Tracking Rate along the Transcontinental Railroad

A cross-disciplinary unit asks students to analyze data in historical information.

s and History

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Over the next few years, train towns along the tracks of the First Transcontinental Railroad will be celebrating 150 years of railroad heritage. This investigation presents a mathematics and history interdisciplinary unit for Algebra 2 students that has proved to be a rich modeling experience grounded in this significant historical event.

The Transcontinental Railroad began laying track east from Sacramento, California, in 1863 and west from Omaha, Nebraska, in 1865; the two lines eventually met in Promontory Summit, Utah, in 1869. One of the train towns, Ogallala, Nebraska, consisted of only a section house and water tank alongside newly laid track. Today a historical marker reads in part: "Track laying was done by crews of Civil War veterans, emigrants, ex-miners, adventurers and gamblers. The crews averaged about two miles of track per day." Rates similar to this one often appeared in newspapers and workers' letters while the railroad was constructed. The accuracy of these rates is the focus of this unit.

This unit highlights four Standards of Mathematical Practices (SMPs) of the Common Core State Standards for Mathematics: "Make sense of problems and persevere in solving them" (SMP 1); "Construct viable arguments and critique the reasoning of others" (SMP 3); "Model with mathematics" (SMP 4); and "Use appropriate tools strategically" (SMP 5) (CCSSI 2010, pp. 6–7). Students

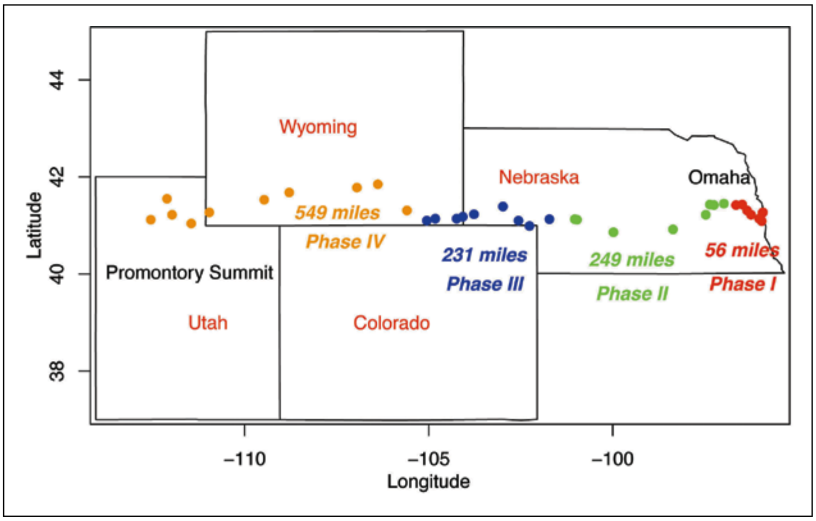


Fig. 1 This map shows the four construction phases of the Union Pacific Railroad from 1865 to 1869.

record their thought processes, use technology, research digitized historical newspapers and books, plot distance-time data, find a best-fit curve, interpret their results, make judgments about historical claims, and share their conclusions.

Because this activity highlights linear and quadratic best-fit curves, an opportune time for teachers to initiate the unit is after students have familiarity with the standard form of the parabola: $y = ax^2 + bx + c$.

GETTING STARTED

Before presenting the unit, we researched and subsequently shared with the students the part of the railroad's history relating to the average rate of laying rails. Two companies simultaneously built the railroad—the Central Pacific Railroad from the west and the Union Pacific Railroad from the east. The U.S. government created competition between them using incentives—namely, money, land, and mineral rights. The faster the companies laid rails, the wealthier their investors became.

We focus on the average rate, in miles per day, that the Union Pacific laid track. Sources indicate that the Union Pacific completed its work in four phases, starting in the spring and ending in the winter each year, except for the last year. Before presenting the unit, we assembled a set of construction data culled from historical letters, books, and websites (Bain 1999; Klein 1987).

This unit exhibits a good deal of flexibility. Although a time line is embedded in the narrative in terms of days and activities in and out of class, it can be altered without affecting essential concepts and skills. For example, some teachers will want to include all five modeling activities; others will find value in choosing fewer.

