

# PK-2-Multiplication and Division

*2014 NCTM Interactive Institute*



NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS

# ‘Words of Wisdom’

“We do not have to fear raised expectations as long as we look first to the child with respect for wherever he or she is on the journey toward deeper understanding of mathematics.”

**Kathy Richardson**

*“Mathematics Standards for  
Pre-Kindergarten through Grade 2”*

October 2000    EDO-PS-00-11

# Welcome

- Introductions
- Learning Goals
- Session ‘Take-aways’
- Instructional Practice and Next Steps

# Assumptions and Beliefs

## PK-2 Teachers

- ‘Set the Stage’
- Prepare for success
- Use ‘best practices’ to develop understanding
- Today: Use and extend ‘assumptions and beliefs’ to build future learning

# Common Core Connections Guiding Questions

**Goal:** Connect Multiplication and Division to  
'Big Ideas' of:

- Counting and Cardinality
- Number in Context
- Addition and Subtraction

## **Session Guiding Questions:**

- What are students learning?
- What's the evidence of learning?
- Instruction: What's next?

# Young Children Solving Problems

- Four pencils are in a pack. You have 3 packs of pencils, how many pencils do you have?
- You have 12 new pencils. You want 3 friends to each have the same number of pencils. How many pencils does each friend get?

# PK-2 Teachers Help Students

- Establish foundational understanding of the meaning of a unit,
- Use language and models to begin developing multiplicative thinking, and
- Help students connect representations, models and language that builds a foundation for learning in grades 3–5.

# Multiplicative Reasoning

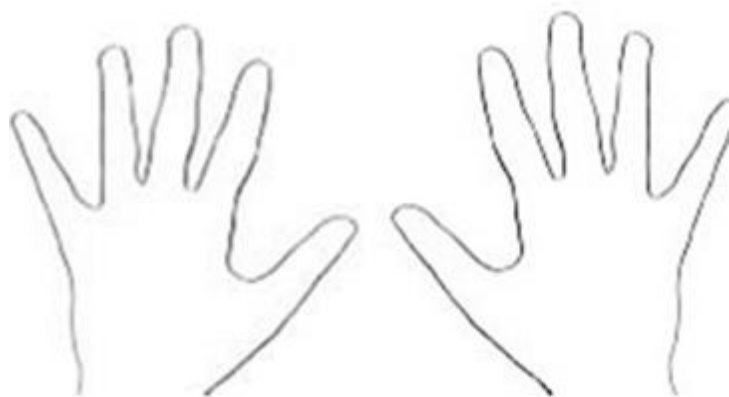
One key aspect that we, as teachers, need to encourage is the development of multiplicative reasoning that extends beyond a view of multiplication as repeated addition.

Jacob and Willis, 2001



# Importance of 'Unit'

- As students learn to count and measure, it is important to emphasize the unit that they are using and help them recognize that the count or measure changes depending on the unit that is established.



# Focus on Multiplicative Thinking

- Determine the distance from A to B in crayons.



- Determine the distance from A to B in cupcakes.



# Unit

- As students engage in situations that involve multiplication and place value, they learn to look at units in a new way, often referred to as unitizing.
- Unitizing means viewing objects or quantities in different-sized chunks (Lannon, 1999).

