

Counting and Cardinality and Place Value

NCTM Interactive Institute, 2014

Friendly Facts

- Write three facts about yourself on a white index card. Each fact must contain a number. When you write the fact, leave out the number.
- Write the three numbers for your facts IN ORDER on a blue index card.
- Do NOT write your name on the white or blue cards.
- Place your white and blue cards in the middle of the table.
- Mix them up.

Friendly Facts Match

- Match the blue cards with the correct white cards
- How were the numbers used in each fact?

Number Sense

- Number sense can be described as a good intuition about numbers. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

(Howden, 1989)

Three Big Number Ideas

- Number Meaning
- Number Relationships
- Number Magnitude

MathWorks/MAEF, 2004

Number Meaning

Cardinal

- How much?
- How many?
- What part of?

Ordinal

- In what order?

Nominal

- Label
- Number used as name

Number Relationships

**Decomposing and
recomposing numbers**

**Ideas about less than, greater
than, in between**

**Subitizing and noticing
patterns**

Number Magnitude

Understanding that numbers are neither large nor small unless you have a context in which to understand them.

Operation Sense

Understanding the relative effect of operating on numbers.

What Number Ideas Did You Hear?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

“Papa,
you’re
almost at
the end!
Did you
start at
1?”

Workshop Goals

Participants will ---

- Represent and solve a problem (Activity 1) requiring an understanding of counting and place value (Essential Understandings 1, 2, and 3).

Workshop Goals

Participants will –

- Discuss and reflect on pedagogy necessary to teach counting, comparing, and representing numbers (Essential Understandings ONE and TWO):
 - Identify counting and sequencing criteria (Activity 2)
 - Match counting and comparing standards to activities appropriate in Prek – 2nd grade (Activity 3)
 - Use number counting and comparing vocabulary to solve a color tile clue problem (Activity 4)

Workshop Goals

Participants will –

– Discuss and reflect on pedagogy necessary to teach place value concepts. (Essential Understandings ONE and THREE):

- Play two different Place value games or activities. Identify the place value concepts that could be addressed (Activity 5)
- Use a variety of strategies to assess students' concepts of place value (Activity 6)

Goal ONE

Participants will ---

- Represent and solve a problem (Activity 1) requiring an understanding of counting and place value (Essential Understandings 1, 2, and 3).

Activity ONE: Problem

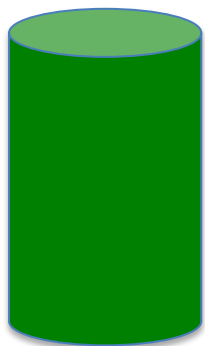


If I read every word in the USA Today newspaper (or the website) would I read more than 1,000,000 words?

Your answer must be represented in at least two ways.

Do you understand the problem?

- Use your CUPS to communicate your understanding as you work on the problem –



We understand
– no problem!



We have
some
questions



We are lost –
help!

Discussion Questions

- How did we address –
 - Essential Understanding One?
 - Essential Understanding Two?
 - Essential Understanding Three?
- How did our solution show that we understood the concepts of place value, counting, and decomposing and composing number?

Goal TWO

Participants will –

- Discuss and reflect on pedagogy necessary to teach counting, comparing, and representing numbers (Essential Understandings ONE and TWO):
 - Identify prenumber, counting and comparing concepts (Activity 2),
 - Match counting and comparing standards to activities appropriate in Prek – 2nd grade (Activity 3), and
 - Use number counting and comparing vocabulary to solve color tile problems (Activity 4).

Prenumber Concepts

- Classification
 - Done with or without numbers
 - Helps children make some sense of things around them and become flexible thinkers
 - Helps children identify what is to be counted

(Reys, Lindquist, Lambdin, & Smith, 2006)



Prenumber Concepts

- Patterns
 - Mathematics is the study of patterns.
 - Exploring patterns require active mental involvement and often physical involvement
 - In early grades, patterns help children develop number sense, ordering, counting, and sequencing
 - In later grades, it helps children develop thinking strategies for basic facts and algebraic thinking.

(Reys, Lindquist, Lambdin, & Smith, 2006)

- **Repeating Patterns** → → Emphasis is on the cyclical nature of the repetition and the id of the elements in the cycle.

(e.g., blue, red, blue, red, blue, red, two elements (blue, red) that repeat)

- **Growing Patterns** → → Shows an arithmetic change between pairs of elements in the pattern or a progression from one step to the next.

(e.g., 2, 4, 6 where each pair differs by 2)

Counting with Patterns



Prenumber Concepts

- Comparisons

- Leads to one-to-one correspondence
- Helps children become aware of relationships such as *more than*, *less (fewer) than* and *as many as*.

(Reys, Lindquist, Lambdin, & Smith, 2006)

- The concept of more is understood first.
- The concept of less is more difficult for children to understand because children have more opportunities to use the word more.
- To help children, pair less with the word more often.

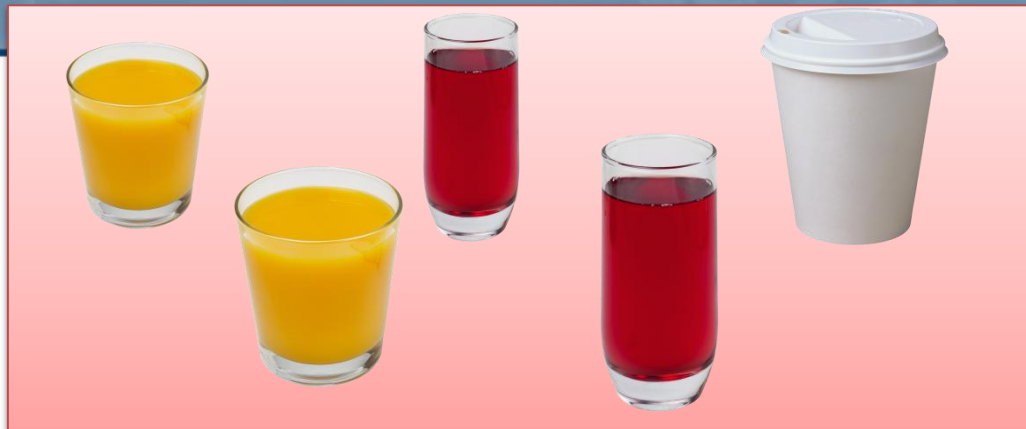
MathWorks/MAEF, 2004



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Who has more cups? Bonnie or Sammy?

