Session 3: Multi-digit Multiplication and Division

John SanGiovanni

Zachary Champagne

Jennifer Suh

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Factor Dazzle

1. Player 1 selects a number (put a chip/counter on the number).

2. Player 2 finds all of the factors for that number (puts a chip/counter on the number).

3. Players take turns until no more spaces are available.

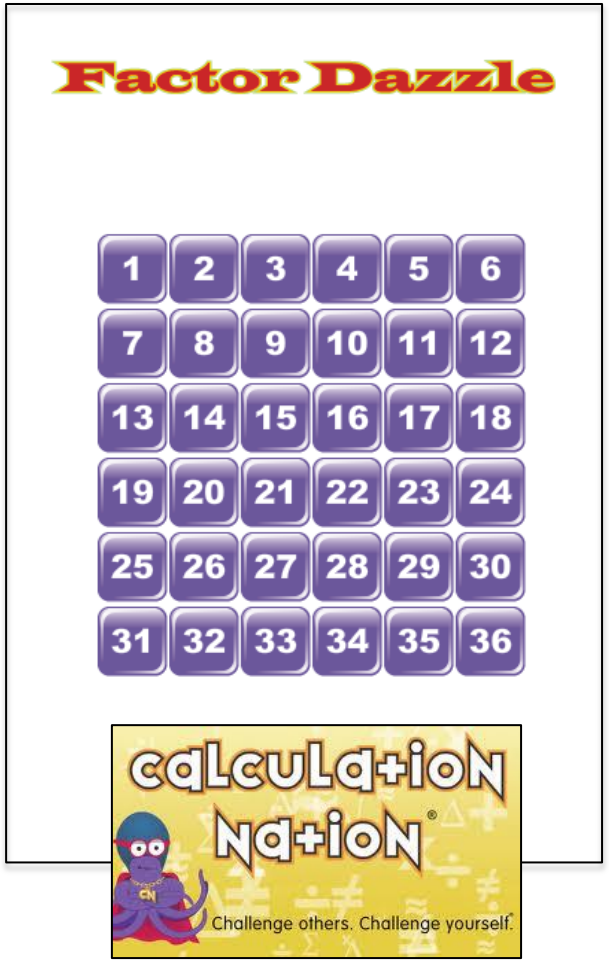
4. Players add numbers covered by their chip.

5. Player with the highest sum wins.

1	2	2	2	2	2
2	3	3	3	3	3
3	4	4	4	4	5
5	5	5	6	6	7
7	8	8	9	9	10
10	11	12	13	14	15
16	18	20	21	22	24
25	26	27	28	30	32

Factor Dazzle

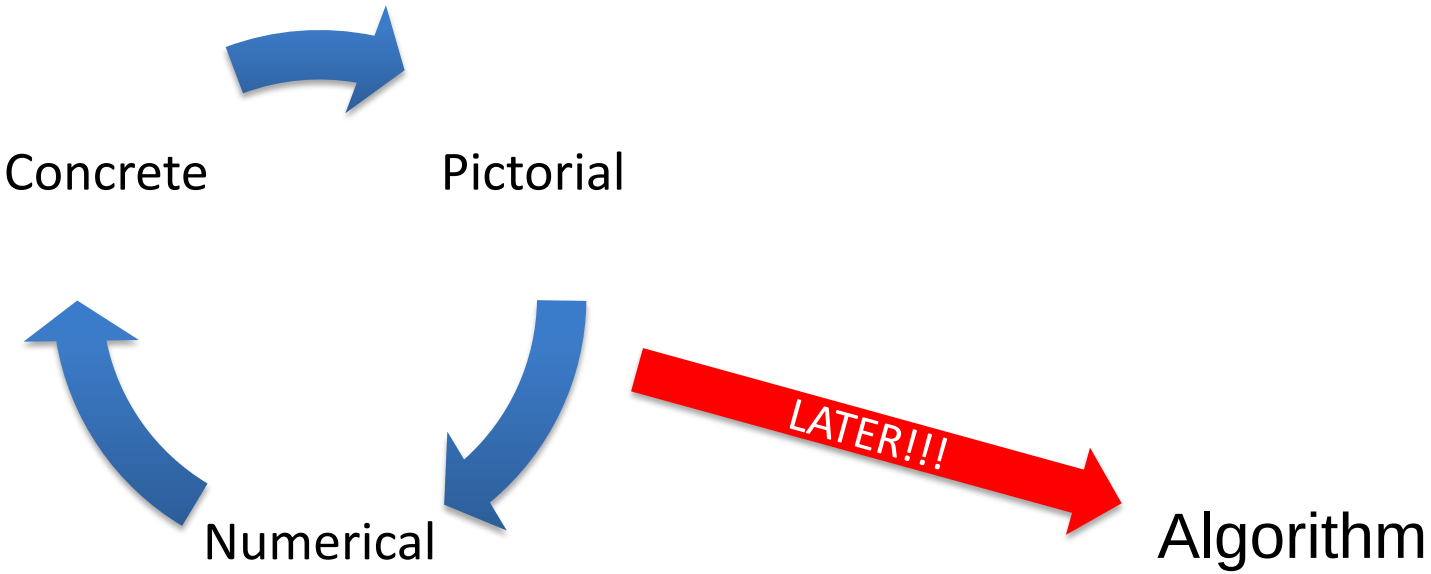
- Factor Dazzle is available on Calculation Nation.
- What would you look for as students played Factor Dazzle?
- How would you know if students are demonstrating understanding of factors?



About this session…

- Recognize the progression of understanding multi-digit computation (concrete, pictorial, numerical)
- Examine the meaning behind the algorithms

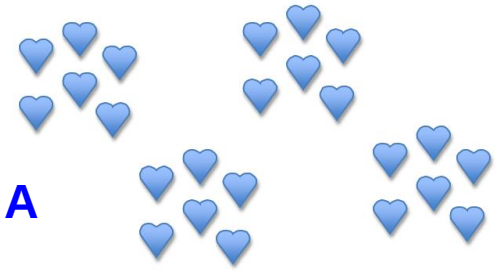
Recognizing A Critical Shift



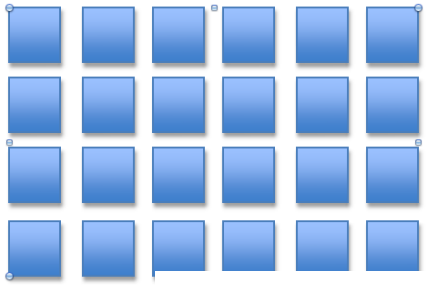
Think to yourself…

Which representation of 4 x 6 do you think is “best”? Why?

Which representation is “best” for 28 x 15? Why?



A

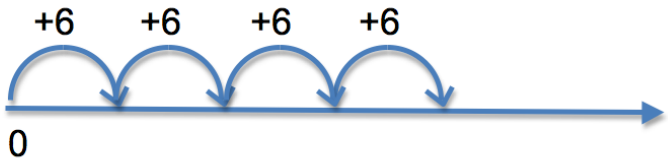


B

6 + 6 + 6 + 6 C

6 “

E



D

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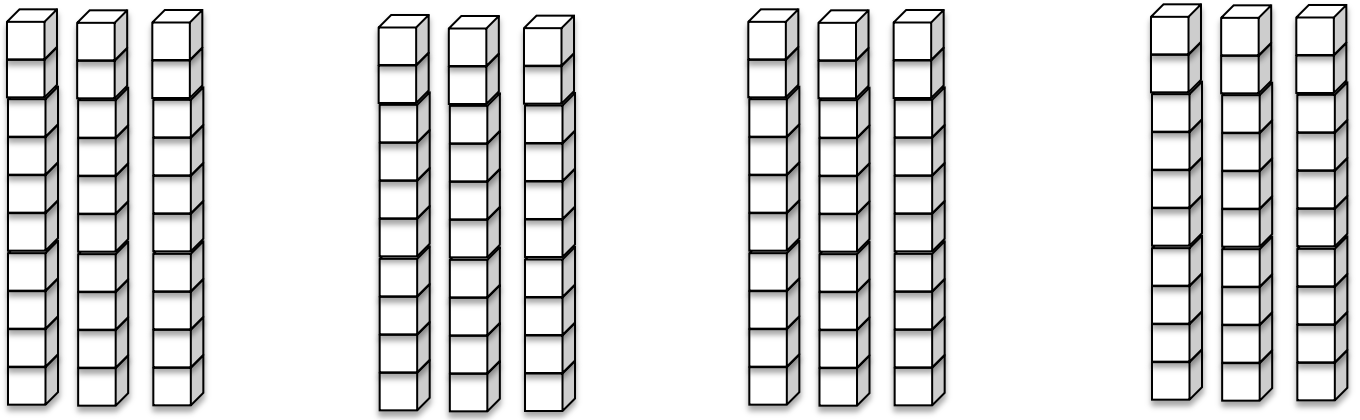
Let's continue this thought: Lots of Rods

- You are modeling a 4 x 30.
- When you look at your model, you notice that only base ten rods (no flats or units) were required.
- Why does this make sense?

