Teaching Geometry to Visually Imp Stu

Teaching a visual subject to a visually challenged student inspires strategies that benefit all students in a class.

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From Euclid's *Elements* to NCTM's *Focus in High School Mathematics: Reasoning and Sense Making in Geometry* (McCrone et al. 2010), geometry teachers know the important geometric elements that high school students should master, those that enable us to understand the world around us. NCTM (2000) described geometry as "a means of describing, analyzing, and understanding the world and seeing beauty in its structures" (p. 309). Dossey et al. (2002) captured the essence of this aspect of visualization by stating that geometry fosters in students an ability to "visualize and mentally manipulate geometric objects." (p. 200). Clearly, visualization is imperative in understanding geometry.

However, not all students have an inherent or physical ability to look at a figure or drawing and make conjectures on the basis of what they see or visualize the figure's properties. One mathematics teacher was challenged by a cognitively able student who did not have the visual capability to fully "see" the beauty of geometry using her eyes. The teacher's experience follows. (Note: The experiences of the primary author are conveyed in first person, and pseudonyms are used throughout.)

THE NEW STUDENT

The greatest challenge of my teaching career occurred when Jessie joined my geometry class. I had taught geometry for sixteen years and considered myself a fairly competent instructor. I knew my subject and was confident in my ability to help any student make the connections from one geometry topic to another. By the end of each academic year, I had rolled the entire geometry curriculum into a neat bundle ready for each student to carry with him or her to whatever mathematics class or life situation they would encounter next.

Most important, as a geometry teacher, I embraced the necessity of visualizing the subject and emphasized the importance of pictorially representing problem situations. My first question to a befuddled student was always, "Have you drawn a diagram to represent this problem?"

One morning in early August, as I was preparing for the new school year, my school district's teacher for the visually impaired, Mrs. Jones, stopped by my room. She informed me that Jessie, who is completely blind, was going to be in my geometry class.

Teaching the visually impaired was not a completely new experience for me. I had taught many students who required large-print text, and I had even taught entry-level algebra to a blind student several years earlier. But I was completely unprepared to teach a subject so dependent on visual cues to someone who could not see. Almost every problem that my students solve involves making a diagram of some sort and gathering information from that diagram to help find solution strategies. Jessie was in a pre-Advanced Placement class designed to be taught as a college prep class; it consisted of about thirty ninth-grade students. Jessie had a high aptitude in mathematics; however, I was skeptical about her chance for success in this primarily visual subject.

I had only one day to prepare for Jessie. There was no time for me to investigate strategies for teaching geometry to the blind. I simply jumped in and hoped for the best.

As I worked feverishly to set up my room for the arrival of my students the next day, Mrs. Jones dropped off some supplies that Jessie would need for my class. These supplies included a Braille ruler and protractor, a geoboard for graphing, and an audiocalculator. Also included was a board with Mylar® polyester film on it; scratching this board with a plastic "pencil" would raise lines, allowing one to draw crude diagrams.

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Although I had some physical tools, I had no instructions on how to teach geometry to a student who could not see. Throughout the school year, as I attempted to teach Jessie, I confronted many impediments. By working toward solutions to these challenges, I began learning new and innovative ways not only to teach this one student but also to improve instruction for all the other students in my classroom.

LOGISTIC CHALLENGES

The Textbook

My district had just adopted a new textbook focused on discovery learning through investigations and problem solving. This textbook had rich examples and descriptions that I had planned to have students read and work through. My initial goal was to have students do this reading silently and then work through the problems in groups. Jessie required a special version of this textbook; unfortunately, the publisher did not yet have a Braille edition available.

Mrs. Jones assured me that the publisher would have the first two Braille chapters by November. However, by November I expected to be well into chapter 5, so the publisher's schedule was not going to do Jessie a whole lot of good. Mrs. Jones was willing to translate a few assignments into Braille but did not have time to translate all the investigative text. As the only secondary school teacher for visually impaired students in the district, she was at that time providing services to eleven children and dividing her time among three campuses.

With no textbook for Jessie to read, I had to find another way for her to be exposed to all the investigative material. I did not want Mrs. Jones just sitting in the back of the class reading the text aloud to her, so I used my favorite teaching strategy—cooperative learning.