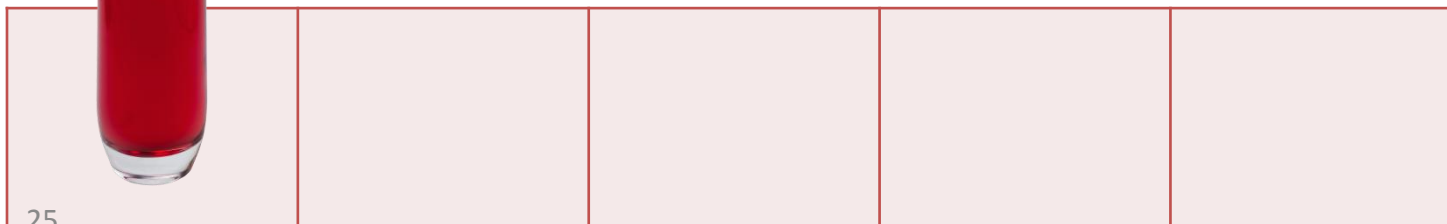
Bonnie's cups:



Sammy's cups:



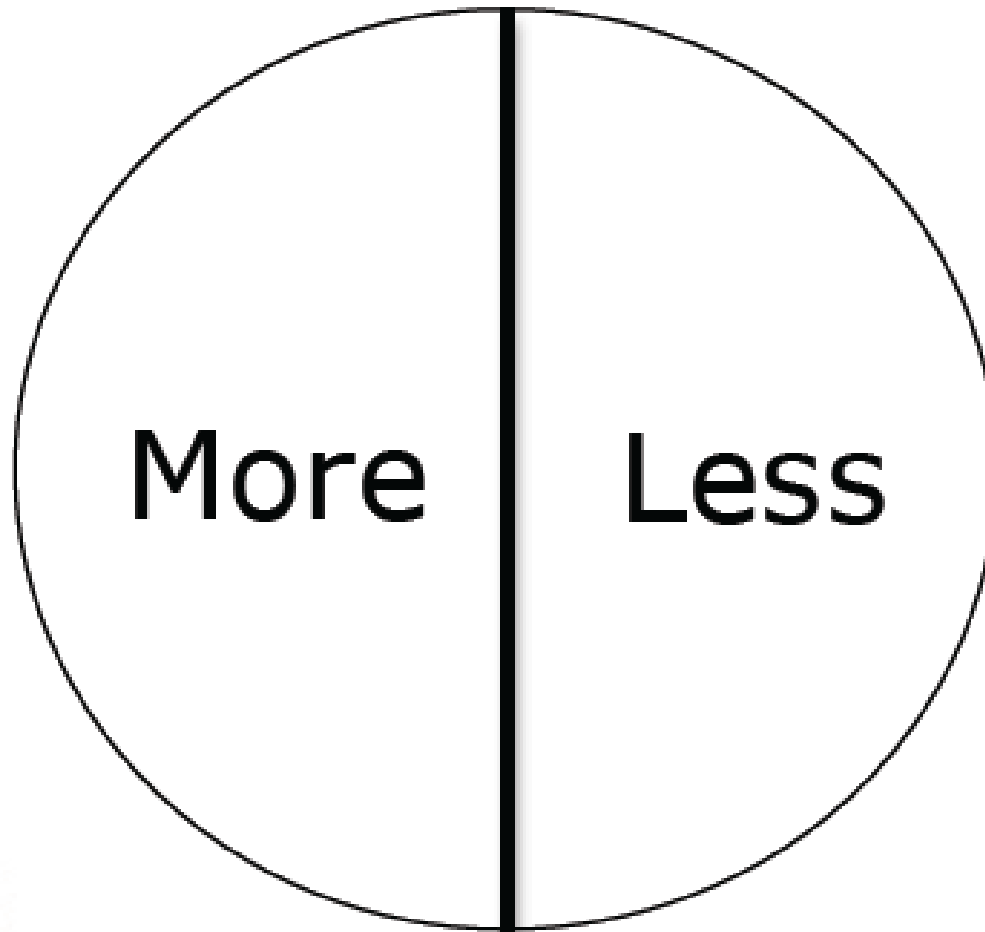
How about this?



When making comparisons, **students must be able to discriminate between important and irrelevant attributes.** For instance, with who has more cups – Bonnie or Sammy?, the cups are very different: their sizes, shapes, colors, contents, etc. Still, the procedure for setting up a correspondence is the same.

- Children need to become familiar with descriptions of relationships such as **more than, less (fewer) than, and as many as**.
 - Bonnie has *fewer* cups *than* Sammy.
 - Sammy has *more* cups *than* Sammy.
 - Bonnie has one *less* cup *than* Sammy.
 - Sammy has one *more* cup *than* Bonnie.
- Note that the child at the prenumerical stage has not yet developed the number concept to know that 6 is one more than 5, there are 5 ones in one 5 (lack of **quantitative reasoning**).