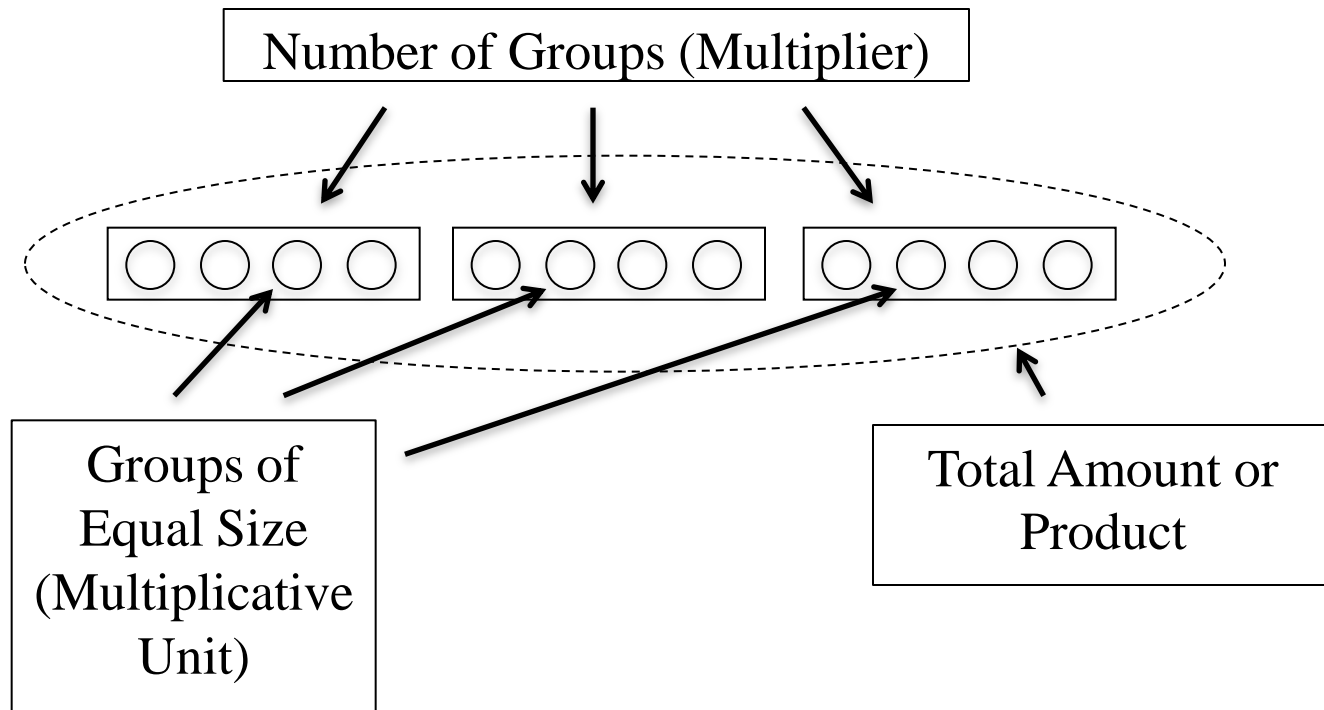
# Three Critical Aspects

Students must first come to recognize multiplicative situations as involving three aspects:

- Groups of equal size (multiplicative unit),
- Number of groups (the multiplier), and
- A total amount (the product)

Kouba (1989), Steffe (1992), Mulligan, and  
Mitchelmore (1997)

# Aspects of a Multiplicative Situation



# Multiplication and Division

- Current instructional practices that connect multiplication and division?
- Talk and share with tablemates-
- Talk with a 'new' partner-
- Group Responses

# Begin with the end in mind...

‘Best practices’ Connected to:

Language Representations Problem types

- Conceptual understanding and ‘making-sense’
- Concrete ‘representations’
- Properties of operations

