*A teacher reflects on the importance of determining what constitutes prerequisite material and the value of limiting the number of skills assessed.*

# INVESTIGATING Diagnostic Pre

As a teacher, I continually try to assess what my students know and where they have gaps in understanding. Because mathematics builds on itself, teachers must determine whether students have the prerequisite skills needed for new material to be connected to prior understanding. The pressure for students to be successful in high school causes engaged parents to ask teachers why their child is not performing well on graded assessments. I have struggled to convey to parents that their child's difficulties often have involved fundamental concepts such as arithmetic skills or ability to work with integers and fractions. To validate my teaching decisions and observations as well as provide concrete data to share with students and parents, I decided to investigate creating and interpreting data from preassessments.

This article is based on my experiences as a second-year teacher at a small private school where I taught all sections of Geometry and Algebra 2. Working with the same students in both courses, I was aware of each one's levels of retention and execution of Algebra 1 concepts before the beginning of the year. I often noticed the specific skills that a student struggled with during class, but, to provide concrete data to back up my informal findings, I wanted to find a way to record each student's understanding of fundamental skills. I hoped that addressing gaps in prerequisite skills


# assessments

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would allow students to focus on new concepts, get correct answers more often, and gain confidence in their mathematical abilities. With this objective in mind, I chose to explore the use of preassessments to determine Algebra 2 students' level of understanding of key algebraic prerequisite skills. The resulting data did not affect students' grades but were instead used to inform my teaching practices.

## TYPES OF PREASSESSMENT

Research shows that formative assessments and teachers' adjustment of instruction based on resulting data are key components to raising overall student understanding and test scores (Black and William 1998; Guskey 2010). NCTM also recognizes the importance of teachers using formative assessments to determine where a student's confusion occurs while solving a problem as well as to direct classroom activities and instruction (NCTM 1991). Formative assessments take on many different forms in the classroom and should be incorporated daily to inform instructional decisions.

Black and William (2009) define formative assessments this way: "Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in

## Understanding the complexity of each problem is crucial to being able to develop diagnostic items and then correctly interpret and use preassessment results.

the absence of the evidence that was elicited" (p. 9). They then clarify the requirement that decisions are either better or better founded than they may have been without the data, because sometimes the data will bring results that confirm teachers' initial instructional plans. I will focus on a specific type of formative assessment called preassessments. When used correctly, I argue that preassessments can accomplish the requirements listed by Black and William.

One type of preassessment is a pretest, with questions covering the content in the upcoming unit. This type of preassessment is designed to highlight what the students learned in previous mathematics classes, with the potential of condensing the unit by focusing only on the topics with which students struggled. Pretests can also be used to differentiate instruction by giving students who demonstrate understanding of the concepts extensions or challenge material during the unit (Brighton 2005).

Another type of preassessment, which Guskey (2010) calls diagnostic preassessments, includes questions designed to determine whether students know the prerequisite material that will be used in the unit. For example, questions on a diagnostic preassessment for a unit on solving linear equations might assess students' competency in working with order of operations, integers, fractions, or decimals. Although a high score on a diagnostic preassessment does not guarantee success in the upcoming unit, this type of preassessment does provide teachers with valuable information about the misconceptions or deficits in students' mathematical understanding that will potentially hinder their success.

## MY FIRST PREASSESSMENT

This article documents my yearlong investigation of diagnostic preassessments and highlights my struggles and insights in creating these preassessments and interpreting results. I will focus on diagnostic preassessments covering prerequisite concepts given to students in an on-level Algebra 2 course. In addition, I will discuss the complications I encountered and thoughts about determining what counts as prerequisite material as well as my method for analyzing the preassessment data and using them to make instructional decisions.

### *Too Much Data*

In my first attempt, I administered a diagnostic preassessment before the unit on solving quadratic functions and equations. Although textbooks such as Glencoe provide ready-made preassessments, I chose to create my own questions because I felt that the textbook's multiple-choice format and emphasis on vocabulary did not give me sufficient information to help me tailor how I would identify my students' specific misconceptions (Glencoe Online Learning Center: Algebra 2 Chapter Readiness Quiz).

My initial objective in writing the preassessment was to create a question that assessed every prerequisite skill that students might encounter in the upcoming unit. Sixteen questions contained problems covering myriad fundamental skills, from simplifying radicals and using order of operations to graphing linear functions and completing *xy*-tables. The result was a preassessment that was both lengthy and disjointed, with no clear focus. I was overwhelmed by the amount of data and challenged by the prospect of picking apart incorrect answers in an attempt to better understand students' misconceptions; the value in informing my teaching practices was lost.