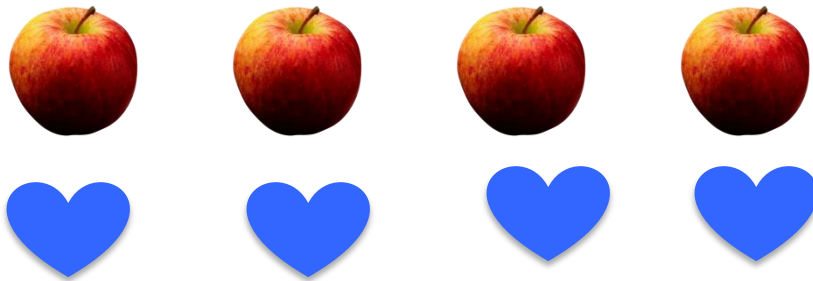
More or Less Game



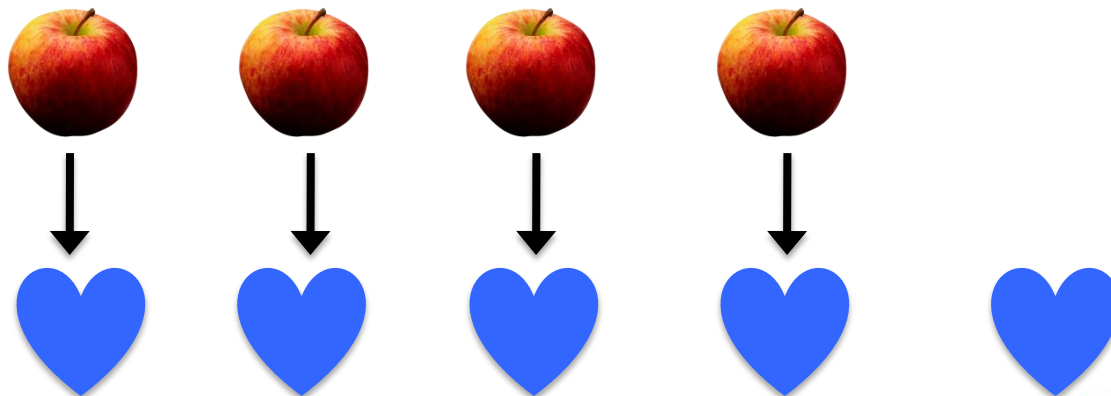
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Three Models for Making Comparisons

1. Counting
2. Physically comparing without counting



3. One-to-one



Prenumber Concepts

- *Subitizing* – Recognizing the Group

The skill to “instantly see how many” in a group is called *subitizing*, from the Latin word meaning “suddenly.” Sight recognition of quantities up to five or six is important for several reasons:

- It saves time.
- It is a forerunner of some powerful number ideas.
- It helps develop more sophisticated counting skills.
- It accelerates the development of addition and subtraction.

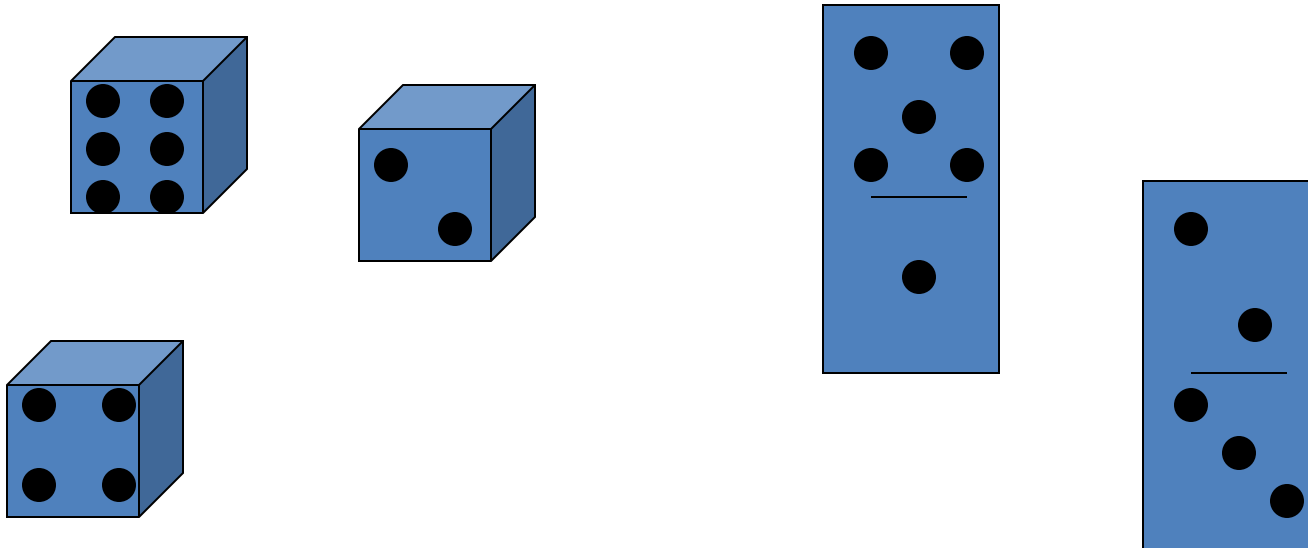
(Reys, Lindquist, Lambdin, & Smith, 2006)

Analyzing mental images can help students:

- Develop a strong visual sense of the structure of a number
- Develop understanding of how a number can be broken into parts
- Develop strategies for keeping track of and combining all parts to find the total number of dots.