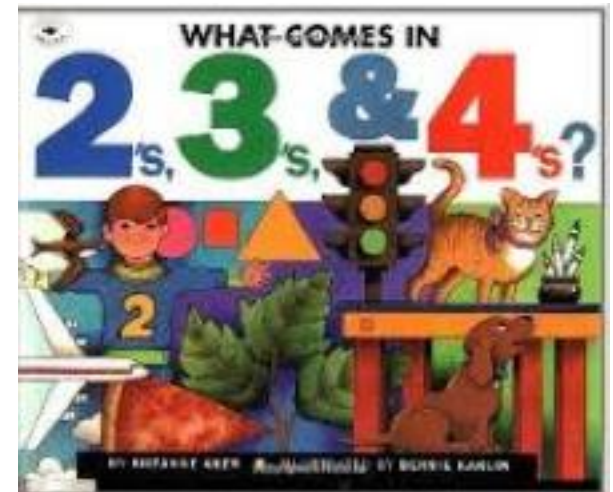
# Skip Counting

*Set and Length Model Connections*



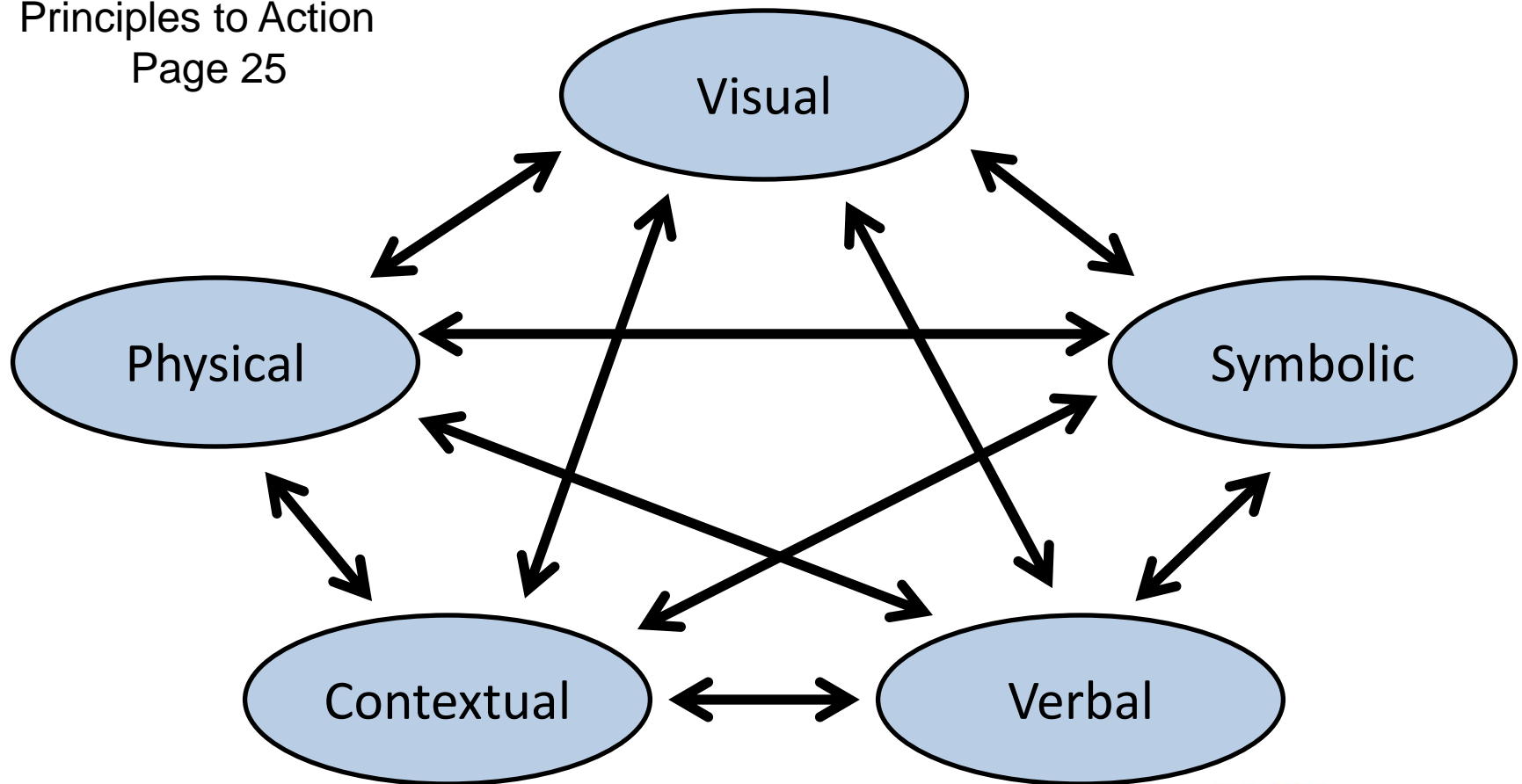
# Skip Counting Connections

- **What Comes in 2's, 3's 4's?**
- How many of us are here today?
- Group by 2's-  
Count: How many groups?  
How many people?
- **This experience-**  
Language? Connections? Representations?



# Lesh Translation Model

Principles to Action  
Page 25



# Grouping By 3's-How Many of Us?

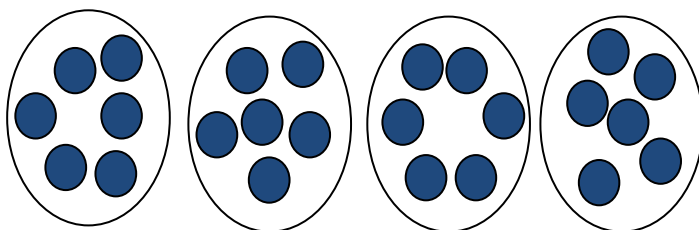
- What is the same when grouping by 3's?
- What's different?
- Process this experience-
  - What's important?  
Language? Connections? Representations?
- Questions?

# Questions?

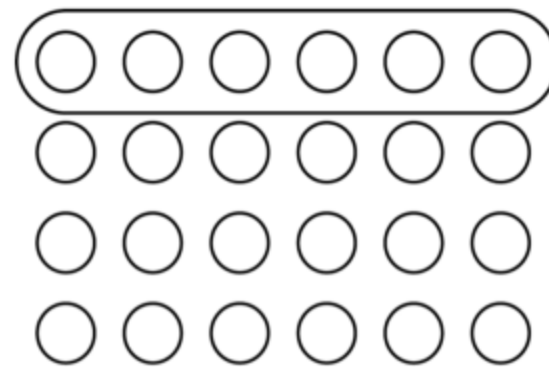
- Richardson's Article
- Deepen understanding?
- Reveal thinking ?
- 'Probe' understanding?
- Develop generalizations?

# Models of Multiplication

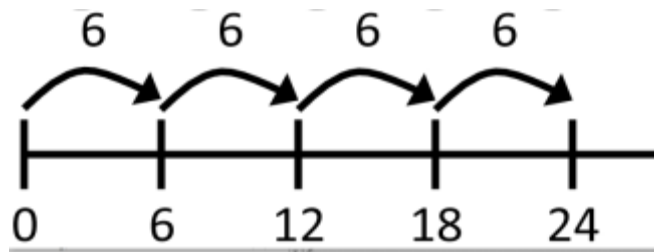
- The set model



- The array model



- The length model



- The combinations model



# Connecting Skip Counting and Multiplication

‘Multiplication as ‘coordinated counting’

Connections with:

- Grouping?
- Skip Counting?
- Understanding Multiplication and Division?

