical to K–12 education

Similarly, building students' sense of *belongingness* in mathematics has been proposed as a critical feature of an equitable K–12 education (Allexsaht-Snider and Hart 2001; Ladson-Billings 1997; National Council of Teachers of Mathematics 2000; Tate 1995). Martha Allexsaht-Snider and Laurie Hart (2001) define *belongingness* as "the extent to which each student *senses* that she or he belongs 'as an important and active participant in all aspects of the learning process' (Ames 1992, p. 263) in mathematics" (p. 97). Allexsaht-Snider and Hart argue that when structural aspects of schools, beliefs about diverse students and the learning of mathematics, and classroom processes (including teaching practices) are aligned in a way that facilitates diverse students' sense of belongingness and engagement with mathematics, we are more likely to achieve the goal of "mathematics for all" so often cited as a goal in reform and policy documents. In a sense, developing a sense of belonging can be conceptualized as the process of becoming a "central participant" (Lave and Wenger 1991) in a community of practice through participation in the practices of the community.

Participating in practices, developing a sense of belonging, and acquiring mathematical knowledge are intertwined strands of learning. This sociocultural view provides a useful framework for analyzing students' opportunities to learn meaningful mathematics. Students who have limited access to any of these dimensions will be inhibited in their opportunities to learn and engage with mathematics.

RECOGNIZING STUDENTS' OBSTACLES TO PARTICIPATION AND BELONGING IN MATHEMATICS

This perspective on learning draws attention away from students' abilities to master content and toward important features of the social culture of the mathematics classroom and how those features can enhance or inhibit student learning. If the goal of promoting diversity is to ensure that all students have access to the learning of meaningful mathematics, then we need to consider the range of opportunities and obstacles that different groups of students have to learning mathematics along each of the three strands of learning (acquiring knowledge, participating in practices, and developing a sense of belonging). In addition to the obstacles that all students may face, students of color, females, English-language learners, students from low-income families, and others face more demanding obstacles than their majority peers.

Mathematically talented students commonly have been stereotyped as lacking social skills

For example, the forms of communication used by some women and students of color may make it particularly difficult for their mathematical communication to be accepted (Orr 1997; Rosser 1995). English-language learners face difficulties understanding mathematics problems that use complex vocabulary or grammar, which has become even more of a concern in some reform curricula that emphasize contextualized problems and student explanations. Concerns have also been raised about the effects of reform-minded pedagogies on students from some cultural backgrounds. While current reform pedagogies in mathematics often call for students to learn to question others and to communicate about their understanding, students from families in which such questioning and communication is considered disrespectful might face harsh consequences, placing them in a double-bind between school-based and community-based norms (Secada and Berman 1999).

Some students find it difficult to engage with problems that are set in unfamiliar contexts or that lack relevance to their lives. William Tate (1995) argues that when African American students are forced to reason about mathematics in "standardized," school-accepted ways, in the context of white middle-class students, these students are distanced from mathematics rather than being engaged with it. Alternatively, mathematics instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level mathematical problem-solving and reasoning and enhance students' engagement with mathematics (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).

Research on the underparticipation of women and people of color in all levels of mathematics has led to some important improvements in our understanding of these issues and some effective interventions, yet it has also had its risks. Mathematics is often considered to be an objective field of study, which has contributed to a cultural blindness to the impact of personal, social, cultural, economic, or political factors on the learning of mathematics. Also, talent is commonly assumed to be required for success in mathematics (Herzig 2002; Love 2002). These beliefs in talent and objectivity, coupled with the small proportions of women and students of color participating in more advanced levels of mathematics, have contributed to the view that those groups of students do not have "what it takes" to succeed in mathematics. These negative stereotypes have added to students' and teachers' perceptions of what students are capable of and have led many students not to engage with mathematics out of a belief that there is no place for them in that field of study.