MathWorks/MAEF, 2004

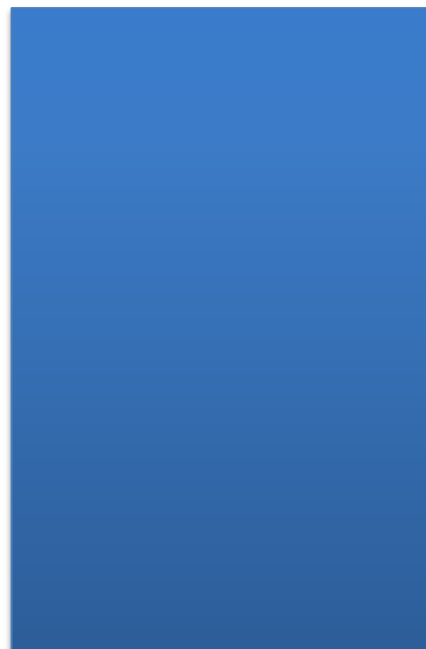
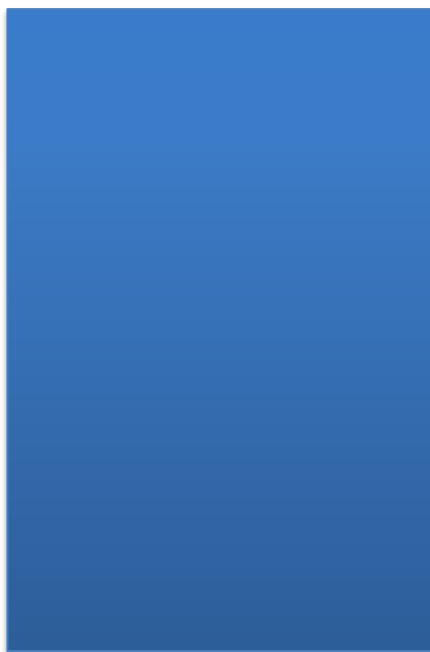
Domino Flash (Spatial Subitizing)



Conceptual Subitizing-









Prenumber Concepts

- Conservation
 - A given number does not vary when rearranged

(Reys, Lindquist, Lambdin, & Smith, 2006)

Counting

- To count successfully, a learner must...
 - Know the verbal sequence
 - Demonstrate one – to – one correspondence
 - Keep track
 - Say the last number to answer “how many” (cardinality)

Examples of Beginning Counting...

“One....two...three...four...

You’re gonna be in BIG trouble! “

“Ten....teen...teen....teen....teen....twenty!”

“PreKindergarten Counting” Video

- What do you observe that they understood?
- What do they not understand about counting?

**For children to orally count to 100,
they need to know:**

- ✓ the single –digit sequence 1 – 9;
- ✓ that 9 signals a transition;
- ✓ the decade vocabulary;
- ✓ each decade term is combined with the single-digit sequence 1 – 9; and
- ✓ there is an exception in the counting term between 10 and 20

Counting Principles

Principle	The child understands that:
One- One Principle	Each item is named with a distinct “tag”.
Stable Order Principle	The items are tagged in a stable, repeatable order.
Cardinal Principle	The final tag represents a property of the set as a whole.
Abstract Principle	These principles apply to any array or collection.
Order Relevance Principle	The same cardinal number results regardless of the order of enumeration.

Counting Strategies

- **Counting ‘What is There’:** Children count groups of objects and answer the “How many?” question.
- **Counting Out:** Children count out a specified number of objects from a larger set of objects.
- **Counting All:** When asked to make and count two groups of objects, children recount all the objects in both groups.

Counting Strategies

- **Counting On:** the child gives number names as counting proceeds and can start at any number and begin counting.
- **Counting Back:** when counting back, children give correct number names as they count backward from a particular point.
- **Skip Counting:** In skip counting, the child gives correct names, but instead of counting by ones, counts by twos, fives, tens, or other values. The starting point and direction are optional.

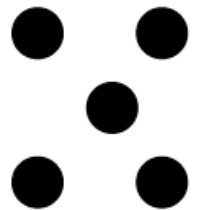
Grab Bag Video

Relationships with numbers

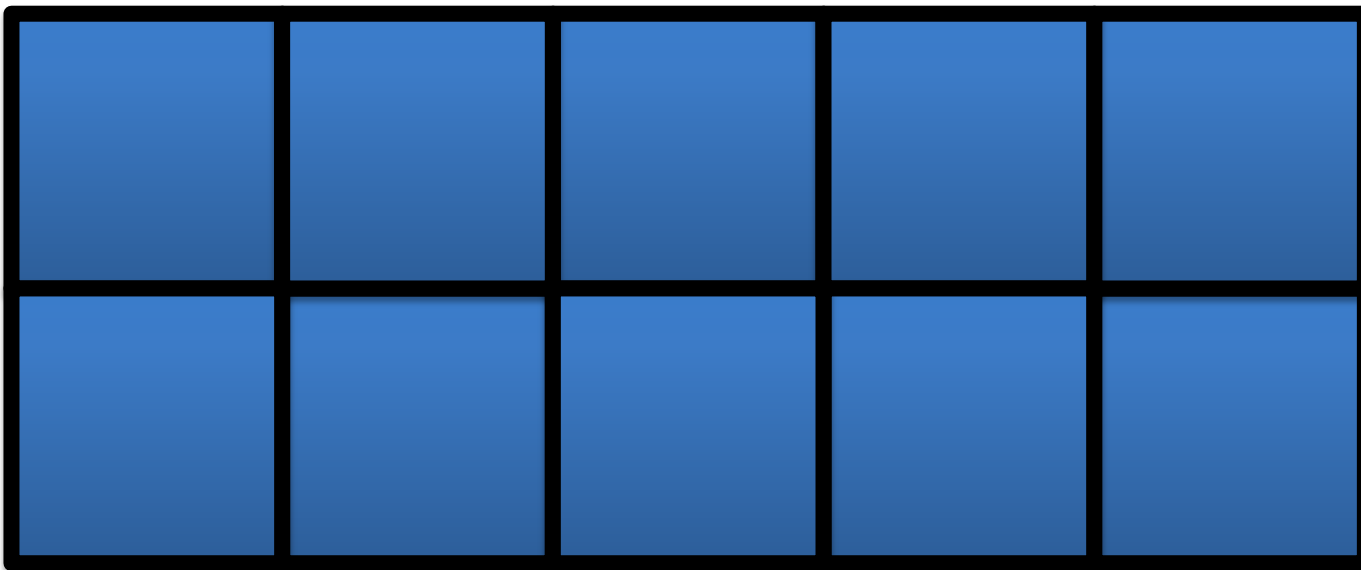
- Anchoring Numbers to 5 and 10
- Part-part-Whole Relationships
- Numbers 10 through 20