# Grade 2: Common Core Content Standards

2 OA: Work with equal groups of objects to gain foundations for multiplication-

**2 OA.3: Determine whether a group of objects (up to 20) has an odd or even number of members, -e.g., by pairing objects or counting them by 2's-**

2 OA.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns.

# Multiplicative Situations

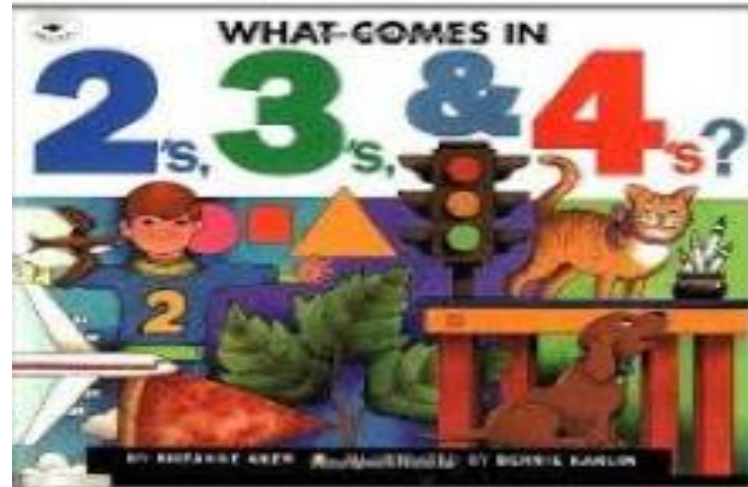
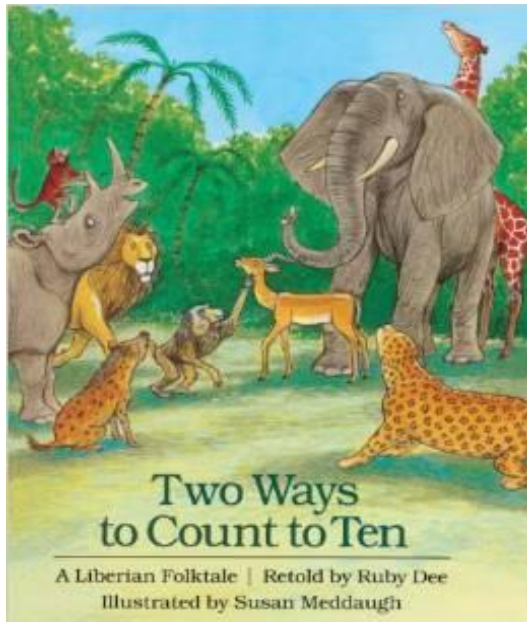
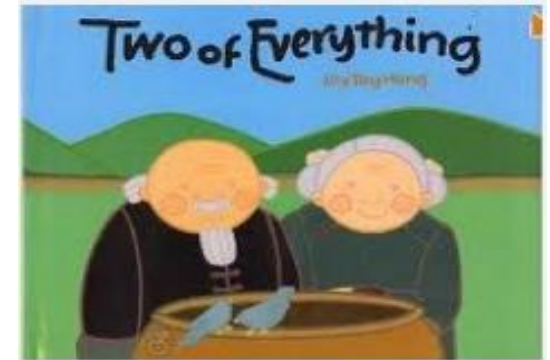
You have 3 pencils. Your friend has twice as many. How many does your friend have?

What other multiplicative language can we develop?



# Counting and Grouping Literature Connections

- Two of Everything
- Two Ways to Count to Ten
- What Comes in 2's, 3's and 4's?



# Things that Come in Groups

*Set and Length Model Connections*

# Things that Come in Groups

- What comes in 2's?

Opportunities for counting by 2's?

Connect: Skip counting

Patterns on 100s Chart      Groups of

- Make a Chart...with things that come in..

2's, 5's, 10's

Optional: 3's, 4's, 6's, 7's, 8's & 9's

# Things that Come in Groups-- Models

Use: Snap cubes or Cuisenaire rods to model counting by 'groups of'

- Highlight pattern on 100s chart-
- Make another 'group of'
- Highlight pattern on same 100's chart-
- Show pattern on t-table
- Discuss: Guiding Questions