In this unit, students use technology to gather information, collaborate, and process data. The teacher can decide how to blend these in-class and out-of-class technology activities. Although the students used an iPad® app to determine the regression model, in past years, students used an Excel® spreadsheet, TI-84 Plus™, and Wolfram Alpha® to accomplish this goal.

On the first day, we used Henry O. Pollak's description of the modeling process as a springboard into the unit: "Mathematical modeling … begins in the 'unedited' real world, requires problem formulating before problem solving, and once the problem is solved, moves back into the real world where the results are considered in their original context" (Pollak 2011, p. viii). After explaining the modeling process, we divided the class into four teams and assigned each of them a Padlet site. (Padlet [http://padlet.com/] is a free bulletin-board app for both the iPad and PC that chronologically posts notes, similarly to a blog.) On the bulletin board, students post their notes and react to previous ones. In the process, the team's understandings and misunderstandings become visible.

Our students in previous settings solved average rate problems on the basis of tables of numbers, graphs, and the formula distance = rate × time. We asked students to consider the text on the historical marker and suggest a method for calculating the accuracy of "about 2 miles per day." We projected a map of the Union Pacific's route (see **fig. 1**) on a whiteboard, pointing out the four construction phases and the approximate location of Ogallala at the eastern edge of phase 3 track. Students listened attentively and appeared confident. (For instructions for creating the map and other student materials cited, see the class website: http://sites.google.com/site/svhstranscon/.) Students need time to reflect on solutions to nonstandard problems. Over the next three days, outside class, the teams recorded their pathways to a solution in Padlet notes.

ATTEMPTS AT A SOLUTION

In class on day 5, we returned to the project. Lesh accurately predicts what a teacher can expect in the students' initial interpretation of a model-eliciting activity: ". . . a hodgepodge of several disorganized and inconsistent ways of thinking about givens, goals, and possible solution steps" (Lesh 2000, p. 6).

Most bulletin board notes for each team had a similar look and displayed an unsettling vagueness. Typically, students correctly identified the problem as involving average rate, used the appropriate distance formula, attempted a solution, and then expressed little confidence in their work. Many attempted—unsuccessfully—to apply previously

Milepost	Town	Date	Working Days (exclude Sundays)	Distance (miles)
Phase I: July 10, 1865 – Feb. 10, 1866				
0	Omaha, NE	July 10, 1865	0	0
1		July 21, 1865	11	1
11	Gilmore, NE	Sept. 25, 1865	67	11
15	Papillion, NE	Oct. 6, 1865	77	15
28		Nov. 18, 1865	114	28
40	Valley, NE	Dec. 21, 1865	142	40
47	Fremont, NE	Jan. 24, 1866	171	47
56		Feb. 10, 1866	186	56
Phase II: April 6, 1866 – Dec. 14, 1866				
56		April 6, 1866	0	0
63.5		April 17, 1866	10	7.5
86.5		May 28, 1866	45	30.5
92	Columbus, NE	June 2, 1866	50	36
100		June 16, 1866	62	44
154	Grand Island, NE	July 7, 1866	80	98
247	100th meridian (Cozad, NE)	Oct. 6, 1866	158	191
291	North Platte, NE	Dec. 3, 1866	207	235
305	O'Fallon's Bluff, NE	Dec. 14, 1866	217	249
Phase III: April 27, 1867 – Dec. 31, 1867				
305	O'Fallon's Bluff, NE	April 27, 1867	0	0
341	Ogallala, NE	May 24, 1867	24	36
377	Julesburg, CO	June 24, 1867	50	72
395		July 10, 1867	64	90
414	Sidney, CO	Aug. 1, 1867	83	109
456	Kimball, NE	Aug. 29, 1867	107	151
481	Pine Bluffs, WY	Oct. 12, 1867	145	176
490		Oct. 17, 1867	149	185
516	Cheyenne, WY	Nov. 13, 1867	172	211
536	Granite Canyon, WY	Dec. 31, 1867	213	231
Phase IV: April 1, 1868 – May 8, 1869				
536	Granite Canyon, WY	April 1, 1868	0	0
573	Laramie, WY	May 8, 1868	33	37
656	Carbon, WY	July 1, 1868	79	120
696	Fort Steele, WY	July 21, 1868	96	160
805	Point of Rocks, WY	Sept. 18, 1868	147	269
845	Green River, WY	Oct. 6, 1868	162	309
955	Evanston, WY	Dec. 4, 1868	213	419
991	Echo City, UT	Jan. 15, 1869	249	455
1032	Ogden, UT	March 6, 1869	292	496
1057	Corinne, UT	March 27, 1869	310	521
1085	Promontory Summit, UT	May 8, 1869	346	549