Experiences that help develop understanding of number concepts and relationships

- Instant recognition of patterned sets (dot arrangements, dot cubes, or dominos)
- 5 as a benchmark (5-frames)
- 10 as a benchmark (10 frames)
- 1 or 2 more, 1 or 2 less (dot patterns, flashing dots, domino trains)
- Part-part-total (shake and spill with bi-colored counters, part-part-total mats)

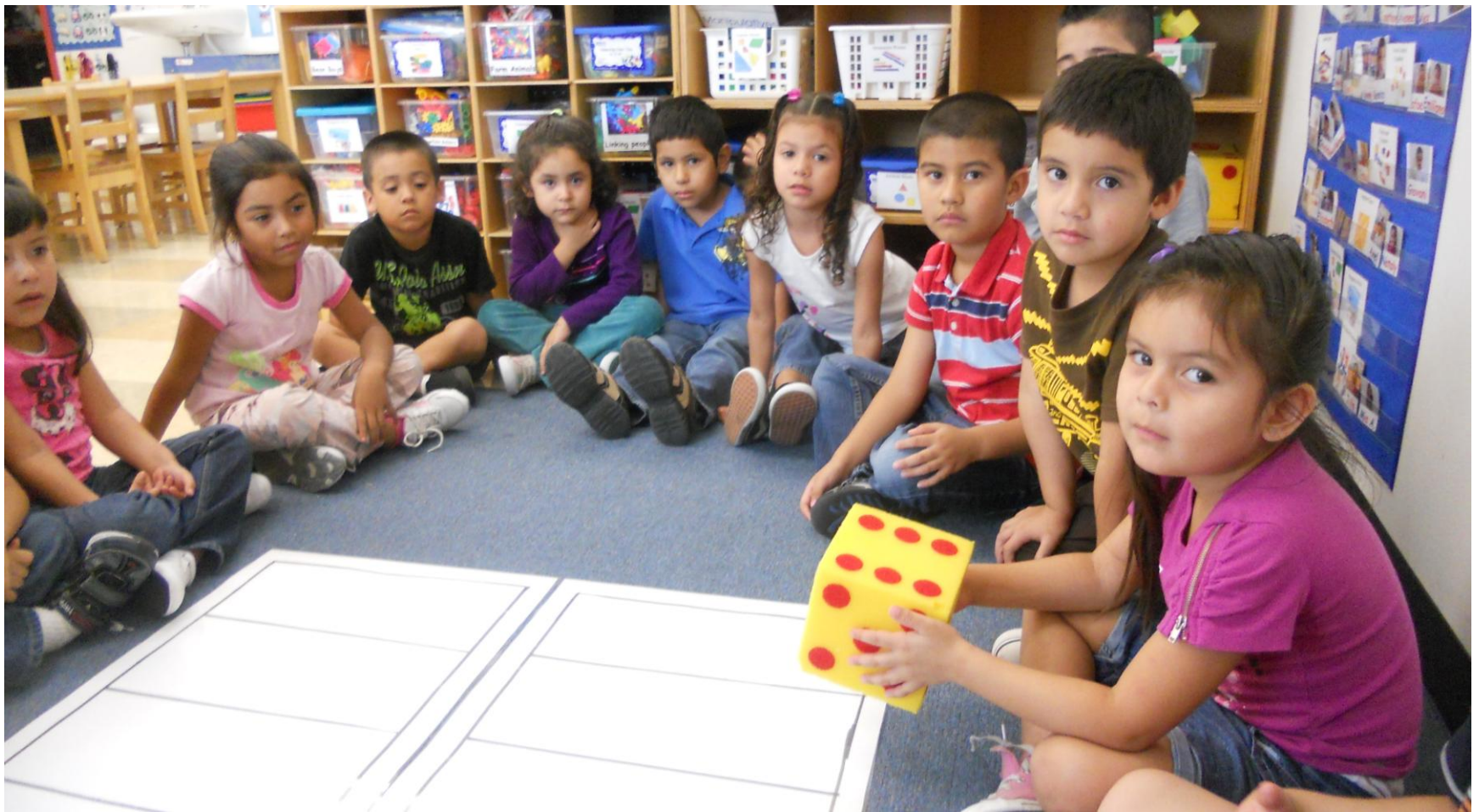


Counting with Models



ten-frame

Counting on a Ten Frame



Counting Using a Ten Frame





“Making 10 with a Die” Video

Counting on the Hundreds Board

Start at
34. Count
by ones to
78. How
many
numbers
did you
say?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Counting on the Hundreds Board

Start at 6.
Count by
tens to 86.
How many
numbers
did you
say?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Find That Number!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Pick a number and count 10 more. Where did you stop?.

