MATHEMATICAL lens

Photograph 1 The Venetian Hotel, Las Vegas

Photograph 2 Store on Orchard Road in Singapore

"Mathematical Lens" uses photographs as a springboard for mathematical inquiry. The goal of this department is to encourage readers to see patterns and relationships that they can think about and extend in a mathematically playful way.

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Larry Ottman, a mathematics teacher at Haddon Heights Junior/Senior High School, took the stunning photograph of a highly unusual clock (see **photograph 1**). This clock is part of the Venetian Hotel in Las Vegas and is based on the clock in the Tower of St. Mark's in Venice, Italy. The clock is unusual for two reasons. First, clocks are rarely seen in casinos; second, it shows the time for an entire twenty-four-hour period. Ottman's photograph invites us to ask the central question in the rock group Chicago's giant hit from the 1970s: "Does anybody really know what time it is?"

1. Why do you suppose the clock is turned so the twenty-fourth hour is on the side, rather than on the top?

2. (*a*) What is the degree measure of the angle swept out as the minute hand moves from one clock number to the next?

   (*b*) Express this angle in exact radian measure.

   (*c*) How many minutes transpire as the minute hand passes through this same interval?

   (*d*) Assuming that the 3 hands in the photograph are on top of one another at 0:00 or 24:00 hours, what time seems to be depicted in **photograph 1**?

(*e*) How many times during a twenty-four-hour period will the hour and the minute hands be on top of each other?

3. On a twelve-hour clock, the hands are all pointing directly up at both noon and midnight. At what time would that occur on this clock?

4. (*a*) Assume that the length of the minute hand is 2 m. Calculate the velocity of a point on the end of the minute hand in meters per second.

   (*b*) If the minute hand is actually shorter than 2 m, will it move faster or slower? Explain.

5. Over the course of a twenty-four-hour period, how many times will the minute and hour hands form a right angle?

6. Ron Lancaster, one of the editors of this column, came across the sign of a "25 Hours Watch Gallery" on a recent trip to Singapore (see **photograph 2**). He and co-editor Brigitte Bentele wondered about some of the changes that could occur if a day were divided into twenty-five hours instead of twenty-four, and if a clock showed all twenty-five hours, from I to XXV. We thought you might enjoy imagining some of these, too.

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MATHEMATICAL LENS solutions

1. The clock in Venice, on which the clock in **photograph 1** is based, was constructed in the fifteenth century at a time when noon and midnight were not the preeminent times during the day. In the Italian system of time, sunset (or thirty minutes after sunset) always occurred at the end of the twenty-fourth hour. The lengths of hours varied throughout the year as the amount of daylight changed. The day was focused on how many working hours were left before the sun would go down and the next day would begin. (See the bibliography for a list of Web sites that discuss time systems, including the Italian.) The editors are not sure why the number 24 is on the right side of this clock. It may have something to do with facing east toward the holy lands, or there may be another explanation. We invite interested readers and students to investigate this subject.

2. (*a*) $\dfrac{360°}{24} = 15°$

   (*b*) $\pi/12$ radians

   (*c*) $60/24 = 2.5$ minutes per number

   (*d*) The time appears to be 14 hours, 12 minutes, and 42 seconds, or 2:12:42 in the afternoon.

   The hour hand is between 14 and 15. Since the minute hand is nearly at 5, we compute:

$$(5)\cdot\left(\frac{1}{24}\right)\cdot(60) \approx 12$$

   The second hand is at about 16 2/3, so compute

$$\left(16\frac{2}{3}\right)\cdot\left(\frac{1}{24}\right)\cdot(60) \approx 42.$$

   The time appears to be 14:12:42, or 2:12:42 p.m. However, since the hour hand is closer to 15 than 14, it is not a logical conclusion for less than 30 minutes to have passed. Perhaps there is an alternate way of reading the clock.

   (*e*) 23 times. At 0:00 (24:00), the hour and minute hands are on top of each other. During the next 24 hours, the minute hand will go around 24 times, while the hour hand will go around once. Therefore, they will be on top of each other 23 times.

3. Unless you assume that the hour hand moves discretely from hour to hour, rather than continuously, this will not occur on this clock. When the minute hand is pointing straight up, the hour hand will already have moved past that point. If we do assume the hour hand only moves once each hour, the time would be 18 hours, 45 minutes.

4. (*a*) The minute hand of radius 2 meters travels 4π meters in one hour. Since one hour contains $60 \cdot 60$ seconds, the hand travels $4\pi/3600 = 0.0035$ m/s.

   (*b*) Since a shorter minute hand will trace out a smaller circle and thus travel a shorter distance in the same amount of time, it must be traveling slower.

5. The minute and hour hands would form a right angle forty-six times.

6. Dividing a clock into 25 hours would mean that one hour would be 1/25 of a revolution, or $360/25 = 14.4$ degrees, a number less convenient than 15 degrees. Since it would be simpler to divide 25 hours into fifths rather than fourths, one might then separate the hour into 12-minute fifth-hours instead of 15-minute quarter-hours. You could meet your friends at a fifth before five instead of a quarter after four! Or one might be tempted to use decimals to give the time: The time 4:36 could be 4 and three-fifths o'clock or 4.6 o'clock. Metrics might win out after all with such a clock.

BIBLIOGRAPHY

Ackerman, Silke, and Robert van Gent. "Time and Date." www.mhs.ox.ac.uk/epact/article.php?ArticleID=19#italian_hours.

"History of the 24 Hour Dial." homepage.mac.com/pete.boardman/24hourclock/history.html.

Ta'bois, Noel. "Hours and Hours." www.sundials.co.uk/tbhou.htm. ∞

Have you ever seen a building, a bridge, a sign, or a natural phenomenon that stimulated mathematical thoughts? Why not take a photograph and send it to NCTM, along with the mathematical questions that the photograph inspires? The questions can be playful, imaginative, curious, and inventive; they can also be mathematical extensions sparked by the photograph.

If the photograph includes identifiable people, the photographer must obtain signed release forms. Photographers must also obtain release forms if trademarked items are shown. Original photographs must be either in hard copy or supplied digitally as 300 dpi images in .jpg format. For details on releases and digital standards, please see the NCTM Web site. Photographs will not be returned.

Send the photographs, diagrams, list of questions, solutions, and completed release forms to the "Mathematical Lens" editors.

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