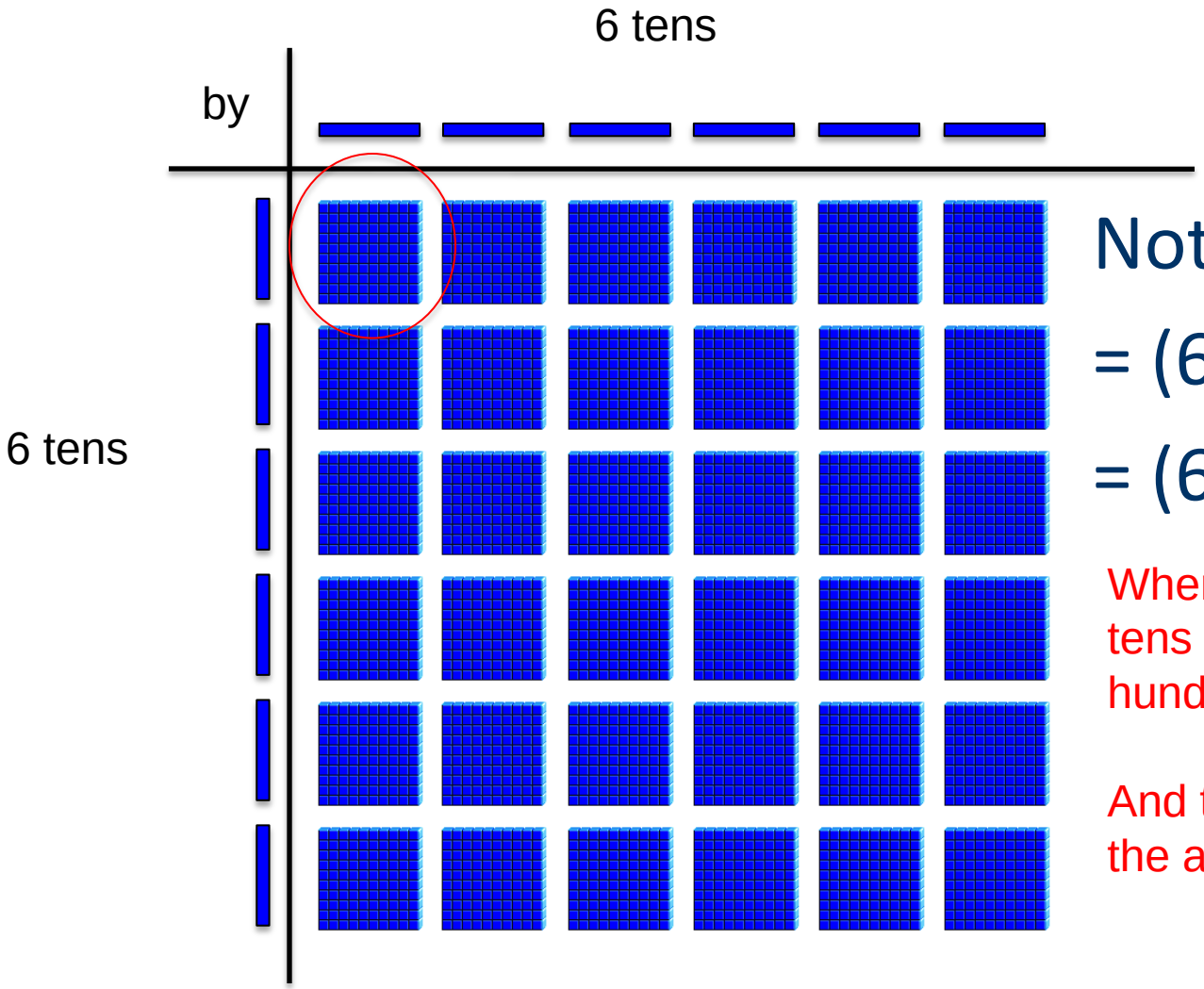
Multiplication involving multiples of 10

- Notice that it makes sense that 4 x 30 ends in a 0 since 4 x 3 tens = 12 tens (and no ones).

IT'S MUCH MORE THAN ADDING ZEROS.



How many seconds are in an hour?



Notice that 60 x 60

= (6 x 10) x (6 x 10)

= (6 x 6) x (10 x 10)

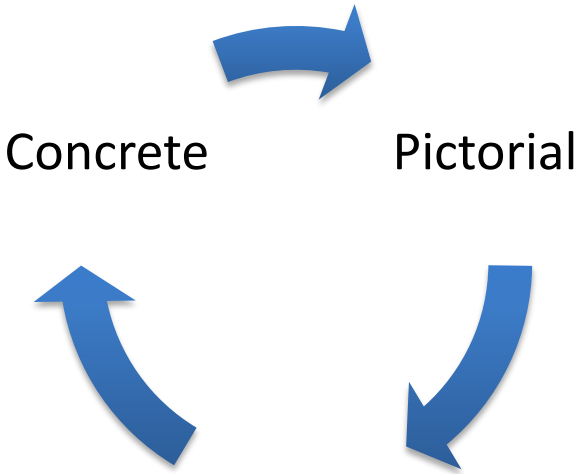
When we multiply tens by tens our product must be in hundreds.

And that we can make use of the associative property!

Model Your Solution

- There are 5 lines of people for an amusement park ride. Each line has 13 people. How many people are in line altogether?

Use base ten models and/or draw a picture to show your thinking.

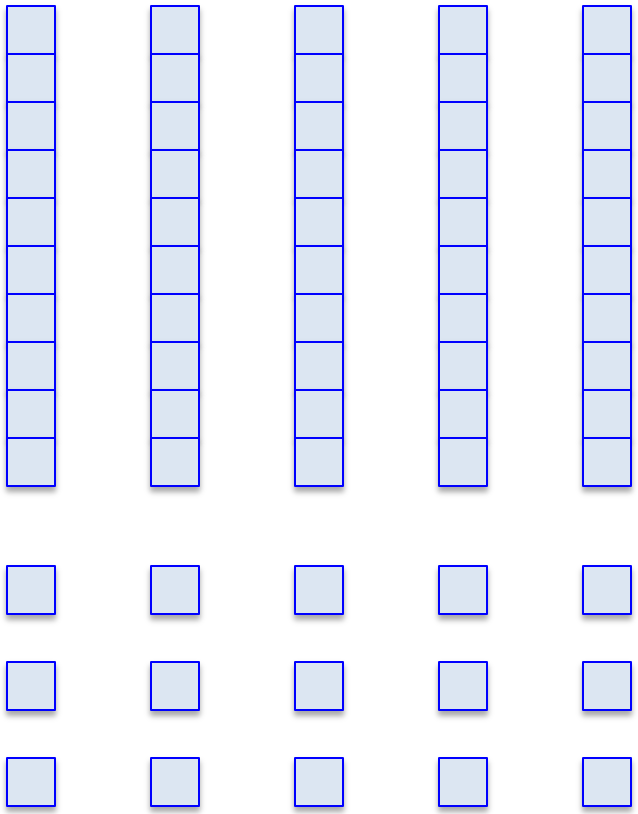


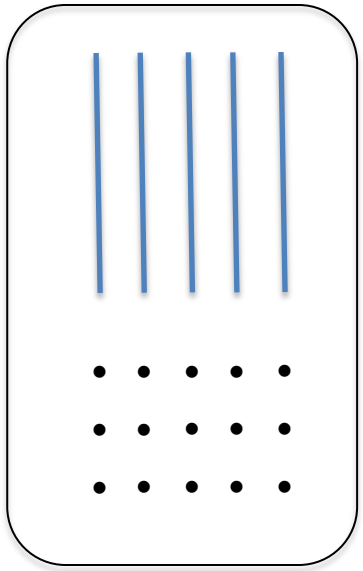
13 x 5

There are 5 lines of people for an amusement park ride. Each line has 13 people. How many people are in line altogether?

- **Concrete: Build with tiles/blocks**
- **Pictorial: Draw a representation**
- Number: What does solution look like with numbers?