Pick a number and count 20 more. Where did you stop?.

Pick a number and count 50 more. Predict where you would stop

Tell how you can use the hundred chart to mentally add 30 to a number.

Here is only a part of a hundred chart:

	D
	46
C	
	B
A	

Use what you know about a hundred chart to find the values:

A _____ B _____ C _____ D _____

Tell how you found C.

Could you find C in more than one way? Explain your answer.

Activity Three: Counting and Comparing Centers

- Complete your assigned center. Answer the questions.
- Then, select one more or all of the centers that relate to your classroom level.
- Share your results.

Activity Three: Centers

- Okta's Rescue from *Illuminations*
- Numeral Comparing Game
- Counting and Data
- Days in School
- Vocabulary Detective
- Bean Out
- Dominoes
- Decomposition Strings

Activity Four: Color Tile Riddles

- I have 9 color tiles and they are connected to make one filled square.
- I did NOT use any yellow tiles.
- The top row contains 1 green tile and 2 red tiles.
- The bottom row includes twice as many green tiles as the top row and half of the red tiles.
- The middle row has tiles that are all the same color. These tiles are NOT colored like the bottom and top row.
- In the whole square there are the same number of blue, red, and green tiles.
- Except for the blue tiles, like-colored tiles are NOT touching.

Goal THREE

Participants will –

– Discuss and reflect on pedagogy necessary to teach place value concepts. (Essential Understandings ONE and THREE):

- Play two different Place value games or activities. Identify the place value concepts that could be addressed (Activity 5)
- Use a variety of strategies to assess students' concepts of place value (Activity 6)



“That’s right, I’ve decided to give myself zero pay raise this year.”

Place Value

An understanding of number involves integrating several key concepts, such as unit, place value, and one-to-one correspondence... Ideas such as unit and decomposition that children encounter in prekindergarten through grade 2 set the stage for their development of fluent use of place value, leading to number flexibility and computational proficiency.

(Dougherty, Flores, Louis, Sophian, 2010, p. 41)

Characteristics of the Numeration System

(Adapted from Reys, Lindquist, Lambdin, Smith, & Suydam, 2004)

- **Place value:** The position of the digit represents its value: for example, the 2 in 23 names “twenty” and has a different mathematical meaning from the 2 in 32 which names “two.”
- **Base of ten:** The term base simply means a collection. Thus in our system, 10 is the value that determines a new collection, and the system has 10 digits, 0 through 9.
- **Use of zero:** A symbol for zero exists and allows us to represent symbolically the absence of something. For example, 309 shows the absence of tens in a number containing hundreds and ones.
- **Additive property:** Numbers can be summed with respect to place value. For example, 123 names the number that is the sum of $100 + 20 + 3$.

Place Value Development

- ***Counting by Ones***

- Physical item worth a count of “one more”
- Each item is a unit of one.
- Items being counted compose the total.
- Number word stand for its numerical value to represent the entire quantity.
- Example: *One, Two, Three, Four...Four blocks*

- ***Counting by Groups and Singles***

- A collection of items can be thought of as a group.
- Groups can contain different quantity of items.
- Example: *One, two, three groups of two, and one, two, three, singles*

Place Value Development

- ***Counting by Tens and Ones***

- “Ten” is used as a unit.
- Count by tens and ones: 10, 20, 30, 40, 41, 42, 43
- Tens and Ones are dealt with separately.
- Example: *When I add 15 blocks with 17 blocks, I have two tens and 12 ones*

- ***Equivalent Groupings***

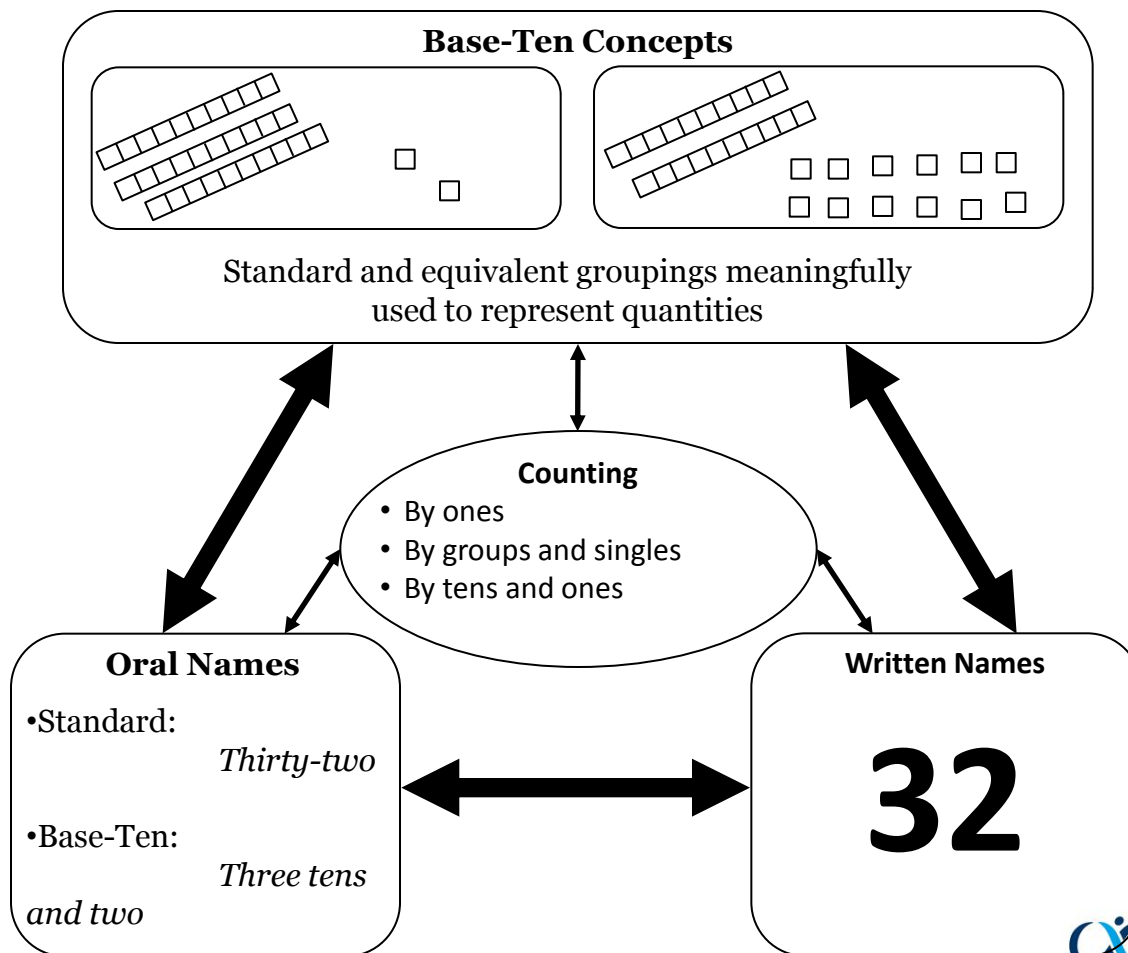
- “Ten” is used as a unit.
- Tens and ones are dealt with simultaneously
- Mental flexibility permits crossing between thinking of tens and ones
- Example: *Three tens and 13 ones is also equal to 43.*