Table 1 Four Construction Phases of Union Pacific Railroad from Omaha, Nebraska, to Promontory Summit, Utah

learned strategies to a nonroutine problem. For example, Maria, using inaccurate data, posted, "I found the rate the workers worked at in the vicinity of Ogallala. The time it took to build the railroad was 204 days, and the distance it covered was 86.6 miles. To find the rate they worked at, I divided 86.6 by 204, and I got 0.42 miles per day. This means that either the sign is wrong or I am wrong because 0.42 is not close to 2 miles per day."

Nick posted, "I don't think we have enough information. We need to know the total length of the railway in Ogallala and the time it took to lay

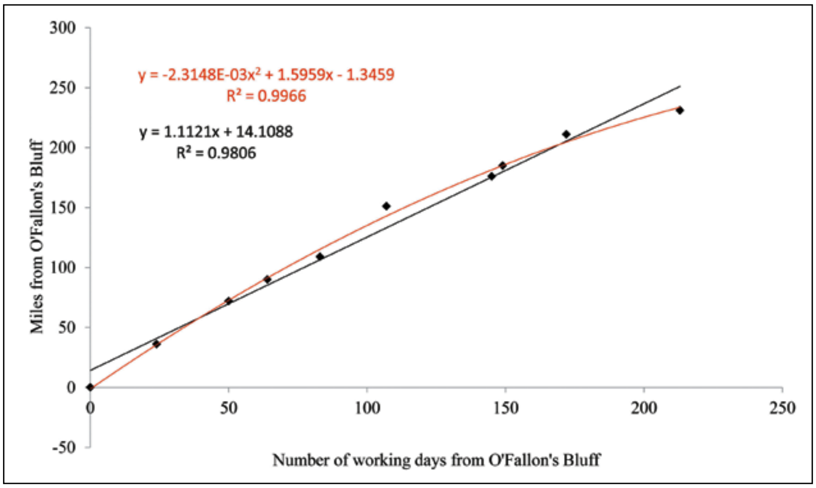


Fig. 2 Students selected the quadratic model because the linear model overestimated milepost data at the beginning and end of the graph.

[the tracks]. Then we can use $d = rt$ to find the average." In class, we used Nick's observation to affirm the value of the notes, emphasizing that they gave our project direction. After hearing Nick's comment, students realized that he had the correct approach.

Solving Using Tabular Data

At this point, we gave the students a spreadsheet containing additional information (see **table 1**), including mileposts along with the closest town, if one existed, and the date that the rails arrived. In class, teams filled in columns showing working days and distance. In calculating the number of working days between posted dates, they used calendars for 1865–69, remembering that workers typically had Sunday free. (Calendars are available at http://www.timeanddate.com/.)

On day 6, outside class, the teams returned to verifying "about two miles per day" using spreadsheet data. Using the data from April 27 to December 31, 1867, they posted on the bulletin board the average rate for laying rails near Ogallala by using time and distance values found in the table. Not all the teams used the same data to make their calculations. Some used data from O'Fallon's Bluff to Ogallala, others from Ogallala to Julesburg or from O'Fallon's Bluff to Julesburg, giving 1.5, 1.4, and 1.4 miles per day, respectively. The consensus was that the text on the historical marker—"about two miles per day"—was generous but not misleading.