Mathematically talented students commonly have been stereotyped as lacking in social skills and without interests outside of mathematics (Campbell 1995; Herzig, October 2004; Noddings 1996). This perception has led to discourses surrounding mathematical ability that describe the mathematically talented as if they are deviant, similar to other "marked" categories in modern history (Damarin, 2000). As Nel Noddings (1996) remarks,

> There seems to be something about the subject or the way it is taught that attracts a significant number of young people with underdeveloped social skills. . . . Because that group so often tends to be exclusive, girls and minority youngsters may wonder whether they could ever be a part of it. But when the group is examined from a social perspective, many talented young people may question whether they *want* to be a part of it. (p. 611)

While labeling the mathematically talented as deviant might affect all students to some extent, females or students of color often feel like outsiders in mathematics classrooms because of their minority status, and they are outcast among their peers because of their mathematical abilities. For students already marginalized by virtue of their race, ethnicity, social class, or gender, belonging in mathematics may actually create a perception of being doubly deviant. While belonging might be a sign of success in mathematics and might facilitate their further learning, it also "marks" students in ways that may be too isolating for those who are already estranged from the mainstream culture of their peers. Given the common perceptions of mathematics students as being white, male, and middle class, it may be that students of other groups recognize tangible ways in which they do *not* fit in with this group. Given perceptions of mathematics students as being socially inept and with no interests outside of mathematics, some students may not *wish* to fit in. "If this impression of students who excel at math [as lacking in social skills] is inaccurate, researchers ought to produce evidence to dispel the notion, and teachers should help students to reject it. If it is true, math researchers and teachers should work even harder to make the 'math crowd' more socially adept" (Noddings 1996, p. 611). Mathematics educators need to pay attention to students' perceptions of the social implications of succeeding in mathematics and work to construct mathematics activities that engage students' true intellectual and social interests. Making mathematics relevant to students' lives is more than just a means of capturing their attention; it is also a critical step in making it socially acceptable—and even desirable—for students to excel in mathematics.

This discussion of the ways that different groups of students experience the social worlds of mathematics instruction differently could go on much further. My intention here is not to provide an exhaustive checklist of all the factors that might inhibit the learning opportunities of diverse mathematics learners, but rather to give tangible examples to mathematics teachers of things they can consider as they work to build a more inclusive educational experience for their students. As we think about the mathematics learning of our students, we need to look beyond their mastery of mathematical skills, facts, and concepts, and examine their opportunities to participate in all the practices that surround mathematics instruction, as well as how students feel about their place in mathematics.

WORKING TO ENHANCE ALL STUDENTS' LEARNING OF MEANINGFUL MATHEMATICS

The research presented in this article points to teaching, not just as helping students acquire knowledge but also as leading them to participate in practices of mathematics learning and to develop a sense of themselves as mathematics learners. This requires us to consider carefully the specific practices in which we want students to engage and to identify the particular obstacles they may encounter. We need to question *which* practices truly enhance *which* students' mathematical understanding, and which practices interfere with that understanding. Research about things that work to engage students of color, poor students, and other groups of students traditionally underserved in our schools suggests the following as some starting points for building opportunities for all students to learn meaningful mathematics:

> For some students, belonging in mathematics may create a perception of being doubly deviant

- Get to know your students as individuals and become familiar with the communities in which they live (Ladson-Billings 1995). Connect school to students' communities and homes and acknowledge that learning occurs in many places (Banks 2005).

- Use this knowledge to link the curriculum to the real interests of the students. Embed specific content and the development of skills within those interests (Sleeter 2005; Tate 1995).

- Implement rich learning opportunities that prepare all students for college (Sleeter 2005).

- Use assessments that give students authentic opportunities to demonstrate their understanding, not confounded by other skills or distractors (Shepard, 2005).

Surely, there are no recipes or magic solutions for building diversity in the mathematics classroom. Each student is an individual with unique experiences, skills, and background, and each classroom has its own distinct physical, social, and cultural context, with its own populations of students, parents, and teachers. The particular obstacles and opportunities students face in learning, and the interventions that will work for them, depend on all of these contextual factors.

The arguments presented here are intended to give teachers a framework for thinking about how to build a more inclusive and inviting educational context that will work for all of their students. By considering the particular practices in which we want students to participate, in addition to the specific knowledge we want them to acquire, teachers can work to minimize the interconnected obstacles students face to participating in classroom practices and to finding paths to belonging in mathematics.

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