Concrete: 13 X 5

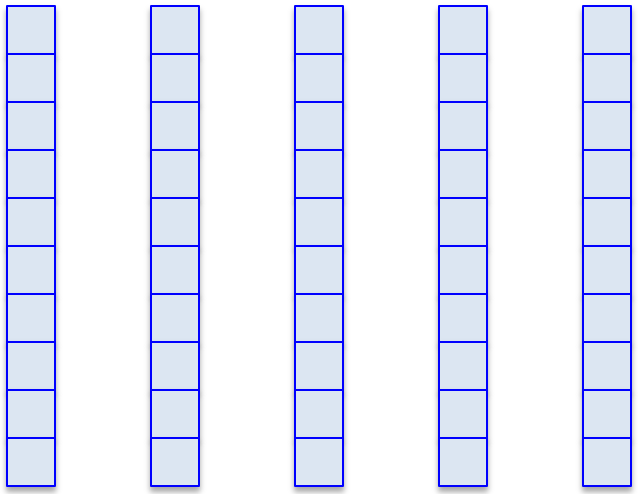




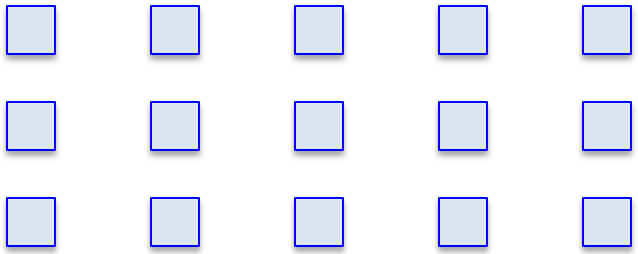
Pictorial – Sticks/Dots

Concrete: 13 X 5

10 x 5

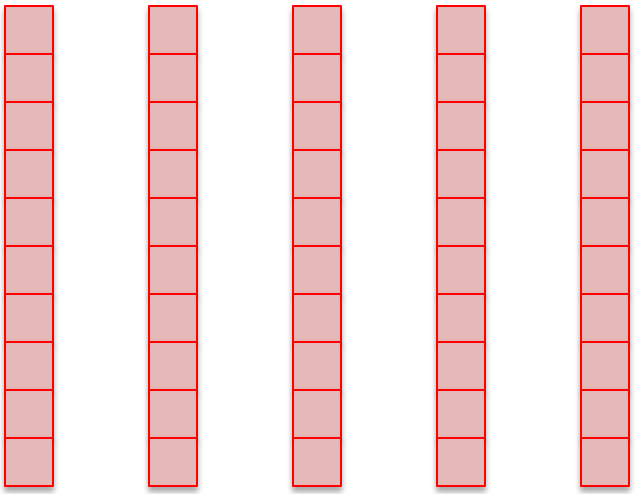


3 x 5



Concrete: 13 X 5

10 x 5



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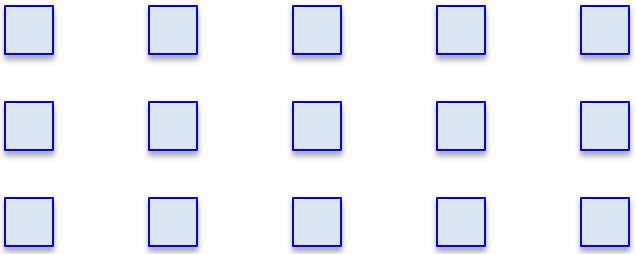
3 x 5



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We were able to think of 13 x 5 as (10 x 5) + (3 x 5). Which of the following would this idea also work with? How might you think of each of these?

34 x 7

61 x 9

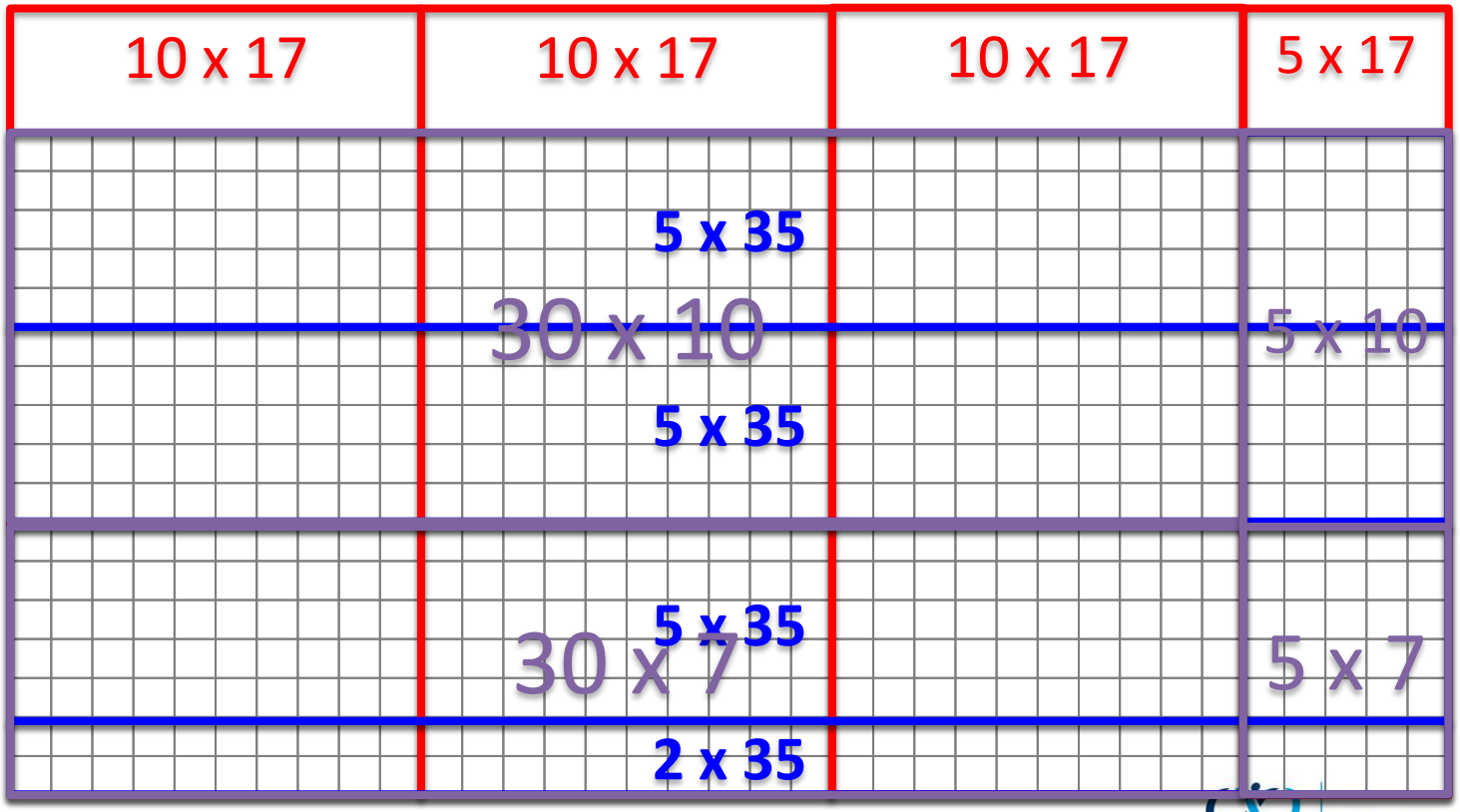
6 x 85

3 x 47

Would our models still work?

Do they lose their efficiency?

Mr. Kern's classroom is 35ft x 17ft. He is planning to partition it into 4 smaller rectangles for stations. What could be the sizes of the 4 stations?



Mr. Kern's classroom is 35ft x 17ft. He is planning to partition it into 4 smaller rectangles for stations. What could be the sizes of the 4 stations?

	30	+	5
10	300		50
+			
7	210		35

46 x 25
What's missing?

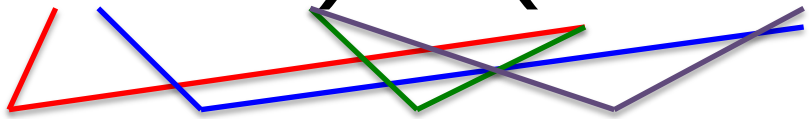
	40	+	6
20	800		120
+			
5	200		30

- How does the model show the answer to the multiplication problem 46x25?
- What do the numbers in the shaded boxes show?
- Is this way easier or harder than the traditional strategy?

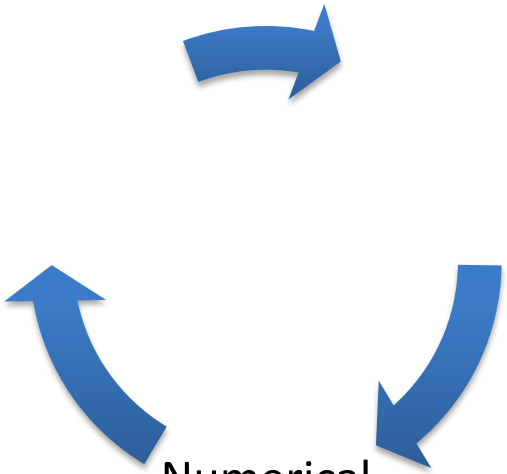
46 x 25?

(x + 3) (x – 8)

(40 + 6) x (20 + 5)