I have always placed my students in groups so that they would have a support system while investigating new concepts and working through problems. This approach has been beneficial for all my students, but it was especially helpful for Jessie. The other students in her group did not mind reading the investigations out loud and including Jessie in the discussion. In this way, we worked through the basic concepts of geometry. As always, these cooperative learning groups were helpful for all my students, whether sighted or visually impaired.

> Thus, I began learning new ways to teach this one student and improve instruction for all the other students.

Note Taking and Problem Solving

Having the text read aloud was an accommodation beneficial to Jessie. However, hearing the text helped with the more basic concepts of the subject. Jessie, like most other students, needed to be able to take notes, complete scratch work, search these notes, and work while attempting to make sense of the more abstract and difficult topics. She needed to find ways of recording the important aspects of a lesson as well as recording her work when solving problems. To do so required the assistance of Mrs. Jones, a Braille typewriter, and me.

Technological Assistance and Its Limitations

Jessie began the school year with a mechanical braillewriter (a braillewriter is a typewriter that stamps the paper with Braille text). Jessie would type on her braillewriter while other students took notes. By feeling what she had typed, she could understand what she had written, so she could read her notes back when necessary. However, the result was mountains of paper notes that were difficult to organize and use.

Braille paper, similar to construction paper, is larger and thicker than regular paper. When text is converted to Braille, much less fits on a single page. Dick and Kubiak (1997) described this increase in page allocation for Braille: "A pocket dictionary, such as the one printed by American Heritage, can occupy a bookshelf 6 1/2 feet high by 3 feet wide!" (p. 346).

The thickness of paper coupled with the increase in page space needed to write or construct in Braille meant that Jessie's notes tended to create stacks of often-unsearchable material. Jessie could not glance through her notes for a definition or an example as other students could. Her notes for a chapter could fill an entire notebook. Looking up a formula or reviewing a process from a previous problem was an extremely tedious process.

About halfway through the school year, Jessie upgraded to an electronic braillewriter—in actuality, more of a computer than a typewriter. She no longer had to print her notes but could store them on this computer and then recall them when needed. The new braillewriter displayed the notes for her in Braille, so she could open a file and read the notes that she had stored there. Jessie's new braillewriter could also be connected to a printer, so her assignments could be printed in regular text for me and turned in without having to be translated by Mrs. Jones. This new technological tool significantly simplified the note taking process for us all.

The new braillewriter benefited Jessie in numerous ways, but it had limitations too. During the class's study of quadrilaterals, I had integrated problems that involved triangles and quadrilaterals and that required an understanding of all the properties of both types of figures and how their angles are related. The drawings often took up a whole sheet of standard typewritten paper; using the drawings, students had to find numerous angle measures to solve a given problem. Typically, students wrote each measurement on the drawing as they calculated each one. This scratch work could then be used to find other measurements in the drawing.

This activity presented a huge challenge to Jessie. She could not take the brailled diagram, place it in her braillewriter, and record the measurements in the drawings when she found them and then use them to find other measurements. I began by breaking the problems into smaller pieces and having her work through them a few at a time, but this approach defeated the purpose of her being able to assimilate all the properties of the different shapes and understanding how they fit together.

The solution that we finally came up with made use of the simple thumbtack. Braille paper is more like card stock than regular paper, and the thumbtack worked well for punching small holes in the card stock from the back, thus raising bumps on the front. Mrs. Jones, who often accompanied Jessie to my class, would write the measurements Jessie calculated in the diagram in Braille using the thumbtack, and Jessie could then read the numbers and move forward through the problem. This was a tedious and time-consuming process, but Jessie successfully completed the unit on quadrilaterals.

Symbols

As the mathematics became more complicated, I realized that Jessie was getting lost. The new braillewriter and the hole punching helped her take notes and record scratch work, but much of the language of geometry consists of symbols that allow students to memorize the vocabulary and aid in note taking. Symbols are used to represent lines ($\leftrightarrow$), segments (—), and rays ($\rightarrow$) as well as triangles ($\triangle$). Symbols are also used to represent relationships: parallel ($||$), perpendicular ($\perp$), and congruent ($\cong$). The list of symbols is extensive.