It is critical that students see the relationship between number names such as “fifty-three” with the grouping of tens concept. They must also see that the way we write numbers (ones on the right, tens on the left of ones and so on) must be coordinated with the idea of groupings.

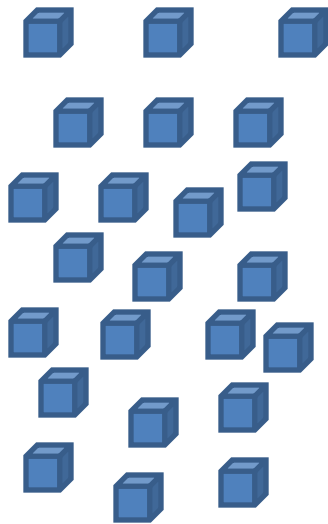
Van de Walle, John A., and LouAnn Lovin. 2006. *Teaching Student-Centered Mathematics: Grades K-3*. Boston: Pearson Education.

Place Value Triangle:

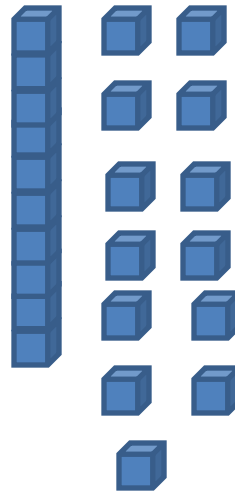
Connecting Concepts with Oral and Written Forms



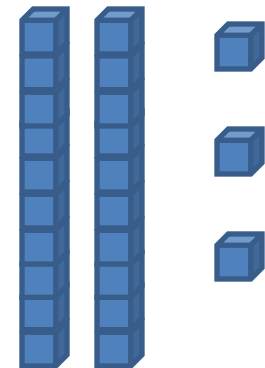
Representations of 23



23 ones



**1 ten and
13 ones**

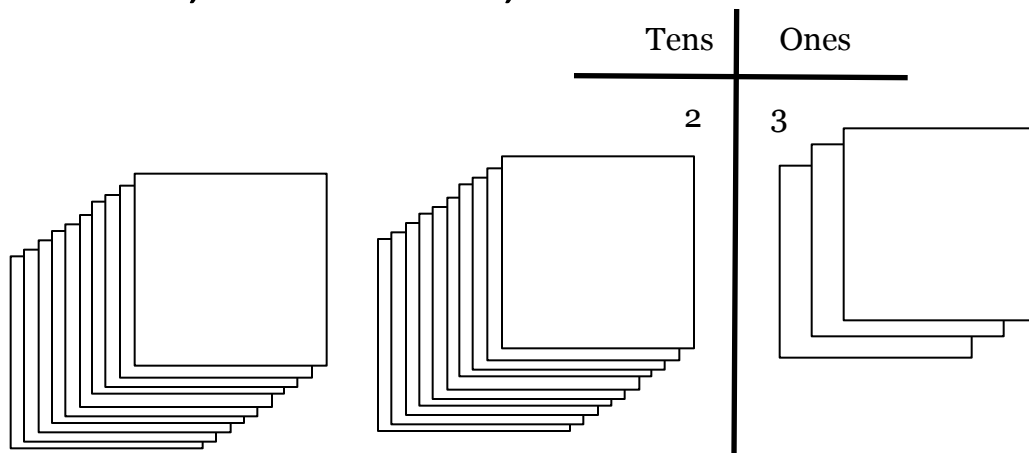


**2 tens and
3 ones**

Place Value Models (for base ten)

Proportional – The material for 10 is ten times the size of 1; 100 is ten times the size of 10.

Ex: base ten blocks, bean sticks, bundled sticks



Nonproportional – The material does not maintain any size relationships.

Ex: money, abacus, color tiles or chips

Activity Five: Games and Books for Place Value

- Select and play a place value game or activity with others at your table.
 - Grouping and Grazing
 - Clear the Board
 - 100s Chart Cover-Up
 - Two Dashes and a Throw Away
 - Seven to 100
 - Books and Place Value
- Answer the questions on the handout.