# Number of Spots on the Grouchy Ladybugs



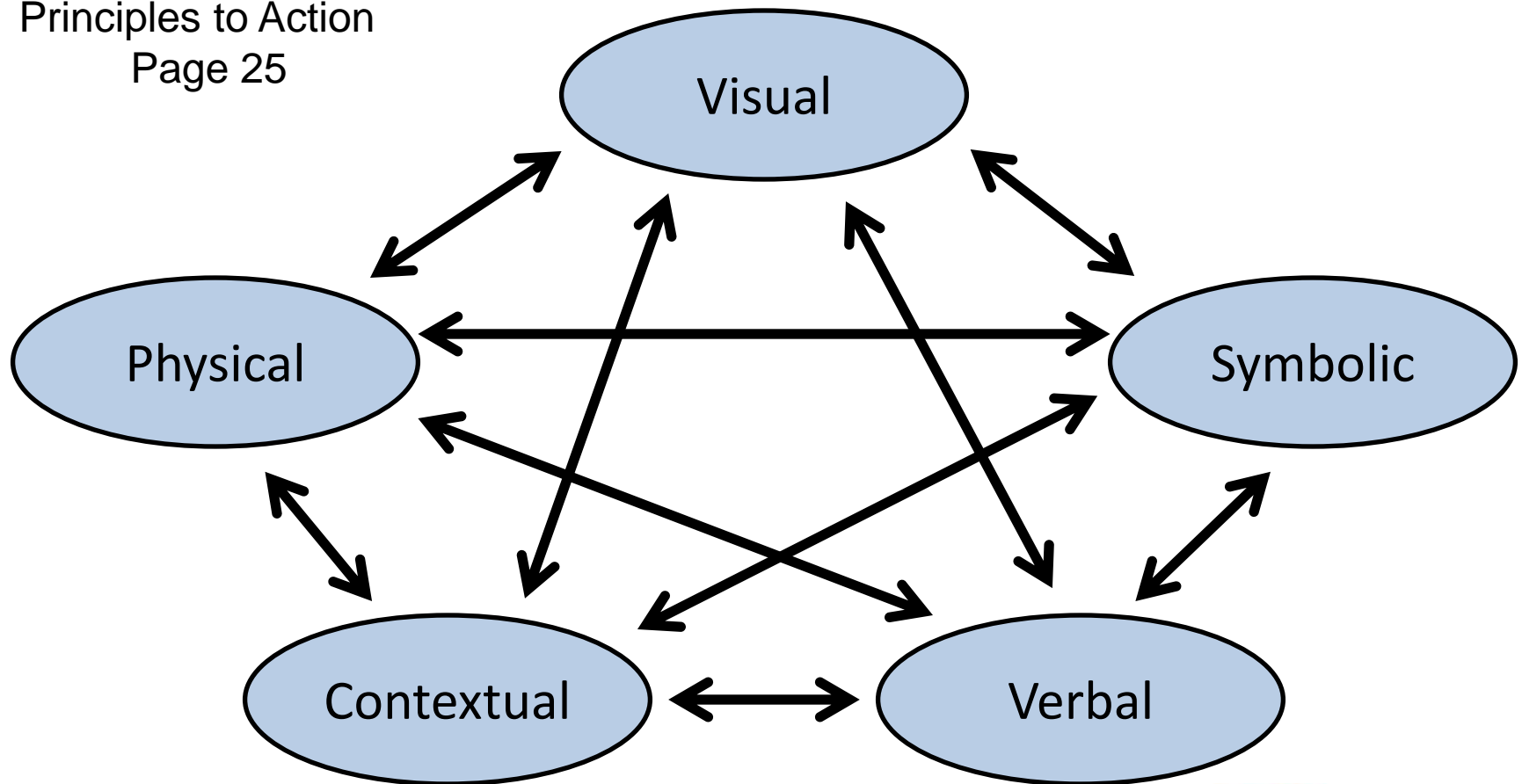
- Show pattern on a t-table
- Discuss: Guiding Questions



# Ladybugs	# Spots on Each	Total # Spots
1	+10	
2	10+10	
3	10+10+10	
4	10+10+10+10	
Total # of Ladybugs		

# Lesh Translation Model

Principles to Action  
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# Circles and Dots

*Set Model Connection*

# Circles and Dots Task

Focus: Groups of Skip Counting Iteration

- Roll a die = number of circles
- Roll again = number of dots in each circle
- Language: I have \_\_\_\_ circles w/ \_\_\_\_ dots each.
- There are \_\_\_\_ dots in total.
- Discuss: Skip counting and Patterns on 100 chart connections?



# Circle and Dots Class Data

- Collect class data
- Record: Number of times each product occurred
- Why did some products/totals occur more often than others?

Why did some not occur at all?

What do you notice?

# Processing Circles and Dots

- What understanding is developed?
- Lesh Connections
- Guiding Questions:

What are students learning?

What might be evidence of learning?

Instruction: What's next?

# OGAP Multiplicative Framework

- Compare the Circle and Dots to:

Additive Strategies

Transitional Multiplicative Strategies

Connections between this task and Framework models?

**OGAP:** Vermont Mathematics Partnership On-going Assessment Project

Funded: NSF (EHR-0227057) and US DOE (S366A020002)

# Name Arrays

*Multiplication-Array Model*

*Division Connections*

# Kindergarten: Counting and Cardinality Standards

Count to tell the number of objects:

**K.CC 5: Count to answer ‘how many?’**

**questions** about as many as 20 things arranged in a line, **rectangular array**, or a circle or as many as 10 things in a scattered configuration;

# Grade 2: Common Core Content Standards

2.OA: Work with equal groups of objects to gain foundations for multiplication

2.OA.3: Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2's;

**2.OA. 4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns.**

# Literature Connection

- 100 Hungry Ants
- Multiplication and Division Connections-
  - Arrays
  - Groups of
  - How many in each group?



# Name Arrays: Part 1

## How many letters in all of our first names?

- Task: Build your name tower with snap cubes

Group by length of the tower

- Record your Name on 1" grid paper strips...

Make 3 copies and share your name strips  
with other tables.

Build and record name arrays

Example: 24 letters in 4 names with 6 letters  
each.



# Cuisenaire Rod Arrays

- Build name with Cuisenaire **unit rods**
- Find **rod (rods) - equivalent** to unit rods
- Use 3-4 of these rods to build an array-

Record: I have \_\_\_\_\_ rods of \_\_\_\_\_ length.