I was left wondering how I could respond to the data. Although reviewing the prerequisite concepts that the majority of the students missed was a logical approach, at least one student had missed every problem. Realistically, teachers must balance reviewing skills that students should have mastered previously and focusing on the concepts in the given course. After this first attempt at creating a diagnostic preassessment, I realized the importance of assessing only a few key concepts to limit the amount of data to interpret. Going forward, I restricted the questions to concepts fundamental to the upcoming chapter, choosing topics for which I was willing to address misconceptions in class.

### *Too Many Embedded Skills*

In addition to covering too many topics, my first diagnostic preassessment also contained items that focused on a specific skill as well as items that were more complex. For example, one question asked students to plot (2, –3) on an *xy*-plane (graphing), while another asked students to graph $y > -2x + 4$ (working with integers, graphing a line in slope-intercept form, and knowing how to graphically apply the inequality sign). Determining the misconceptions of those who incorrectly plotted (2, –3) was easy; assessing which aspect of graphing the inequality confused students was much more

difficult because of the multiple skills embedded within the single question. If students incorrectly completed an $xy$-table for the equation $y = (x - 1)^2 + 3$, with provided integer $x$-values of $-1$ to 3, was it because they struggled with order of operations, made arithmetic errors, or multiplied by 2 instead of squaring the number? What did it mean if a student correctly completed half the table?

When assessing prerequisite concepts, teachers must decide how far they want to break down each problem into its individual concepts, according to the type of information they want to glean and their informal observations about the classes' current level of mathematical understanding. To truly determine prerequisite knowledge hidden within a problem, teachers must take a close look at each question through the mind's eye of a student and carefully reflect on the aspects that he or she might find challenging and the nuances of mathematical language in the questions themselves.

Reviewing textbooks for prerequisite courses can also help teachers understand how each concept was built up. By pairing a question asking students to create an $xy$-table for $y = x^2$ with one for $y = (x - 1)^2 + 3$, teachers will be able to differentiate between errors in misinterpreting the squared notation and those surrounding basic arithmetic and order of operations. Understanding the complexity of each problem, even going back to its elementary roots, is crucial to being able to develop diagnostic items and then correctly interpret and use preassessment results.

### *Unclear Interpretation*

The first diagnostic preassessment data did not confirm the informal observations I had collected for more than a year but suggested the opposite. Students who struggled with basic algebra scored much higher on the preassessment than I had anticipated, while other students who consistently demonstrated a high level of understanding on end-of-unit assessments performed poorly on the preassessment. Did the disconnect between my expectations and the data highlight the weakness of the preassessment, or did it reflect students' lack of effort on the preassessment, which was not to be graded? Or was I incorrectly interpreting what the students were trying to convey through their answer (Black and Williams 2009)?

Another difference between diagnostic preassessments and chapter tests is that the students were not told about the preassessment and did not review the material ahead of time. Do higher student scores on end-of-chapter tests than on preassessments reflect the fact that the content is not transferring from short- to long-term memory? I was left with more questions than answers.

Regardless of the stakes, a single assessment cannot provide a complete window into the mathematical minds of students and must not replace the information that teachers collect daily.

## A SECOND ATTEMPT AT PREASSESSMENTS

Applying what I learned from my first diagnostic preassessment, my second attempt consisted of a single, complex question. By focusing on only one question, I hoped to reduce the pressure on students and also limit the amount of data to analyze. For the unit on exponential functions, I asked students to create a table and graph the function $y = 3^x$, using integer $x$-values from $-3$ to 3. My primary goal was to determine how many students could correctly apply the negative exponent rule, an important skill that they would need to build on when graphing exponential functions. Assessing a single concept provided the opportunity to record students' answers and tease apart the different misconceptions reflected in their work.

When interpreting the student work, I tracked the data two ways. First, I tracked student answers using a binary system: 0 for a correct answer and 1 for an incorrect answer (see **fig. 1**). The data from this preassessment revealed that 15 of 17 students incorrectly applied the negative exponent rule. In addition, I recorded all incorrect answers for further analysis. Analyzing students' work revealed that 8 of 17 students stated that $3^{-3}$ was $-27$. Reflecting on the data allowed me to dissect answers and determine which misconceptions to address in class and which to address individually.

## DOCUMENTING PREASSESSMENT DATA

Excel® is a useful tool for keeping track of and analyzing data, regardless of the number of students being assessed or the number of prerequisite skills being analyzed. To track the students' answers, I used a binary system, which allowed me to use the sum formula at the end to tally the number of incorrect answers. Excel can also count the number of

Overall, the information gleaned from the diagnostic preassessments has helped shape the ways that I introduce new concepts and the types of questions that I ask in class.