Solving Using Best-Fit Curves

In the next phase of the unit, students solved the Ogallala problem using data analysis tools—namely, scatter plots, the best-fit curve and its equation, and interpolation. These tools allowed students to calculate the average rate of track laying using data points closer to Ogallala than those listed in the table.

On day 7, after commenting on the students' bulletin board solutions, we demonstrated a free scientific data analysis app for the iPad (http://www.scidataanalysis.com). Using the app, the class created a scatter plot for data from April 27 to December 31, 1867 (see **fig. 2**). We then duplicated their iPad scatter plot on the SMART board™. Since the class had little experience with curves of best fit, a paper-and-pencil problem was proposed: Find the equation of a line that passes through two data points such that the rest of the points cluster closely about the line. Volunteers were eager to draw the line on the SMART board. Eventually, we settled on the points (24, 36) and (145, 176), which determine the equation $y = 1.16x + 8.23$.

Without difficulty, students fitted a line to their iPad scatter plot and recorded its equation: $d = 1.1121t + 14.1088$ with $R^2 = 0.981$. We explained that R^2 is a measure of how closely the points are clustered about the line, with 1 being a perfect fit. We also discussed how the linear function reveals a relationship between the distance and the time variables, using the terms *linear rate*, *increasing rate*, and *constant rate*. The students were pleased with their pencil-and-paper work and amazed at the speed of the iPad app.

After fitting a line to the data, the students in class on day 8 fitted a parabola through the scatter plot and recorded its equation: $d = -0.0023148\,t^2 + 1.5959t - 1.3458$ with $R^2 = 0.997$. They selected the parabola as the better fit on the basis of its higher R^2 value and the tight clustering of the points about the curve. This form of the equation was familiar to students because they had recently completed a unit on the parabola's standard equation.

The sections of the parabolas here, both concave up and concave down, graph increasing functions. Consequently, the slope of a segment connecting any two points on the parabola is the average rate of laying track for the time interval. By using this concept, students are visually able to compare average rates for various time intervals along the curve of best fit. We posed this question: Did the crew work faster at the beginning or at the end of the construction period, and how do you know? Students realized the need to compare slopes of segments visually drawn between pairs of data points—for example, the first and last 100 working days. They concluded that the crew worked faster at the beginning of the construction period.

The iPad app has a feature that allows students to solve for t and d. The students used pairs $(t_1, 26)$ and $(t_2, 46)$ with the equation $d = -0.0023148t^2 + 1.5959t - 1.3458$ to calculate $t_1 = 18$ and $t_2 = 31$, where 26 and 46 are arbitrarily chosen positions 10 miles on either side of Ogallala. Students then

calculated the average rate: (46 – 26)/(31 – 18) = 1.5 miles per day.

Whenever students work with decimals, they always ask, “How many places should we keep?” From the scatter plot through interpolation, the students stay within the app’s computational environment but then need to make rounding choices. We instructed them to round the decimal so that it made sense in the context of the problem.

Using the best-fit curve, the students calculated other average rates based on distance intervals of their choice—for example, 2, 5, and 7 miles on either side of Ogallala. For each choice, their rounded results were 1.5 miles per day. As in the case of the tabular calculations, they accepted as reasonable the text “about two miles per day” found on the historical marker.

The students now had two methods for analyzing a “miles per day” statement—tabular and curve of best fit.

SEARCH FOR HISTORICAL RECORDS

When we introduced this unit, we told students that they would have the opportunity to test the accuracy of “miles per day” statements found in historical newspapers and books. On day 9, as an in-class activity, we demonstrated search techniques for using Chronicling America’s digitized historical newspaper project (http://chroniclingamerica.loc.gov/) and Google Books Advanced Search (https://books.google.com/advanced_book_search). To keep students’ workload reasonable, we assigned each team to one of the four construction phases and required the team to discover only one “miles per day” statement, but they found more. On days 10, 11, 12, as outside-class assignments, the teams began their quest using the phrases “Union Pacific” and “miles per day” in search engines and posted their results on the bulletin board. The search clearly energized the students.

The students’ solution to the Ogallala problem provided strategies for testing other “miles per day” statements. Examples of their work, which formed the basis of team media presentations at the end of the unit, follow.