Sighted students become accustomed to the symbols as visual clues to use when creating a diagram, labeling parts, or drawing conclusions. When translated into Braille, these symbols take up more space than the words that the symbols represent, so Jessie simply had to memorize the words and their meanings without the benefit of the visual clues that other students enjoyed. For example, when the symbol $\perp$ is used, a student can visually associate the perpendicular lines intersecting to create the "square" 90 degree angles, but Jessie could only use the word "perpendicular" as her clue. Such symbols help students bridge the gap between terminology and concrete representations. Jessie's inability to see or use these symbols created a greater challenge for her in learning the material.

CONTEXTUAL CHALLENGES

One of the toughest challenges was guiding Jessie to an understanding of concepts for which, because of her blindness, she had no context. As mathematics teachers, we often try to connect the abstract topics of the subject with examples of real-life situations to help students better understand them. The available contextual clues for a blind student are greatly decreased from that of a sighted student.

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Perspective

One such contextual issue arose during a lesson using perspective drawing of three-dimensional figures. Jessie needed to feel what she could not see. When we studied polygons, I gave her pattern blocks to feel and trace. When the other students drew tessellations, she built hers using the blocks. Then we began working with three-dimensional solids. Jessie had worked with solids in previous mathematics courses, but, like many students, she seemed to have a hard time equating two-dimensional perspective drawings with the three-dimensional figures that they represented.

Perspective drawings give a two-dimensional account of how a three-dimensional figure is viewed. Jessie had no visual context for this perspective. For example, a *prism* is defined as containing two parallel congruent bases. Often, when a prism is drawn on two-dimensional paper, the drawing looks as if the bases intersect (see **fig. 1**). To make this idea clear to Jessie, we had to trace each face of the prism on the paper and then indicate to her which face of the prism it represented. This was a laborious process. On paper, to indicate perspective, rectangles are sometimes drawn as parallelograms, and circles are sometimes drawn as ovals—another confusing point for Jessie. However, throughout the unit on surface area and volume, the solid figures were always available for her to touch and manipulate.

Solids of Rotation

A final example of Jessie's limited context occurred during the last unit of the school year, in which I

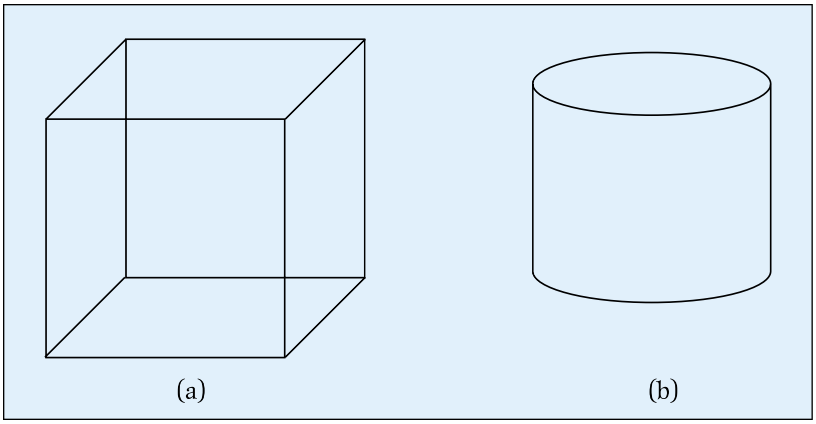

Fig. 1 When drawn on paper, a rectangular prism (**a**) has bases that appear to intersect (the front and back faces, for example). Some of the faces appear to be parallelograms rather than rectangles. This cylinder (**b**) appears to have a base that is shaped like an oval rather than a circle.

combined several topics: graphing lines in the coordinate plane and finding the area and perimeter of the bounded region that they create. Jessie had done most of her graphing on a geoboard with rubber bands, an acceptable process for finding area and perimeter. The last part of this unit involved revolving those bounded regions about an axis and drawing the solid that was created.

This concept, difficult for my sighted students, was overwhelming for Jessie. No amount of verbal explanation from me seemed to help her wrap her brain around this idea, so I had to find a way to make it concrete for her. I needed to come up with some way of constructing a manipulative that would represent this concept.