Activity Six: Assessments of Place Value Understanding

- Review children's responses to the place value problems –
 - What does the child understand about place value?
 - What does the child still need to know?
 - What activities/games would be helpful to increase his/her understanding?

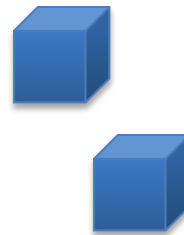
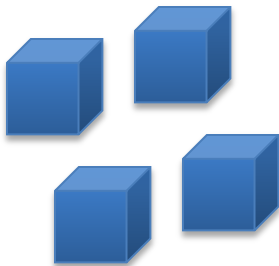
Assessment ONE

- Problem: Yesterday we wrote the numeral “39” on our “days in school” chart. How would we write today’s number?

Answer: “410” “forty ten”

Assessment TWO

- Problem: (Show the numeral 42. Point to the digit 4) Use your counters to show how many this is. (Point to the digit 2) Use your counters to show how many this is.
- Answer:

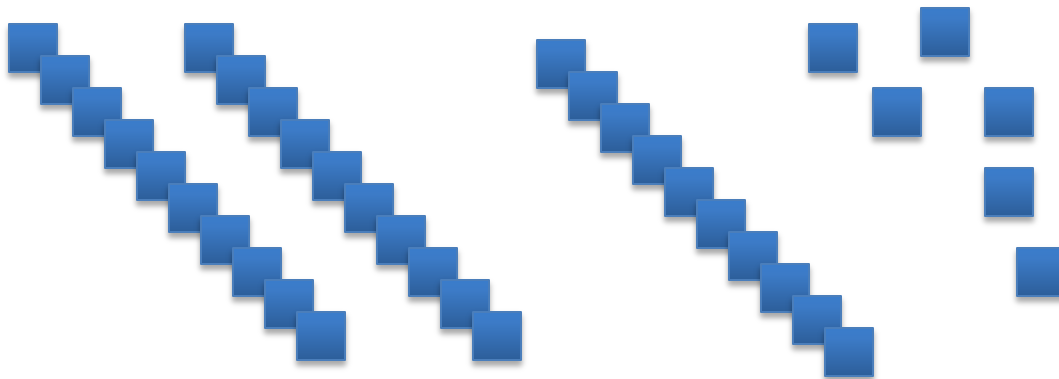


Assessment THREE

- Problem: How many ways can you show 125 using place value blocks?
- Answer: There are only 5 ways.

Assessment FOUR

- Problem: (Place 36 tiles on the table) How many groups of ten can you make? How many are left over? How many would you have if I gave you ten more tiles? How could you find out?
- Answer: Student made groups correctly. She counted out ten more to answer the question, and correctly said 46.

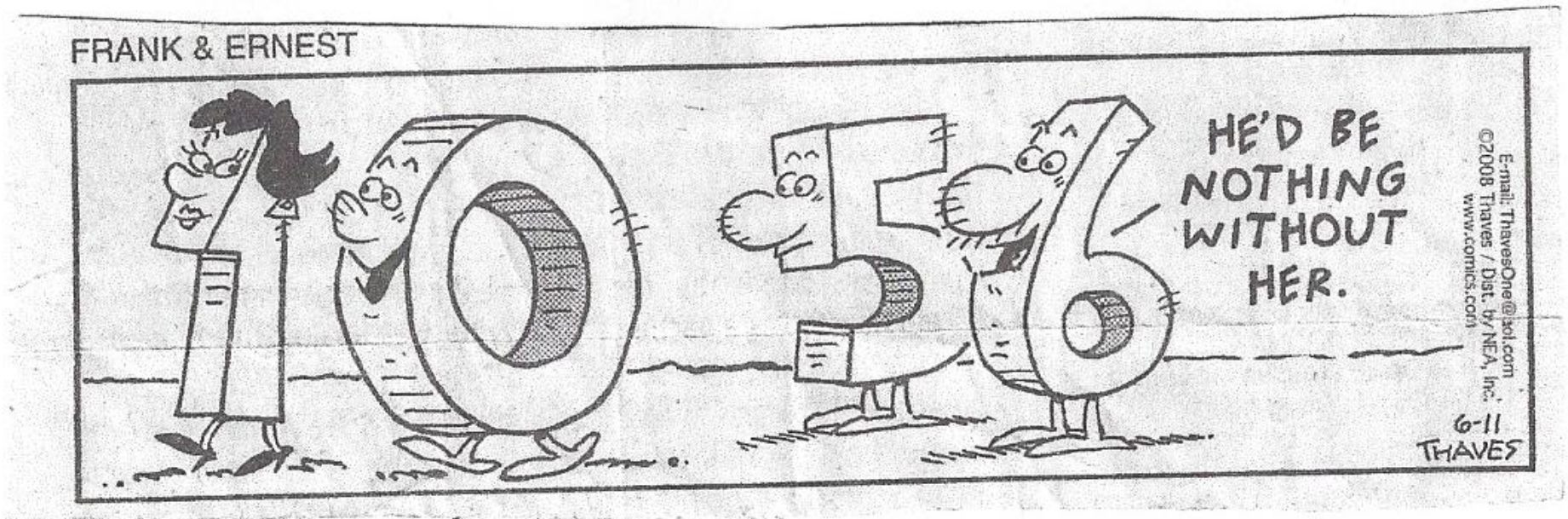


Assessment FIVE

- Problem: Find 37 on the hundreds chart.
How could you count up from 37 to 58?
- Answer: Begin with 37, then say... 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58.

Assessment SIX: “The Man Who Had 10 Children” Video

Would your students understand this comic?



Because of this workshop --

- I will keep doing –
- I will start doing –
- I will stop doing -

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