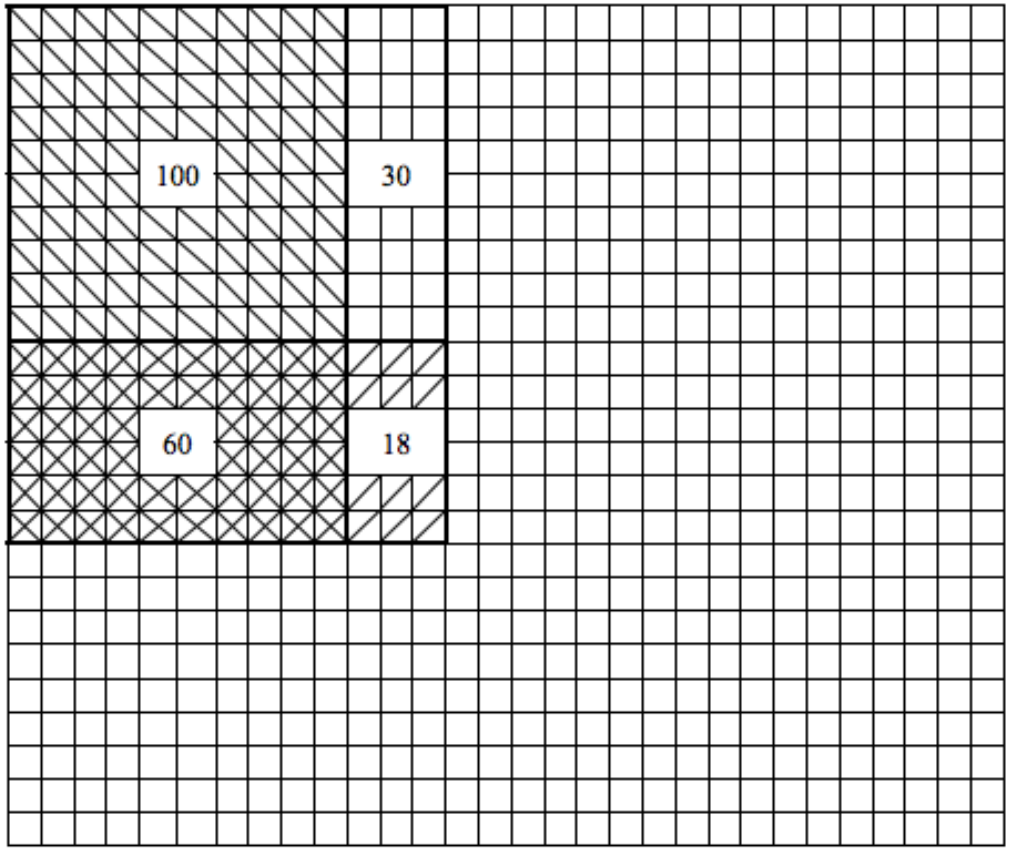
800 + 200 + 120 + 30 = 1,350



Opportunities for Practice

1. 13×16

	$10+3$
$\times$	$10+6$
	18
	60
	30
$+$	100
	208



Multiply and Conquer, Illuminations NCTM
http://illuminations.nctm.org/Lesson.aspx?id=3210

Oscar was at the top of a stadium looking at the parking lot. He wondered how many cars there were in a particular lot. He could see there were 48 rows of 32 cars.

How are the algorithms similar? How are they different?

Partial Products Algorithm

$$\begin{array}{r} 48 \\ \times\ 32 \\ \hline 16 \\ 80 \\ 240 \\ +\ 1200 \\ \hline \end{array}$$

Partial Products Algorithm

$$\begin{array}{r} 48 \\ \times\ 32 \\ \hline 1200 \\ 240 \\ 80 \\ +\ 16 \\ \hline \end{array}$$

Traditional Algorithm

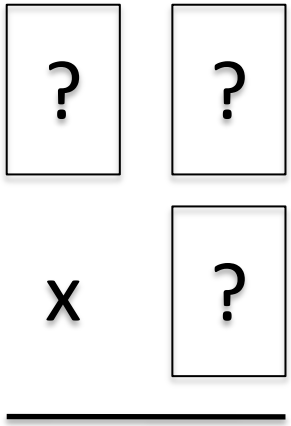
$$\begin{array}{r} {\scriptstyle 2}\ {\scriptstyle \cancel{1}} \\ 48 \\ \times\ 32 \\ \hline 96 \\ 1440 \\ \hline \end{array}$$

Practice-Rich Task

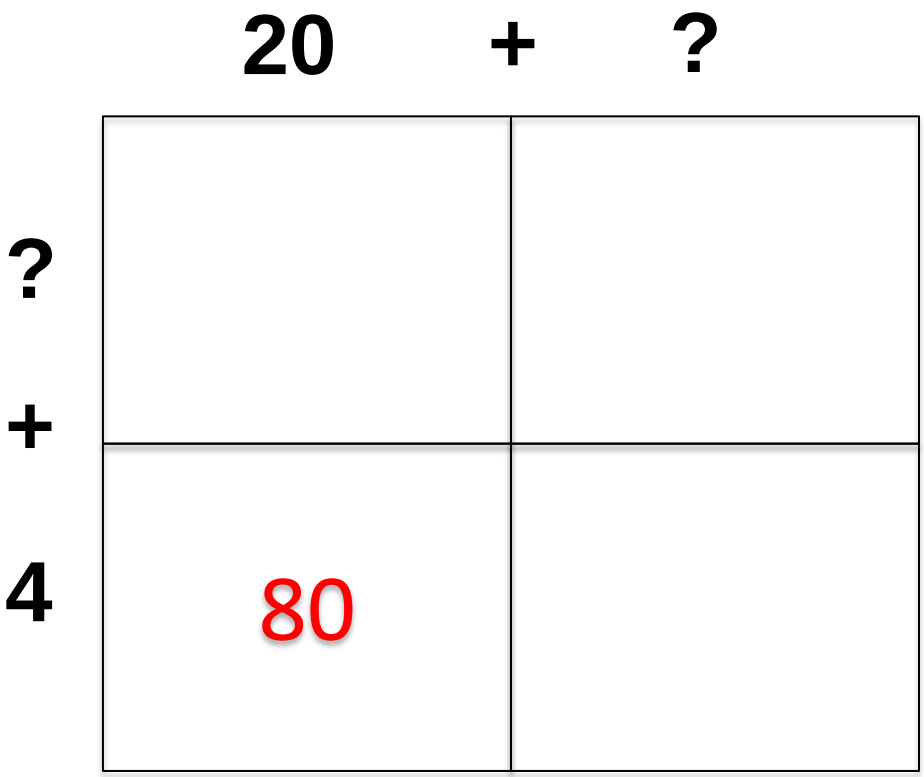
Using dice or digit cards generate 3 digits.

Create the largest product and the smallest product possible.

Justify your products.

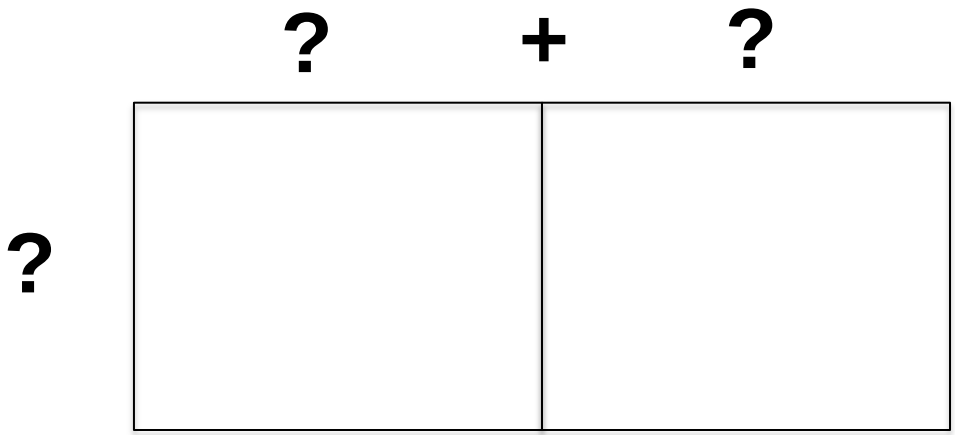


Missing Factors, Missing Products



Does It Work With Any Factor?

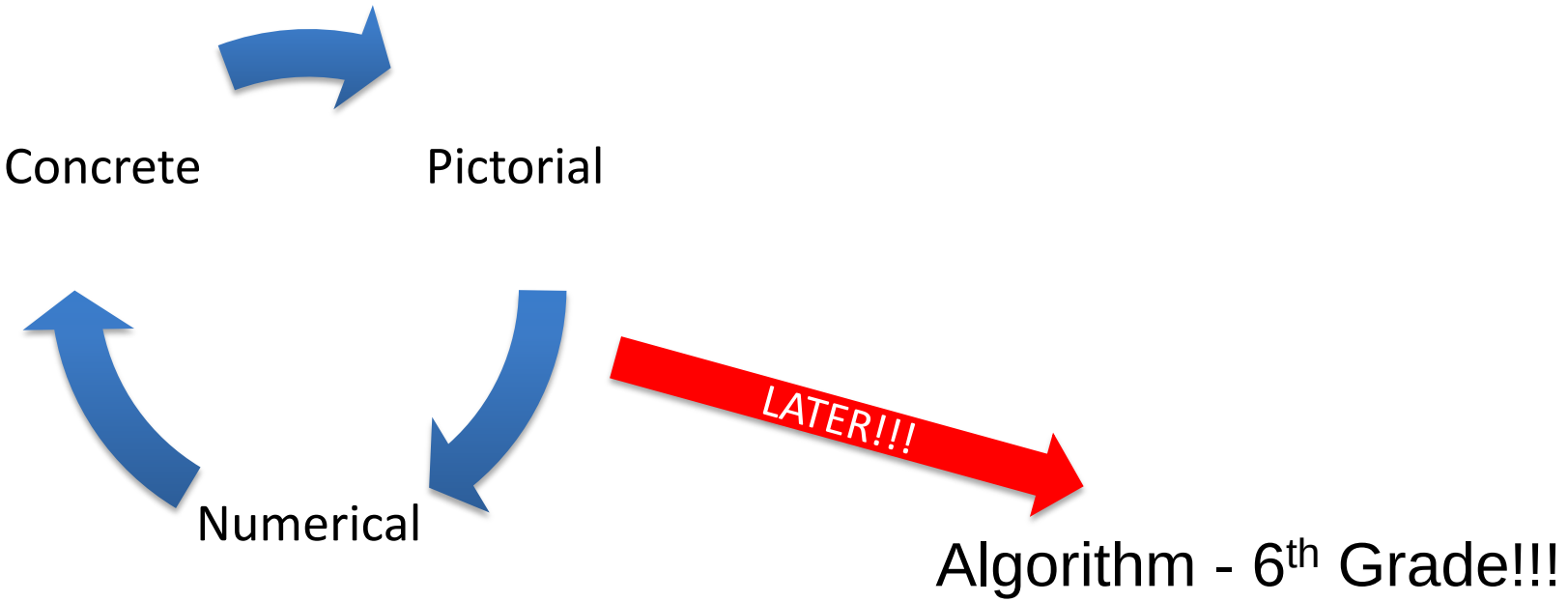
Basketballs weigh 1.4 pounds. There are 6 balls on the rack. What is the total weight of the basketballs on the rack?