Silas Seymour, Consulting Engineer

In a Web search, team 1 discovered a statement made by Silas Seymour about the very slow initial progress of the railroad. He wrote that when 12 or 15 miles were laid, “it was struggling along at the rate of one-quarter to one-half mile per day” (Seymour 1867, p. 50).

To test the accuracy of Seymour’s statement, this team, using data from July 10, 1865, to February 10, 1866 (see **table 1**), calculated the average rate between mileposts 11 and 15:

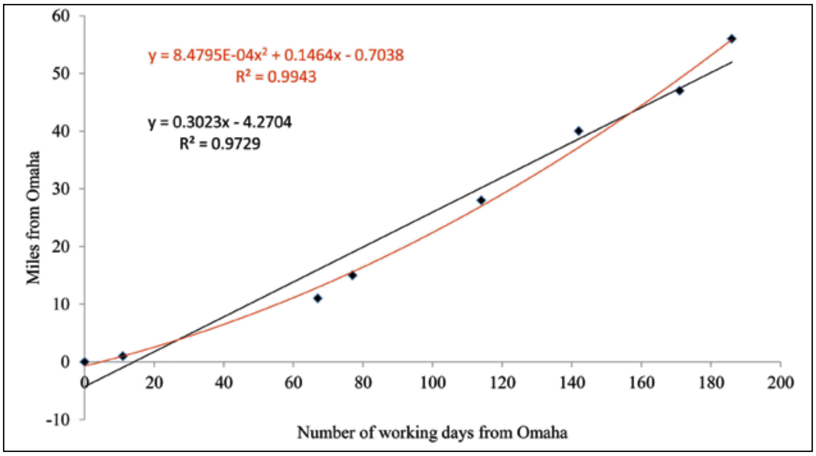


Fig. 3 Students selected the quadratic model because the linear model underestimated milepost data at the beginning and end of the graph.

(15 – 11)/(77 – 67) = 0.4 miles per day—a value that the team felt validated Seymour’s statement.

This team also devised a method to measure the crew’s improvement over time. They plotted the data, found the best-fit curve, and compared rates for the first and last halves of the construction period (see **fig. 3**). In their calculations, they used three data points, (0, 0), (93, d), (186, 56), since $d = 0.00084795t^2 + 0.1464t - 0.7038$, $d = 20$ when $t = 93$. For the first half, the average rate was (20 – 0)/(93 – 0) = 0.2 miles per day; for the second half, the average rate was (56 – 20)/(186 – 93) = 0.4 miles per day. They attributed the increase to workers’ experience and organization.

John Carbutt, Photographer

Team 2, in viewing a set of online stereophotographs taken by the Chicago photographer John Carbutt, discovered a notation on one of them: “Laying the rails of U.P.R.R. — 2 miles per day, October 1866” (Carbutt 1866; see **fig. 4**). The students plotted data from April 16 to December 14, 1866, and compared both the linear and quadratic

Source: Stereophotograph by John Carbutt, Union Pacific Railroad Museum

Fig. 4 The photograph caption notes laying rails at two miles per day.