From card stock, I cut out a right triangle and a rectangle and taped each to the end of a pencil. The pencil was my axis of rotation. When I rotated the pencil, Jessie was able to feel the triangle going around in her hand. She could trace the circular base that the triangle created and understood that the three-dimensional figure swept out was a cone. The rectangle became a cylinder in her hands (see **fig. 2**). Jessie eventually understood that all solids of rotation have circular bases and that she could find the surface area and volume of each. We even managed to revolve a trapezoid to create a frustum.

LOOKING BACK

My experience in teaching Jessie taught me that any student can reach his or her cognitive potential when instruction is tailored to individual needs. I began that school year with unrealistically high expectations of what Jessie could do. In the beginning, I expected her to keep pace with the rest of the class. After all, she was in an advanced class, wasn't she? As I came to understand Jessie's special needs, I learned that reading and understanding problems, drawing diagrams, and recording information all take more time, space, and organizational strategies. I learned that Jessie *could* reach my high expectation of learning the material, just not as quickly as my sighted students.

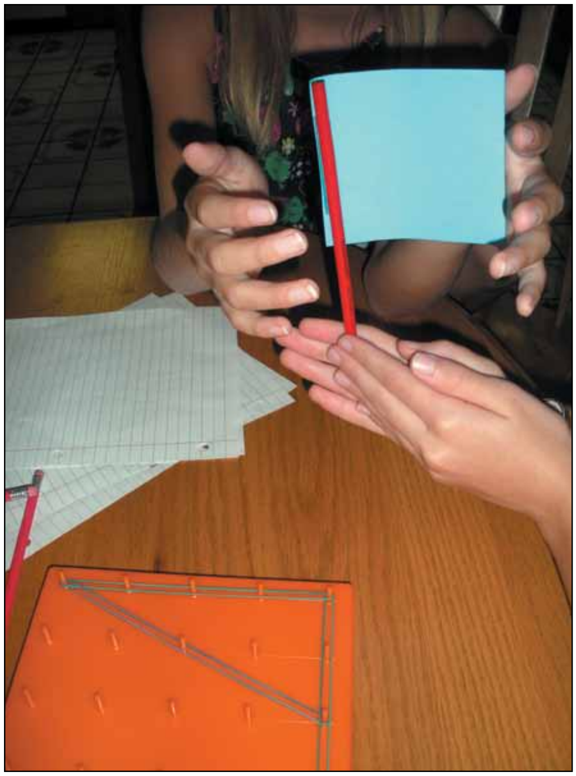

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Fig. 2 Affixing a cutout rectangle to a pencil and rotating the pencil gave Jessie a tactile means of understanding volumes of revolution.

I sometimes underestimated Jessie's capabilities. I doubted that a blind student could fully understand a visual subject like geometry. Yes, geometry is a visual subject, but visualization is not reliant solely on one's eyes. Jessie could see the beauty of this subject with her hands. Small bumps on the paper became pictures in Jessie's mind. Thumbtack punctures became scratch work for problem solving. Twirling rectangles became cylinders in Jessie's palms. In essence, Jessie saw mathematics through her "mind's eye" (this phrase is drawn from Fisher and Hartmann's article [2005]).

Each student learns in his or her own way. Every year I differentiate my instruction to meet students' needs. I have used multiple representations, had students act out problems, scheduled silent study time, had cooperative group activities, and tried to understand and implement teaching strategies that catered to all possible learning styles. Through my experience with Jessie, I learned that when instruction was differentiated for her individual needs, she too could learn without the use of her eyes.

Although I did not have time to research the best ways to help Jessie "see" geometry, I believe that she left my class with a solid understanding of the subject. She was successfully able to complete all geometry objectives during this school year beside her sighted peers. This success I attribute to the persistence of all of us—Jessie, Mrs. Jones, and myself—in finding ways that helped Jessie make sense of the material.

Understanding geometry is not reliant on seeing objects and images with one's eyes. Problems can be represented "visually" in many ways; they can be spoken, read, drawn, and felt. All students can learn; however, they all learn differently.

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