There are \_\_\_\_\_ units all together.

- Commutative Property
- Discuss: Guiding Questions

# Array Investigation

What numbers have only 2 arrays?

Which quantity has the most arrays?

Record arrays on grid paper.

Organize your information.

- Guiding Question Discussion-

Mathematical understandings being developed?

Patterns?

Connections to commutative property?

Division Models?

# Looking Ahead: Grade 3

## Area of Rectangles

- 3 OA 3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement.....
- **3 MD 7: Relate area to the operations of multiplication and addition.**
  - a) Find the area of rectangles by tiling
  - d) Recognize area as additive.

# Processing: Name Arrays

- Grade 3 Connections?
- Alignment with current practices?
- What can I include in my instruction?
- Guiding Questions

# More Array Investigations

- How Long? How Many?
- How many squares on a checker board?
- Connect 4 Game: Another array model

# Models and Representations

The effective use of physical models and representations is a key component of successful mathematics teaching (Clarke and Clarke 2004). Thus, developing a deep understanding of multiplication and division requires that we consider the various representations and interpretations of these models.

# Division Connections to Arrays

*Repeated Addition or Subtraction*

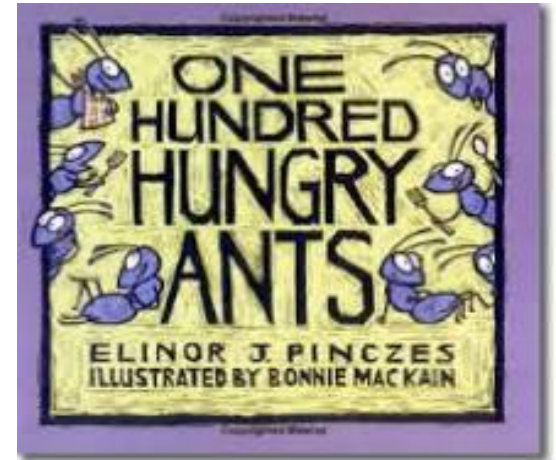
*Array and Skip Counting*

*Division Models*

# One Hundred Hungry Ants

## Literature Connection

- What if there were 12 hungry ants?
- What are the different arrays that they could form?
- How would the arrays look?
- Commutative Property?





$$12 \div 3$$

- Model with manipulatives
- Discuss models with tablemates
- What do you notice?

# Division Models: $12 \div 3$

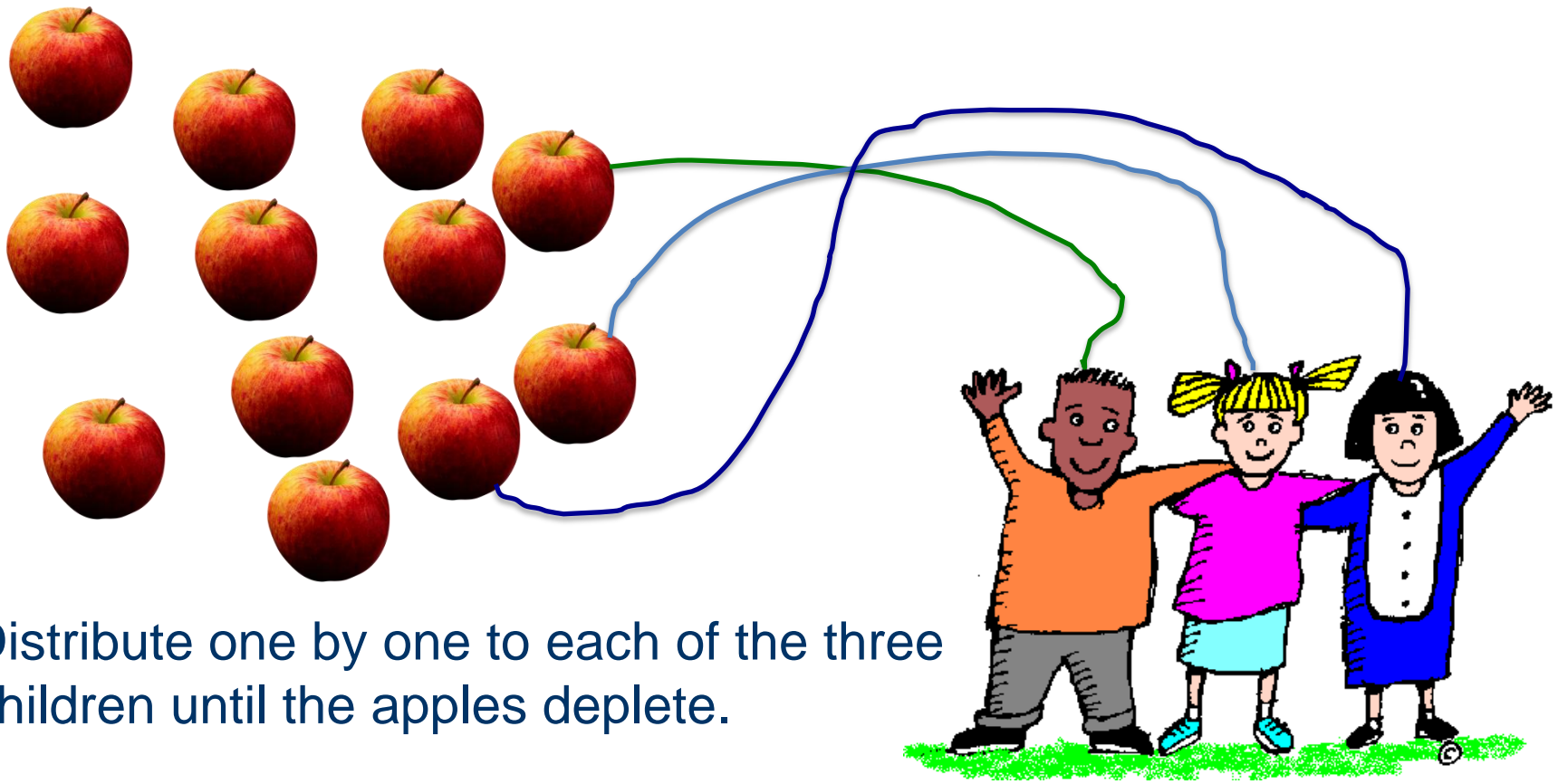
- There are 12 apples shared equally by 3 students. How many apples does each student get?