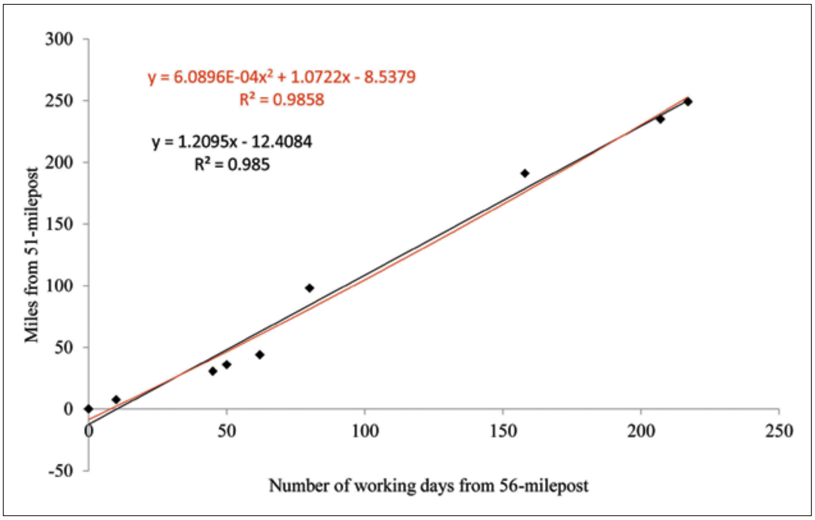
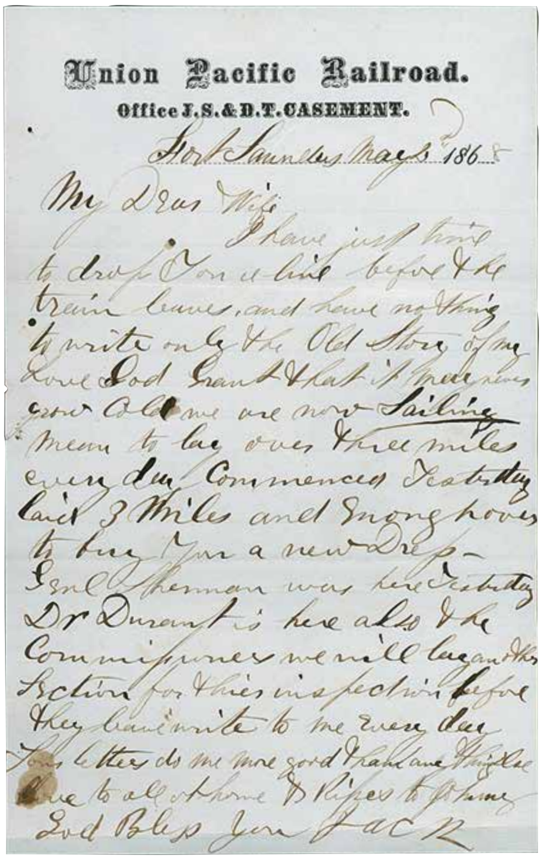


Fig. 5 Students selected the simpler model, the linear model, because the graphs are almost coincident.

fits (see **fig. 5**). They chose the linear fit because the graphs and R^2 values were almost identical. They were uncertain about which data points to use to make a judgment about 2 miles per day. A student discovered that the Union Pacific hired Carbutt to take pictures for the 100th meridian celebration (near Cozad, Nebraska); the railroad reached Cozad on October 6, 1866. With that information, students calculated the average rate for an arbitrarily chosen twenty-day working period ending on October 6. This decision required the use of the data points (158, 191), from the table, and (138, d), where $d = 155$ was calculated from the best-fit line, $d = 1.2095t - 12.408$. They calculated that the average rate was $(191 - 155)/(158 - 138) =$ 1.8 miles per day. The stereophotographs fascinated the students, so they were pleased when their calculations verified Carbutt's value.



Union Pacific Railroad.
Office J.S.&D.T.CASEMENT.
Fort Saunders May 2 1868
My Dear Wife
I have just time to drop You a line before the train leaves, and have nothing to write only the Old Story of my Love God Grant that it may never grow Old we are now Sailing mean to lay over three miles every day Commenced Yesterday laid 3 Miles and Strong hopes to buy You a new Dress
Genl Sherman was here Yesterday
Dr Durant is here also & the Commissioners we will lay another Section for their inspection before they leave write to me every day
Your letters do me more good than anything else
Love to all at home & Kisses to Johnny
God Bless You Jack

Source: Jack Casement Collection, American Heritage Center, University of Wyoming

Fig. 6 Casement, who had worked on the Ohio Railroad before the Civil War, was hired along with his brother Daniel to lead the construction of the Union Pacific line.