Agree or Disagree? Why?

Jackson says he used the distributive property to solve 721 ÷ 7 because it is the same as 700 ÷ 7 + 21 ÷ 7.

Recognizing A Critical Shift



Can you “Build-Up” to this solution?

A librarian has 413 books to put on 8 shelves in the public library. Each shelf holds the same amount of books. How many books will she put on each shelf?

What if she started by putting 2 on each shelf?
How many groups of 2 could she put on each shelf?
Are there any leftover?
Is there a better (another) size of the group she could begin with?

Can you "Build-Up" to this solution?

A librarian has 413 books to put on 8 shelves in the public library. Each shelf holds the same amount of books. How many books will she put on each shelf?

51 books on each shelf with 5 books leftover.

51

8 shelves

10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1
10	10	10	10	10	1

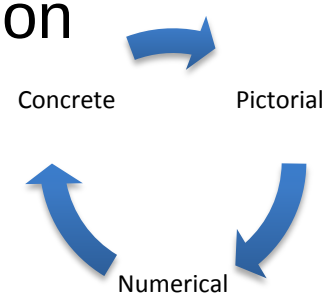
(80)	(160)	(240)	(320)	(400)	(408)
(10)	(20)	(30)	(40)	(50)	(1)

Dividing with Representations

8 dinosaurs shared 221 bones. How many bones did each dinosaur get?

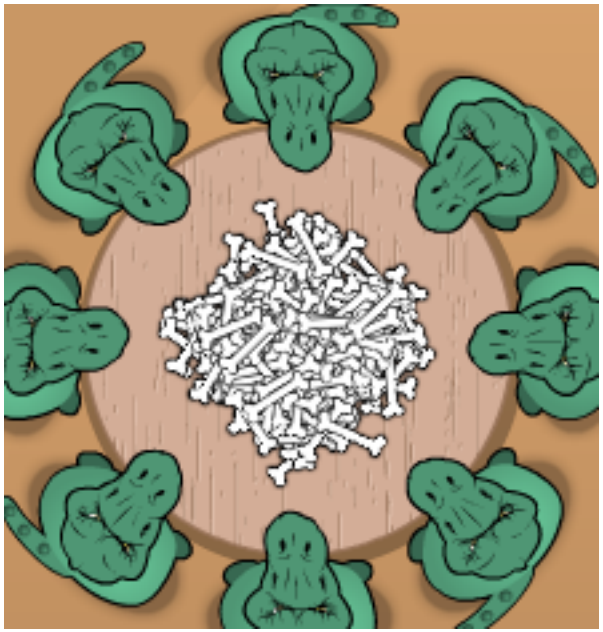
Concrete: Use base-ten blocks to represent your solution

Pictorial: Draw a representation of your solution



Dividing with Representations

8 dinosaurs shared **221** bones. How many bones did each dinosaur get?



8 x **10** = 80

8 x **10** = 80 (160)

~~8 x 10 = 80 (240)~~

8 x **5** = 40 (200)

8 x **2** = 8 (216) with 5 leftover

Each could get **27** (10+10+5+2) bones.

Quotient Café, Illuminations NCTM

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Are there "partial dividends" in Jackson's problem: 721 ÷ 7?

How many groups of 7 are in 700? -----> 100 (100 x 7 = 700)
How many groups of 7 are in 21 (what's left)? -----> 3 (3 x 7 = 21)

So, there are 100 groups of 7 + 3 groups of 7 -or- 103 groups of 7

What “partial dividends” might you use for these?

- 645 ÷ 5
- 786 ÷ 6
- 1384 ÷ 50

A note about partial quotients…

- So Jackson decomposed the dividend.
- Would it work if we decomposed the divisor?

- 721 ÷ 7 ---> (721 ÷ 5) + (721 ÷ 2)

Session 4: Estimation, Mental Math, and Basic Facts

John SanGiovanni

jsangiovanni@hcpss.org

@John SanGiovanni

July 10 – 12, 2014

San Diego, CA

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Think-Pair-Share

How are **estimation** and **rounding** similar or different?

Estimation vs. Rounding

- These are two very different ideas.
- Rounding is a convention for deciding which landmark number a given number is closest to based on place value.
- Estimation is involves mental computation with approximate numbers and focuses on understanding the meaning and properties of arithmetic operations.
- In short rounding is one way to estimate and is often not the most beneficial.

Maximizing the Impact of These Tasks

- Students engage mentally
- Students share with partners
- Group share
- Teacher records and facilitates BUT does not influence or dictate

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17
17 x 23 x 30	17 x 30 x 25

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17
17 x 23 x 30	17 x 30 x 25
197 ÷ 50	20 ÷ 5

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17
17 x 23 x 30	17 x 30 x 25
197 ÷ 50	20 ÷ 5
14 x 71	41 x 17

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17
17 x 23 x 30	17 x 30 x 25
197 ÷ 50	20 ÷ 5
14 x 71	41 x 17
1,000 ÷ 600	80 ÷ 40

Reasoning about Number:
Which is greater A, B, or too close to call?

Column A	Column B
9 x 8	6 x 20
19 x 3	7 x 17
17 x 23 x 30	17 x 30 x 25
197 ÷ 50	20 ÷ 5
14 x 71	41 x 17
1,000 ÷ 600	80 ÷ 40
700 ÷ 76	523 ÷ 25

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45			

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	**7 x 40**		

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	**59 x 1,000**	

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	59 x 1,000	**560 ÷ 8**

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	59 x 1,000	560 ÷ 8
15 x 4			

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	59 x 1,000	560 ÷ 8
15 x 4	**34 x 47**		

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	59 x 1,000	560 ÷ 8
	34 x 47	**25 x 48**	

Paper or Head?

Is it easier in your head or on paper? Why?

450 ÷ 45	7 x 40	59 x 1,000	560 ÷ 8
15 x 4	34 x 47	25 x 48	**18,000,000 ÷ 3,000,000**

Journal: Estimation Range

975 ÷ 12

(a) What are the different strategies for estimating this quotient.

(b) Give an acceptable range for possible answers to this problem.

Journal: Estimation Range

281,034 ÷ 18

(a) What are the different strategies for estimating this quotient.

(b) Give an acceptable range for possible answers to this problem.

Which is best? More or Less?

(Estimate and consider the actual)

$29 \times 32 \approx$

a. 90 b. 900 c. 9,000

$54 \times 126 \approx$

a. 5,100 b. 6,000 c. 6,800

$56 \times 72 \approx$

a. 3,500 b. 4,000 c. 4,200

Which is best? More or Less?

(Estimate and consider the actual)

$893 \div 4 \approx$

a.22 b. 200 c. 225

$975 \div 12 \approx$

a.10 b. 80 c. 1,000

$1{,}795 \div 58$

a. 30 b. 40 c. 50

Mental Math: Missing Digits

$$\_\_5 \times 7 = 175$$

$$168 \div 1\_\_ = 12$$

$$46 \times 3\_\_ = 1{,}748$$

$$\_\_42 \div 18 = 19$$

Why Estimate?

Formulate an argument as to why we should estimate in computation (specifically in multiplication and division)

More estimation

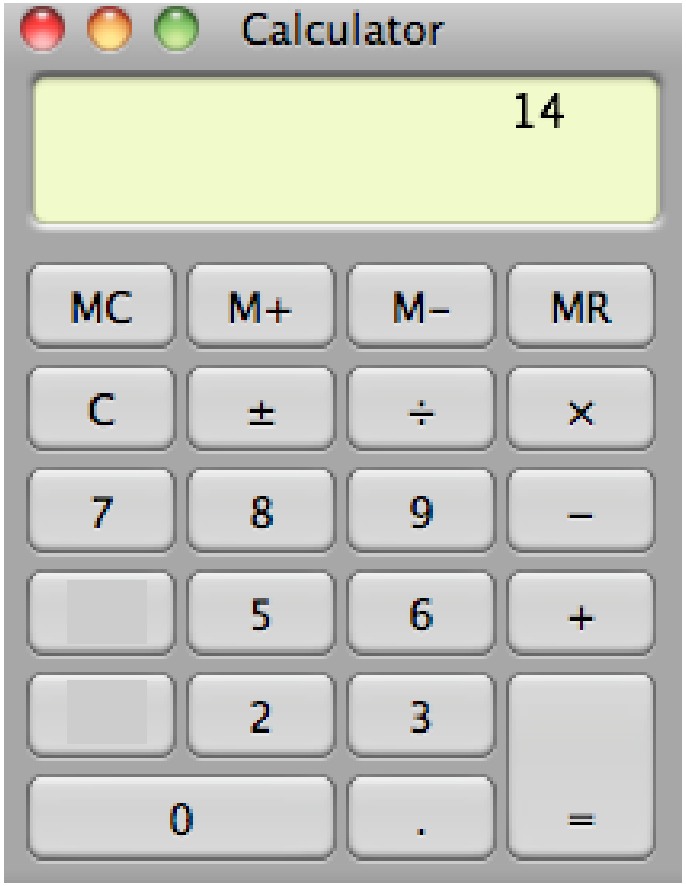
You multiplied two numbers and the product was almost 400. What could the two numbers have been?