*Partitive or Sharing Model*

- There are 12 apples. Each bag has 3 apples. How many bags?

*Quotative or Measurement Model*

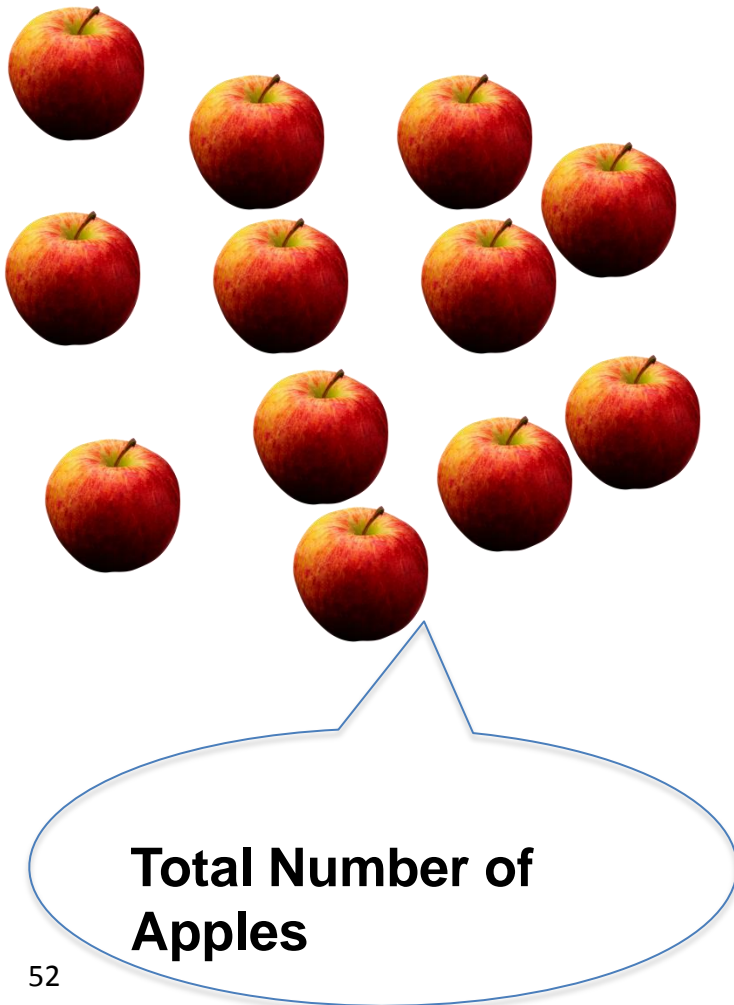
# MODELING THE PARTITIVE DIVISION



Distribute one by one to each of the three children until the apples deplete.