Ashtabula Weekly Telegraph

Team 3, in searching historical newspapers, found information about the rate of laying track west of Julesburg, Colorado. The *Ashtabula* (*OH*) *Weekly Telegraph* of July 6, 1867, reported, "The Union Pacific has now got to Julesburg. . . . It crawls along at the rate of three to five miles per day" ("Pen and Paste Brevities" 1867). Since Julesburg is immediately west of Ogallala, this team reviewed the tabular and curve-fitting work that they had completed for the Ogallala problem. The students were immediately skeptical of the newspaper's report because of their previous calculations and the concave-down shape of the curve. Using the table, they calculated the average rate between Julesburg and milepost 395: $(90 - 72)/(64 - 50) = 1.3$ miles per day. This calculation confirmed what they already suspected about the newspaper's rate values.

Jack Casement, Construction Leader

Team 4 found a copy of a letter dated May 2, 1868, in which Jack Casement wrote to his wife Frances, "We are now sailing mean to lay over three miles every day" (Casement 1868; see **fig. 6**).

Using the table, students chose the average rate between May 8 and July 1, 1868, as a test of Casement's statement, which probably referred to the near future. They calculated the average rate: $(120 - 37)/(79 - 33) = 1.8$ miles per day.

In addition, this team chose to investigate the behavior of the rate after July 1, 1868. They plotted the data for April 1, 1868, to May 8, 1869 (see **fig. 7**). Since the curve was concave down, they reasoned that the crew worked at a rate slower than 1.8 miles per day as time increased. The team concluded that Casement's crew could have reached 3 miles per day on any given day but that his sustained performance did not match his enthusiasm.

TRACKS LAID

At the end of the unit, each team shared its results with the rest of the class using Google Presentations.

The class viewed tables, graphs, equations, calculations, and heard how these were used to make decisions about track-laying rates over 1085 miles and parts thereof. The class saw photographs of Thomas Durant (1820–85), vice president of Union Pacific and operational leader of the railroad; Grenville Dodge (1831–1916), chief engineer for the Union Pacific Railroad from 1866 to 1869; and Casement (1829–1909), who had been a Union brigadier general in the Civil War; they also read brief descriptions about their role in building the Union Pacific. The teams made the case that mathematics can be used to reveal new information about historical topics and that mathematical representations can be learned in the context of other disciplines—in this case, history.

We asked students to include in their presentations what they learned from this experience. One student said, "I learned not to believe everything I read." Another commented on how the table and the graph are used differently during mathematical analysis. Others spoke about using websites to search historical documents. There was consensus that fitting models without technology would be impossible. Technology produced the plot and the regression equations as well as interpolated values. Technology also provided an R^2 value, which helped students justify their choice of a best-fit curve.

Applying mathematical representation to the history of the First Transcontinental Railroad heightened students' interest in both topics, more than if each were treated independently. For this mathematics class, the modeling experience was a wonderful ride.

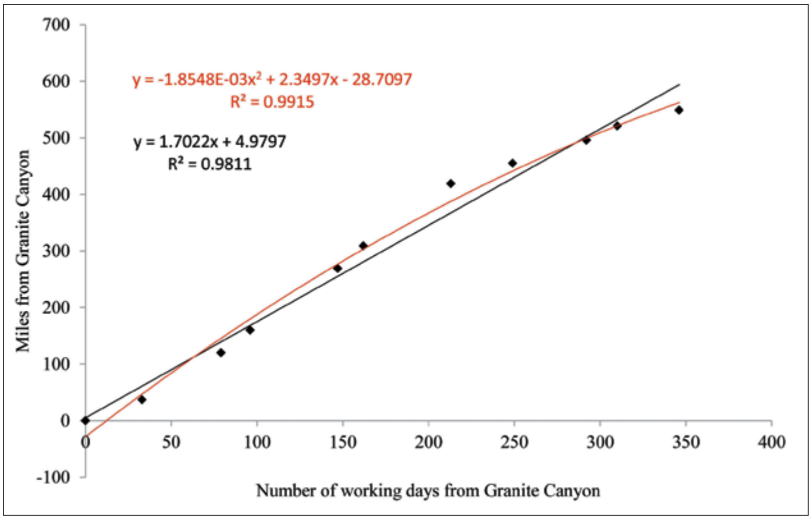


Fig. 7 Students selected the quadratic model because the linear model overestimated milepost data at the beginning and end of the graph.

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