You multiplied two numbers and the product was a bit more than 400. What could the two numbers have been?

From Good Questions: Great Ways to Differentiate Mathematics Instruction

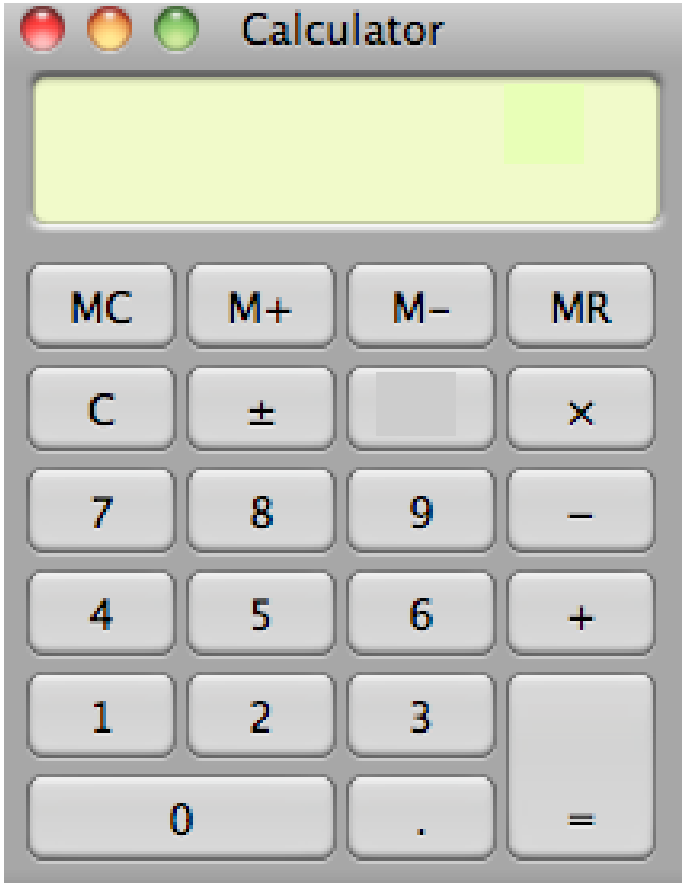
Broken Calculator

"I want to make 14 using my calculator, but the *1 and 4 keys are broken!* How can I make my calculator display show 14?"



Broken Calculator

He wanted to check his answer to 54 ÷ 9 = 7. However, the ÷ key on his calculator was stuck. How could he check this problem using the × key? What calculator keys would he press?



More About Martin's Broken Calculator

- What keys would you need to press? Write the sequence down.
- Explain how you can do a division problem, using the multiplication key instead.

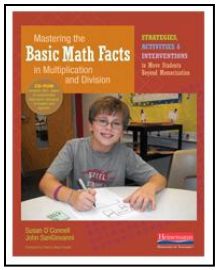
Automaticity and Basic Facts

Let's take a test.

E x D	B x C	G x A	F x B	D x A	B x B	A x E
C x F	A x 1	10 x D	D x D	B x A	1 x D	10 x C
B x G	E x B	C x G	F x A	B x 1	1 x C	E x 10
E x F	D x F	G x G	A x G	C x C	G x G	C x B
10 x C	E x F	G x G	A x D	C x B	F x C	E x E
B x C	1 x G	D x D	G x B	F x F	E x G	D x A
B x 10	G x G	B x A	1 x 1	E x 1	C x C	C x G

Traditionally, how have we taught (or learned) basic facts?

“They don’t know their facts!”



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Challenge: Can you remember every word?

blocks two the crossed get little ran and
the the school street girl to to

the little girl ran two blocks and crossed
the street to get to the school

Which is easier to remember? Why?

Memorizing things that make no sense can be very difficult!

Mastering Basic Facts Requires:

1. Understanding the meaning of the operations
2. Understanding the properties of the operations
3. Developing strategies to find answers while developing automaticity
4. Instruction and practice activities develop quick recall

Teaching Basic Facts

1. Instructional activities that present concepts in context and provide opportunities for making connections
2. Intentional sequence
3. Meaningful activities for investigation, discussion, and practice
4. Practice and patience
5. Purposeful assessment

Teaching Basic Facts

Which facts are easiest to master?

Foundation Facts for Multiplication

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3											
4											
5	0	5	10	15	20	25	30	35	40	45	50
6											
7											
8											
9											
10	0	10	20	30	40	50	60	70	80	90	100

Why are these easier?

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3											
4											
5	0	5	10	15	20	25	30	35	40	45	50
6											
7											
8											
9											
10	0	10	20	30	40	50	60	70	80	90	100

Why are these easier?

- Patterns
- Experience with skip counting
- Simplicity of numbers
- Connections to real world (pairs, fives, tens)

Foundation Facts with the Commutative Property

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6			15					30
4	0	4	8			20					40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12			30					60
7	0	7	14			35					70
8	0	8	16			40					80
9	0	9	18			45					90
10	0	10	20	30	40	50	60	70	80	90	100

x 2 Facts (Doubles)

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

x 10 Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

x 5 Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

X1 Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

x 0 Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Foundation Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6			15					30
4	0	4	8			20					40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12			30					60
7	0	7	14			35					70
8	0	8	16			40					80
9	0	9	18			45					90
10	0	10	20	30	40	50	60	70	80	90	100

Building on Foundation Facts

What is left?

Commutative Property

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8		16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12			30	36	42	48	54	60
7	0	7	14			35		49	56	63	70
8	0	8	16			40			64	72	80
9	0	9	18			45				81	90
10	0	10	20	30	40	50	60	70	80	90	100

x 3 Facts (Double and 1 More • 2n + n)

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

X4 Facts (Double – Doubles • 2(2n))

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30					60
7	0	7	14	21	28	35					70
8	0	8	16	24	32	40					80
9	0	9	18	27	36	45					90
10	0	10	20	30	40	50	60	70	80	90	100

X6 Facts - "Double Threes" • 2(3n)

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42				70
8	0	8	16	24	32	40	48				80
9	0	9	18	27	36	45	54				90
10	0	10	20	30	40	50	60	70	80	90	100

X6 Facts - “1 more than 5” • 5n + n

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42				70
8	0	8	16	24	32	40	48				80
9	0	9	18	27	36	45	54				90
10	0	10	20	30	40	50	60	70	80	90	100

X9 Facts – 1 less than 10 • 10n - n

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42			63	70
8	0	8	16	24	32	40	48			72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

x 8 Facts – Double, Double, Double

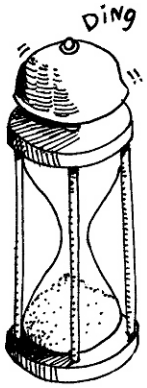
X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42		56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

