MATHEMATICAL lens

Ponte della Maddalena, near Borgo a Mozzano, Italy

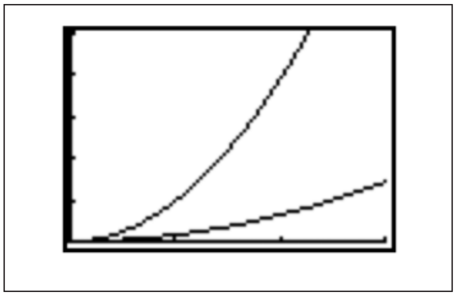

Fig. 1

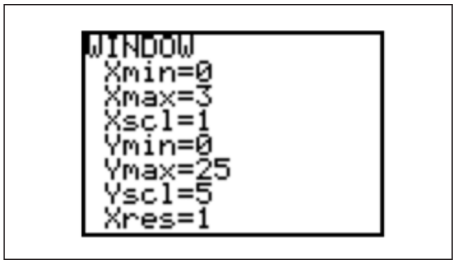

Fig. 2

Ron Lancaster, an editor of this column, walked across the spectacular asymmetric Ponte della Maddalena over the river Sergio near Borgo a Mozzano, Italy, in June 2006. Built in the eleventh century, it was the only bridge along this river that was not bombed by the Germans during World War II. They let it stand because they thought it was too narrow for American tanks to cross. The Germans, however, were mistaken: The bridge was wide enough for the tanks to cross, and many Italians were liberated as a result.

"Mathematical Lens" uses photographs as a springboard for mathematical inquiry. The goal of this department is to encourage readers to see patterns and relationships that they can think about and extend in a mathematically playful way.

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The bridge is also known as the Ponte del Diavolo because of a legend that the devil built it in a single night.

Standing at the highest point on the bridge, Ron dropped several stones into the water, recorded the time it took each stone to reach the water, and then challenged co-editor Brigitte Bentele to use the average of these times to find the height of the bridge. How would you respond to the challenge? Before proceeding with the questions, think about how you could find this height knowing only how long it took a stone to reach the water.

1. (*a*) The average time for a stone to reach the water was 2.1 seconds. Use this time to determine the height from which the stone was dropped.
   HINT: Use the formula
   $$d = \frac{1}{2}gt^2,$$
   where d is the height of an object (in meters), g is the acceleration due to gravity (approximately 9.8 m/sec.^{2}), and t is the time (in seconds).

   (*b*) If the stone had taken twice as long to reach the water, would the bridge be twice as tall? Support your answer with some calculations.

   (*c*) How high would the bridge be if the stone did take twice as long to reach the water?

   (*d*) If this bridge were located on the moon, how long would it take for the stone to reach the water?

2. Both graphs shown in **figure 1** were created on a TI-84 graphing calculator using the formula
   $$d = \frac{1}{2}gt^2,$$
   one with the value of g for the earth and the other one with the value of g on the moon. Identify each graph and explain how you were able to make this determination. Is it possible to identify these graphs without knowing what WINDOW settings were used (**fig. 2**)?

MATHEMATICAL LENS solutions

1. (a) The height of the bridge is slightly more than 21.5 m.

$$d = \frac{1}{2}(9.8)(2.1)^2$$
$$= 21.609 \text{ m}$$

(b) No, the bridge would not be twice as tall.

$$d = \frac{1}{2}(9.8)(4.2)^2$$
$$= 86.436 \text{ m}$$

Actually, it is not necessary to do any calculations. Since the variable, t, is squared, doubling the time would yield four times the distance:

$$(4.2)^2 = (2 \cdot 2.1)^2 = 4 \cdot (2.1)^2$$

Thus, the ratio of the distances is 1:4, not 1:2.

(c) 86.436 m

(d) The acceleration due to gravity on the moon is 1.6 m/sec.^{2}. Therefore,

$$21.609 = \frac{1}{2}(1.6)t^2$$

and so $t = \sqrt{\dfrac{21.609}{0.8}}$

$$\approx 5.2s.$$

Thus, it would take the stone approximately 5.2 seconds to reach the water below. But this was a trick question. There is no water on the moon!

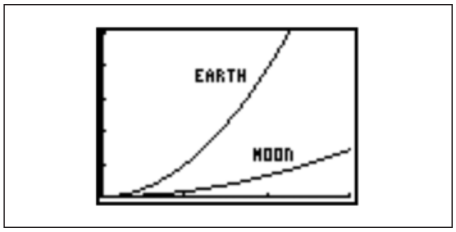

Fig. 3

2. The graphs can be identified (see **fig. 3**) without the WINDOW settings by noting that the coefficient of t^2 in the formula for the earth ($d = 4.9t^2$) is larger than the coefficient of t^2 for the moon ($d = 0.8t^2$). ∞

Members who wish to use this month's photograph in a classroom setting can download the image from NCTM's Web site, www.nctm.org. Follow links to *Mathematics Teacher*, and choose Current Issue. Then select Mathematical Lens from the Departments, and look for the link to the image.

Have you ever seen a building, a bridge, a sign, or a natural phenomenon that stimulated mathematical thoughts? Why not take a photograph and send it to NCTM, along with the mathematical questions that the photograph inspires? The questions can be playful, imaginative, curious, and inventive; they can also be mathematical extensions sparked by the photograph.

If the photograph includes identifiable people, the photographer must obtain signed release forms. Photographers must also obtain release forms if trademarked items are shown. Original photographs must be either in hard copy or supplied digitally as 300 dpi images in .jpg format. For details on releases and digital standards, please see the NCTM Web site. Photographs will not be returned.

Send the photographs, diagrams, list of questions, solutions, and completed release forms to the "Mathematical Lens" editors.