$$12 \text{ apples} \div 3 \text{ children} = 4 \text{ apples / child.}$$

# SOLVE THE PROBLEM USING THE PARTITIVE DIVISION MODEL

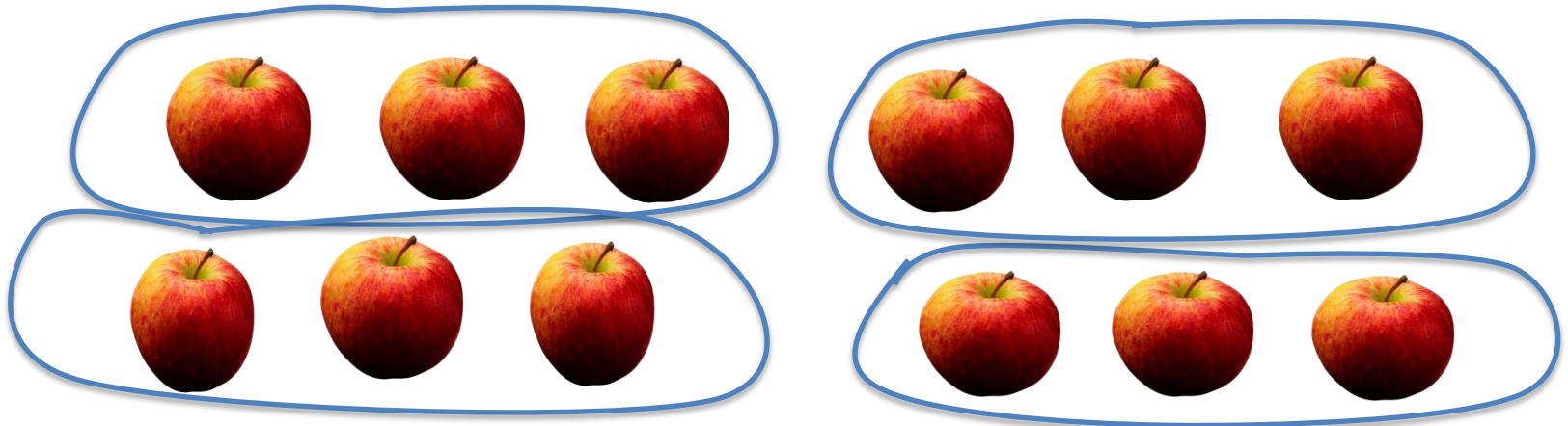


# Division Model: $12 \div 3$

- There are 12 apples grouped in bags with 3 apples in each bag. The question is how many bags?

*Quotative or Measurement Model*

# MODELING THE MEASUREMENT DIVISION



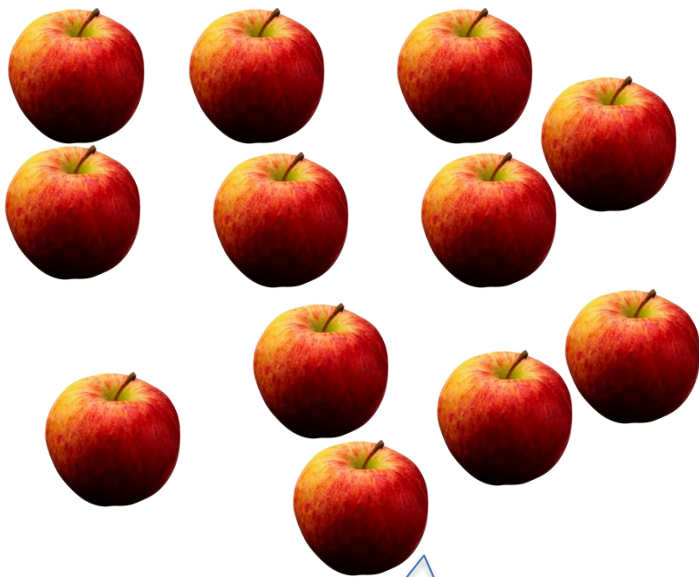
$$12 \div 3$$

Measure out the number of apples per each bag to produce the answer, which is the number of bags.

12 apples  $\div$  3 apples per bag/how many bags?

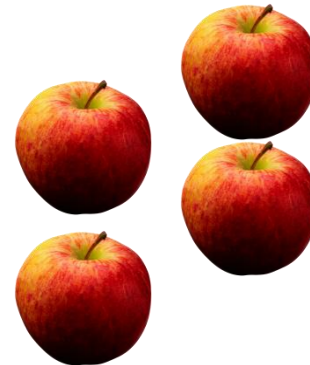
# SOLVE THE PROBLEM USING MEASUREMENT

## DIVISION



**Total Number of Apples**

**Number of Apples for Each Person**



# Two Models of Division

## Partitive (Fair-Sharing)

- Also known as “the How Many in Each Group? Interpretation”
- $A \div B$  means the number of objects in each group when **A objects are divided equally among B groups.**

## Measurement (Repeated Subtraction)

- Also known as “the How Many Groups? Interpretation”
- $A \div B$  means the number of groups when **A objects are divided into groups with B objects in each group.**



- The distinctions between the measurement and partitive division problems are critical because children initially solve them in very different ways, reflecting the different information given in the problem.

# Multiplication and Division Problems

# Write Multiplication/Division Problems

- Use the numbers: 2, 4 and 8
- Write a multiplication and division problem
- Model and Analyze the problem
- Write ‘problems’ that model other problems types
- Discuss: ‘How are these experiences preparing students Grade 3? Beyond?’

# Looking Ahead: Grade 3

## Area of Rectangles

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# Multiplication and Division Problem Types

- Connect Problem Types to:
  - Skip Counting
  - Things that Come in Groups
  - Patterns on 100 Chart
  - Circles and Dots
  - Name Arrays

# Begin with the end in mind...

‘Best practices’ Connected to:

Language Representations Problem types

- Conceptual understanding and ‘making-sense’
- Concrete ‘representations’
- Properties of operations

# THANK YOU

For your time and dedication to mathematics  
teaching and your students' learning.  
All the best in the 2014-15 school year!

# Disclaimer

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