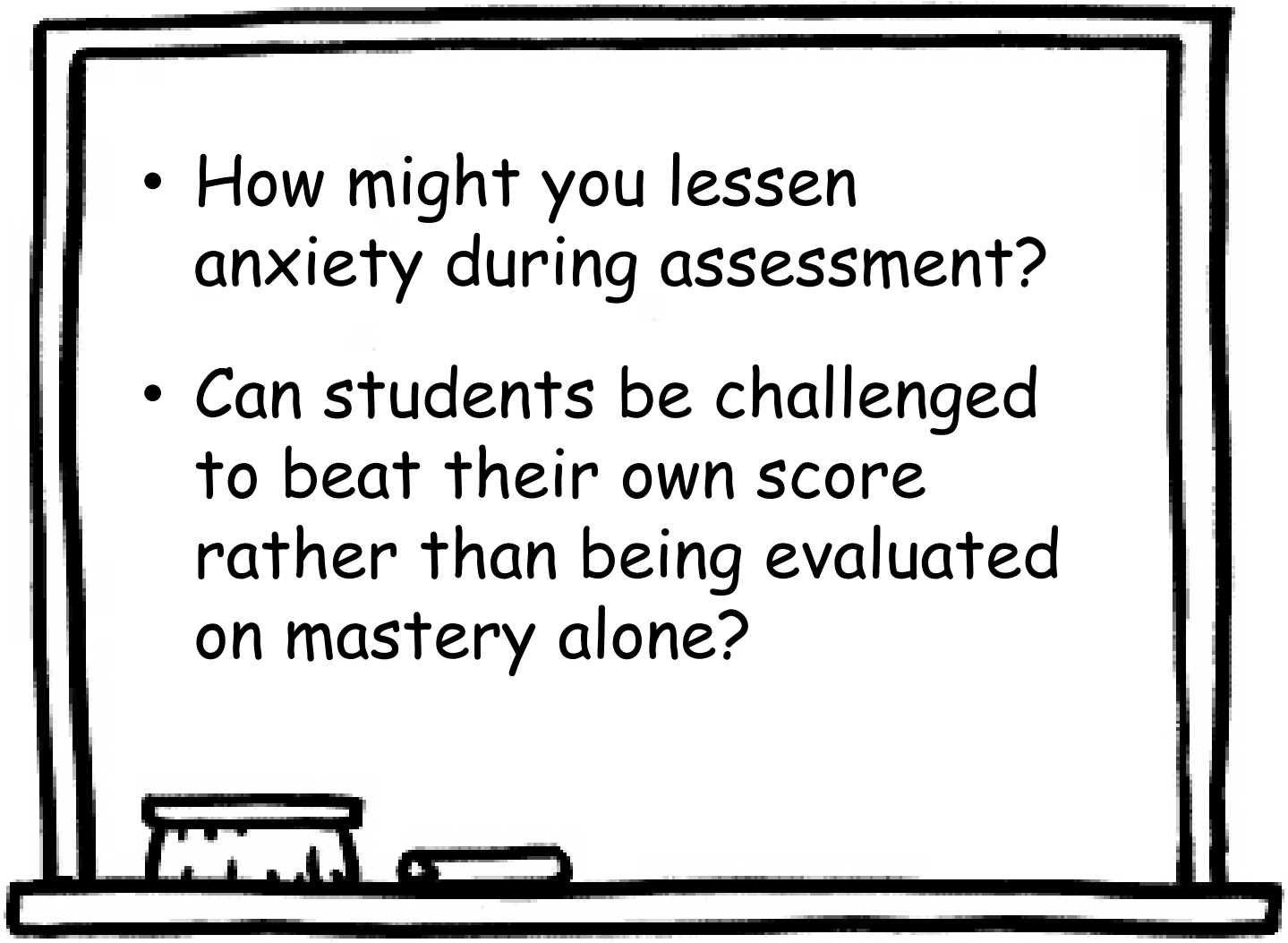
x 7 Facts

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Assessing Basic Facts

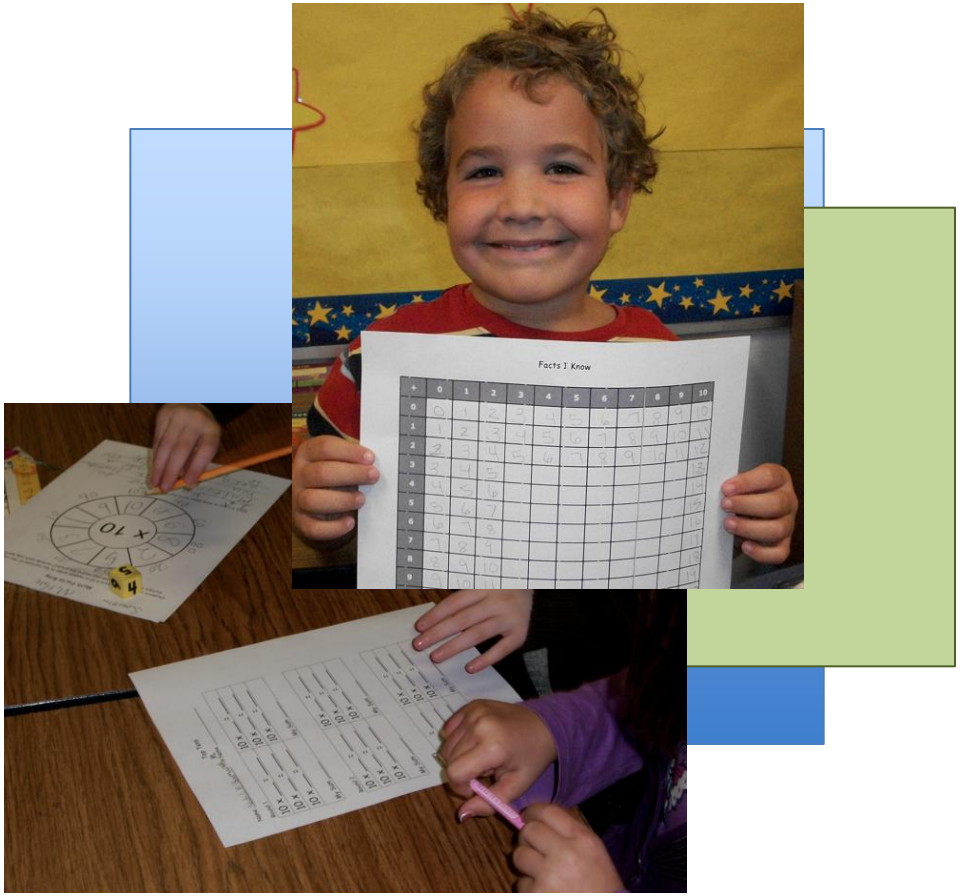
What role does time play in math fact mastery?




• How might you lessen anxiety during assessment?
• Can students be challenged to beat their own score rather than being evaluated on mastery alone?

Challenges with Traditional Timed-Tests

- Negative impact
- They don't teach anything
- Can reinforce inefficient strategies
- Avoid overuse



Monitoring Progress

- Individual Fact Checks
  - Brief, low-stress
- Individual Conferences/Interviews
  - Discuss fact check, set goals
- Teacher-Administered Fact Checks
  - Verbal, put aside missed cards, set goals
- Teacher Observation
  - Automatic, strategies, neither?
- Progress Graphs
  - Keeping track of own progress

Name:______________________________ Inventory Date:__________ Operation:_____ Score:___

Interview (circle one): Level 1 Level 2 Level 3 Level 4

18-20 points: Student demonstrates automaticity.
15-18 points: Student demonstrates understanding of strategy(ies) continued practice is needed for automaticity

<u>18-20 points:</u> Student demonstrates automaticity.
<u>15-18 points:</u> Student demonstrates understanding of strategy(ies) continued practice is needed for automaticity
<u>12-15 points:</u> Student demonstrates limited understanding of strategy(ies). Further exploration and/or reteaching is needed
<u>Less than 12 points:</u> Student demonstrates basic understanding of some facts/strategy(ies). Concept/strategy must be presented again with exploration and reinforcement.

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Fact is: Fact: __________
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments:

Name: [redacted] Inventory Date: 5/30 Operation: X Score: ___

Interview (circle one): Level 1 Level 2 Level 3 Level 4

18-20 points: Student demonstrates automaticity.
15-18 points: Student demonstrates understanding of strategy(ies) continued practice is needed for automaticity
12-15 points: Student demonstrates limited understanding of strategy(ies). Further exploration and/or reteaching is needed
Less than 12 points: Student demonstrates basic understanding of some facts/strategy(ies). Concept/strategy must be presented again with exploration and reinforcement.

Fact is: Fact: 7x4
Automatic ✓ (2) Known ___ (1) Unknown ___ (0)
Comments: "28"

Fact is: Fact: 8x6
Automatic ___ (2) Known ___ (1) Unknown ___ (0)
Comments: 8+8=14 then added that to 24

Fact is: Fact: 8x3
Automatic ___ (2) Known ✓ (1) Unknown ___ (0)
Comments: "8+8=16 so I added 8 more"

that

Fact is: Fact: 7x7
Automatic ___ (2) Known ___ (1) Unknown ✓ (0)
Comments:

Fact is: Fact: 6x4
Automatic ✓ (2) Known ___ (1) Unknown ___ (0)
Comments:

Name: [redacted] Inventory Date: 5/30 Operation: X Score:

Interview (circle one): Level 1 Level 2 Level 3 Level 4

18-20 points: Student demonstrates automaticity.
15-18 points: Student demonstrates understanding of strategy(ies) continued practice is needed for automaticity
12-15 points: Student demonstrates limited understanding of strategy(ies). Further exploration and/or reteaching is needed
Less than 12 points: Student demonstrates basic understanding of some facts/strategy(ies). Concept/strategy must be presented again with exploration and reinforcement.