**Primary Goal Assessed**
Student correctly applies the negative exponent rule (i.e., 3^(–1) = 1/3)

| Student No. | 0 or 1 | Explanation |
|---|---|---|
| 1 | 1 | B |
| 2 | 1 | C |
| 3 | 0 | A |
| 4 | 1 | B |
| 5 | 1 | F |
| 6 | 1 | B, H |
| 7 | 1 | D |
| 8 | 1 | C |
| 9 | 1 | B |
| 10 | 1 | B |
| 11 | 1 | D |
| 12 | 1 | B |
| 13 | 1 | E |
| 14 | 0 | A |
| 15 | 1 | B |
| 16 | 1 | B |
| 17 | 1 | G |
| **Total Incorrect:** | **15** | |

**Key**

0 Correctly applied negative exponent rule
I Incorrectly applied negative exponent rule

A Student correctly applied the negative exponent rule
B Student said 3^(–3) = –27 and so forth
C Student ignored the negative exponent –3(–3) = 27
D Student did not fill in the y-values on their table
E Student wrote 3^(–3) = 3/3 = 1.
F Student turned it into a multiplication problem –3^(–2) = 3(–2) = -6
G Student put answer as a fraction, but then multiplied –3^(–3) = 1/9
B, H As in B, student switched the base and exponent 3^(–2) = –8

**Analysis of Data**

Number of students who correctly answered problem: 2 (out of 17)
Most common error: B (8 out of 15)
Second most common error: C (2 out of 15)

**Fig. 1** Preassessment data show student responses to a single question.

times each misconception was encountered through the use of the COUNTIF feature. Because I cared about how many students incorrectly applied the negative exponent rule as well as the resulting types of errors, I created a key that highlighted their misconceptions. The creation of the key caused inputting the data to take longer but ultimately made the table more valuable in helping inform my teaching decisions.

In my first preassessment, I chose not to keep track of misconceptions and recorded only whether or not a student correctly answered the question. As with my second assessment, I used 1 to indicate a wrong answer; the table uses blanks instead of 0 entries (see **fig. 2**). The sum formula in Excel counted the number of students who missed each question as well as the total number of questions that each student missed. This type of table is helpful when trying to track the most-missed questions.

According to the data, I needed to address questions 5 and 15 with the entire class. I also determined that I did not need to spend time reviewing the material in questions 11 and 12, because nearly every student correctly answered those problems. This preassessment data could also be used to form groups to differentiate the types of prerequisite information that each student reviewed.

Tables tracking the number of students who missed each problem provide teachers with only limited information. For the information to best inform teaching practices, it is important that teachers analyze and record errors to track trends in student misconceptions (see **fig. 1**) to accurately address the misunderstandings in class.

## ADDRESSING PREASSESSMENT RESULTS IN THE CLASSROOM

Although the topics on my diagnostic preassessments were not always Algebra 2 concepts, they all played a part in whether or not my students could arrive at the correct answer when solving a problem. I chose not to immediately address the misconceptions identified in the preassessment; I wanted time to thoughtfully look through the students' work, pick apart their answers, and think about the best way to engage students in correcting the error I wanted to highlight.

To maintain an appropriate level of rigor, I did not devote entire class periods to addressing the

| Student No. | Question No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | Total by Student |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | | 1 | | 1 | | 1 | 1 | | 1 | | | 1 | 1 | 1 | 1 | 10 |
| 2 | | | | | 1 | 1 | 1 | 1 | | | | | 1 | 1 | 1 | | 7 |
| 3 | 1 | 1 | | 1 | 1 | | | | | | | | | 1 | 1 | 1 | 7 |
| 4 | | | | | | | 1 | | | | | | 1 | | 1 | 1 | 4 |
| 5 | | | | | 1 | | 1 | | | | | | 1 | 1 | 1 | | 5 |
| 6 | 1 | 1 | 1 | 1 | 1 | | 1 | | | | | | 1 | 1 | 1 | 1 | 10 |
| 7 | 1 | | | | 1 | | 1 | | 1 | | | | | | 1 | 1 | 6 |
| 8 | | 1 | | | | | | | | | | | 1 | | 1 | | 3 |
| 9 | | 1 | | | 1 | 1 | | | | | | | | | 1 | 1 | 5 |
| 10 | 1 | | | | 1 | | 1 | | | | | | | | 1 | 1 | 5 |
| 11 | 1 | 1 | | 1 | 1 | | 1 | | | 1 | | | 1 | | 1 | 1 | 9 |
| 12 | 1 | 1 | | 1 | | | 1 | | | | | | 1 | 1 | 1 | 1 | 8 |
| 13 | 1 | | | 1 | 1 | | | | | 1 | 1 | 1 | 1 | 1 | | 1 | 9 |
| 14 | 1 | 1 | 1 | | 1 | | 1 | | | 1 | | | | 1 | 1 | 1 | 9 |
| 15 | | | | | 1 | | 1 | 1 | | 1 | | | | 1 | 1 | | 6 |
| 16 | | 1 | | | 1 | | 1 | | | | | | 1 | 1 | 1 | 1 | 7 |
| 17 | | | | | 1 | | | | 1 | | | | 1 | | | | 3 |
| | | | | | | | | | | | | | | | | | |
| **Total by Question** | 9 | 8 | 3 | 5 | 14 | 2 | 12 | 3 | 2 | 5 | 1 | 1 | 11 | 10 | 15 | 12 | |