Fact is: Fact: 7x9
Automatic ____ (2) Known ____ (1) Unknown ____ (0)
Comments:

Fact is: Fact: 9x8
Automatic ____ (2) Known ____ (1) Unknown ✓ (0)
Comments:

Fact is: ✓ Fact: 9x5
Automatic ____ (2) Known ____ (1) Unknown ____ (0)
Comments:

Fact is: 45 Fact: 9x5
Automatic ____ (2) Known ____ (1) Unknown ____ (0)
Comments:

Fact is: Fact: 9x9
Automatic ____ (2) Known ✓ (1) Unknown ~~(0)~~
Comments: 10x9=90−9=81

Fact is: 7 Fact: 7x1
Automatic ____ (2) Known ____ (1) Unknown ____ (0)
Comments:

Fact is: Fact: 7x9
Automatic ____ (2) Known 27 (1) Unknown ____ (0)
Comments: ~~9~~ 9x2 = 18 + 9

Fact is: Fact: 9x4
Automatic ____ (2) Known 36 (1) Unknown ____ (0)
Comments:

Fact is: ✓ Fact: 9x9
Automatic ____ (2) Known ____ (1) Unknown ~~(0)~~
Comments: 10x9=90−9=81

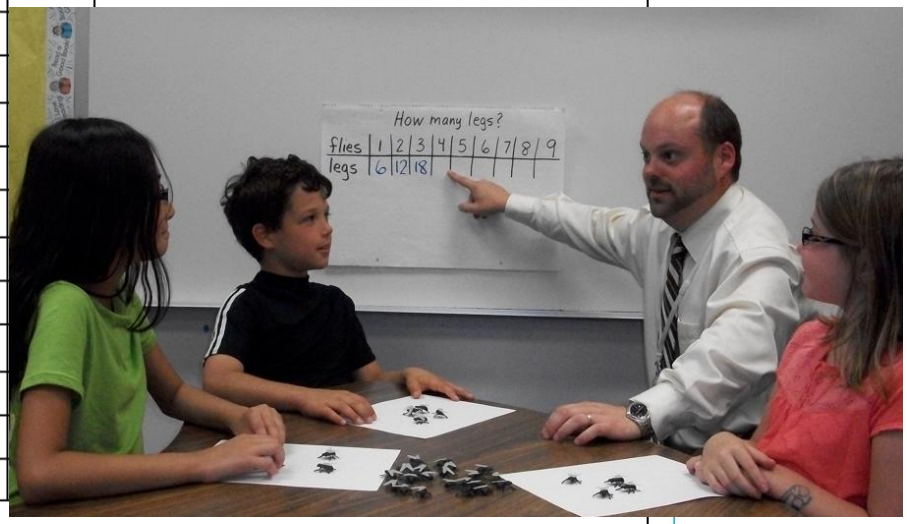
Data from Observation

Classroom Observation of Automaticity

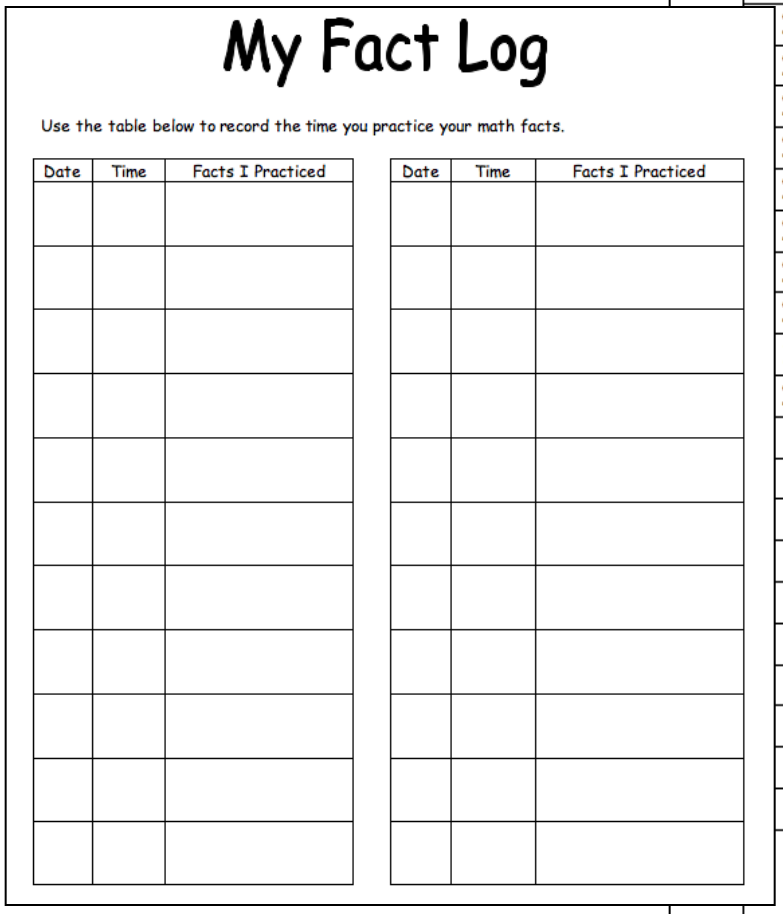
Student	Date	Date	Date	Date	Date

Observation Rubric

3	Student demonstrates quick recall of all (or all but a few) facts during independent or group work.
2	Student demonstrates quick recall of some facts and recalls other facts with an appropriate strategy.
1	Student applies appropriate strategies to find facts but does not demonstrate quick recall.
0	Student does not apply appropriate strategies or demonstrate quick recall of facts.



Student Progress

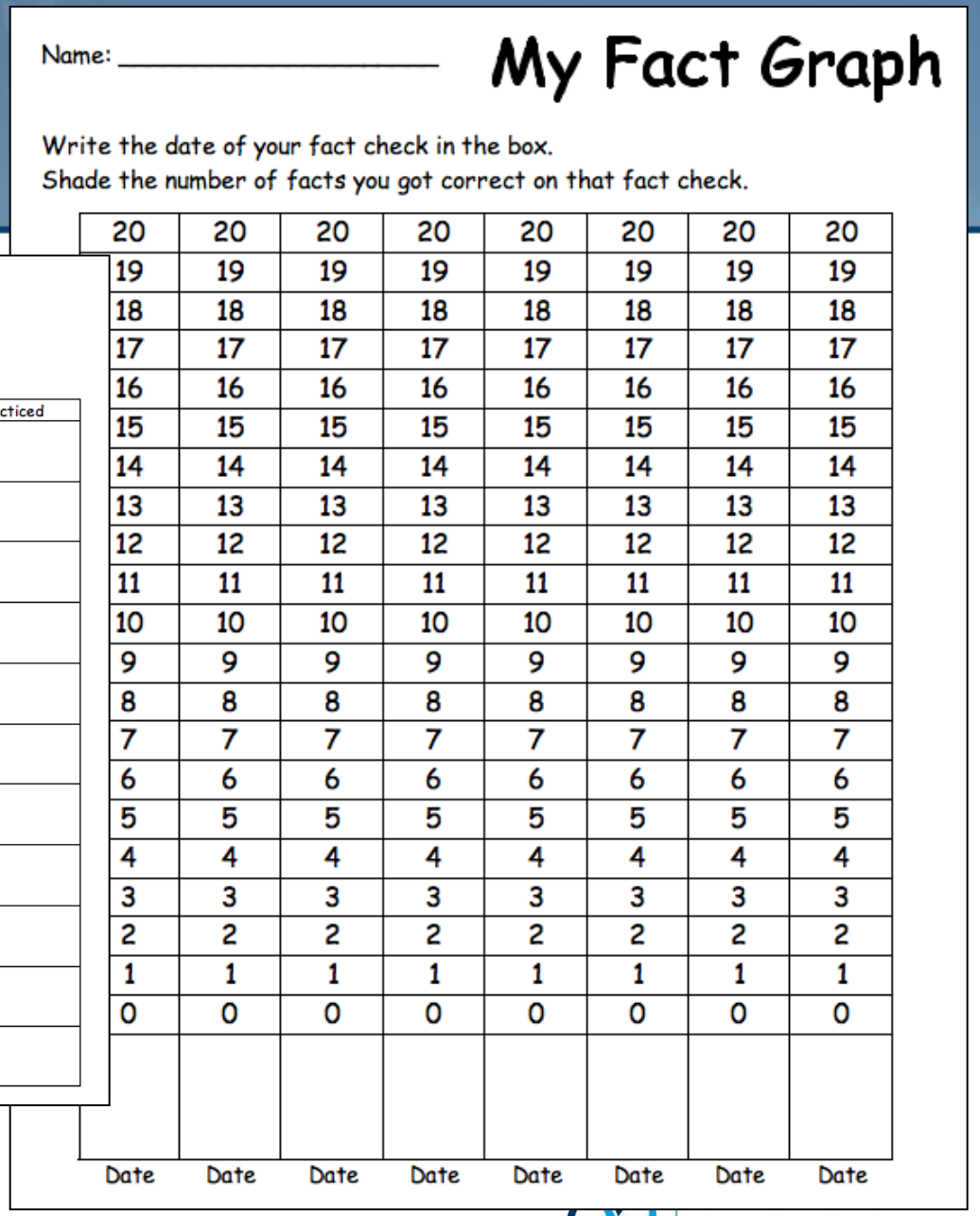


My Fact Log

Use the table below to record the time you practice your math facts.

Date	Time	Facts I Practiced

Date	Time	Facts I Practiced



Name: _____

My Fact Graph

Write the date of your fact check in the box.
Shade the number of facts you got correct on that fact check.

20	20	20	20	20	20	20	20
19	19	19	19	19	19	19	19
18	18	18	18	18	18	18	18
17	17	17	17	17	17	17	17
16	16	16	16	16	16	16	16
15	15	15	15	15	15	15	15
14	14	14	14	14	14	14	14
13	13	13	13	13	13	13	13
12	12	12	12	12	12	12	12
11	11	11	11	11	11	11	11
10	10	10	10	10	10	10	10
9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8
7	7	7	7	7	7	7	7
6	6	6	6	6	6	6	6
5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
Date	Date	Date	Date	Date	Date	Date	Date

Start • Stop • Keep

Think about your multiplication/division instruction.

- What is something you will START doing?
- What is something you will STOP doing?
- What is something you will KEEP doing?

Where did you see the Standards for Mathematical Practices in these ideas about multiplication and division?

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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