**Fig. 2** Preassessment data are used for a unit on solving quadratic functions and equations.

students' gaps in understanding that were highlighted in the preassessments. Instead, I incorporated the review into the lessons in which that skill was directly applied to a new concept. On my first preassessment, for example, most students missed question 15, which asked students to graph $y = |x + 3| - 2$ using function transformations. I wanted students to make connections between the way transformations occurred in absolute value and in quadratic functions, so I began the lesson on quadratic function transformations by reminding students of absolute value function transformations and by looking at *xy*-tables before transitioning to graphing quadratic functions. I also realized that additional class time would be needed to investigate and practice transformations of quadratic functions because students would not be able to immediately apply their knowledge of absolute value transformations to the new concept.

On the second preassessment, the most common student error involved incorrectly interpreting negative exponents by making the final answer negative. I addressed this error in two ways in class. First, I highlighted the correct use of negative exponents using an *xy*-table when we evaluated exponential functions. Second, I emphasized the shape of graphed exponential functions and highlighted the range, giving students a pictorial reminder of why $-27$ was not a reasonable output for the function $y = 3^x$. By doing so, I hoped to provide students with a tool for catching their error on future problems.

Overall, the information gleaned from the diagnostic preassessments has helped shape the ways that I introduce new concepts and the types of questions that I ask in class. The preassessments were also a time saver, because they reduced the need to review concepts students already understood. For example, on the first preassessment, 14 of 17 students missed simplifying $(x + 3)(x - 2)$, while only 3 students missed simplifying $2x(x + 5)$. A common incorrect student answer on $(x + 3) \cdot (x - 2)$ was $x^2 - 6$. If we look at both questions together, the data suggest that students had some knowledge of the distributive property and the fact that $x$ times $x$ equals $x^2$, but they needed reminders of what to do when multiplying two binomials.

By knowing the specific gaps in their prerequisite skills, I made sure not to assume students'

knowledge of a concept or skill. Although this approach meant that we often took one or two steps backward before moving forward in our learning, the reminders and review process helped to strengthen understanding without significantly diminishing the level of rigor in the class.

### THE VALUE OF DIAGNOSTIC PREASSESSMENTS

As I reflected on my struggles in interpreting diagnostic preassessment data, I slowly examined my own expectations for preassessments as well as my beliefs of what a preassessment should look like and how the data should be analyzed. My initial belief in the power of preassessments, I realized, was a simplistic approach to a complex problem. I concluded that diagnostic preassessments were not a quick fix and, on their own, would not be the solution for students struggling in Algebra 2. The convoluted data from the first assessment reminded me that students are complex people and that many variables are at play every day in the classroom. A single, written preassessment cannot accurately assess a student's understanding nor can it take the place of collecting informal observations over a longer period of time.

Despite the pitfalls of written diagnostic preassessments, I believe that they should be used to identify students' misconceptions and gaps in understanding. They can support informal observations or help teachers become more aware of specific areas with which a student is struggling. Teachers can use the results of diagnostic preassessments to tailor their lessons to the skills of the students in each class, incorporate needed review or reteaching, and include activities that help students recognize common misconceptions.

Further, diagnostic preassessments can be an excellent way for a mathematics department or a group of teachers who all teach the same course to track and analyze what prerequisite material students tend to forget or common misconceptions they have retained. Collecting this sort of data over time could help teachers anticipate student misconceptions and work to address gaps in understanding through additional focus in the previous course or planned review in individual lessons.



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