# Five Keys for Teaching MENTAL MATH

*A comprehensive look at mental math in the light of technology, the Common Core State Standards for Mathematics, and brain-based learning.*

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For a number of years, I have been teaching mental math strategies as part of my mathematics courses, and my students have found (sometimes to their surprise) that the strategies are useful. As students develop mental math proficiencies, they complete homework assignments faster, their confidence increases, and their overall understanding of numbers, operations, and algebraic thinking improves. After studying the Common Core State Standards for Mathematics (CCSSM) and brain-based learning research, I believe we need increased attention to mental math instruction in secondary school mathematics (grades 7–12) and in teacher education programs. The purpose of this article is to share some keys for teaching mental math. Before doing so, I will share my rationale for my recommendation to increase mental math instruction in secondary school mathematics and in teacher education.

First, experts and policymakers are calling for mental techniques. Cathy Seeley, NCTM President for 2004–2006, began her “Do the Math in Your Head!” article (2005) by cogently stating:

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What does it mean to know mathematics? This is a complex question, but there is strong agreement that facility with numbers and skill in problem solving play important roles. *Principles and Standards for School Mathematics* [2000] calls for students to be proficient with tools that include pencil and paper and technology, as well as mental techniques. I would like to make a case for raising the importance of mental math as a major component in students' tool kits of mathematical knowledge. Mental math is often associated with the ability to do computations quickly, but in its broadest sense, mental math also involves conceptual understanding and problem solving.

Mental math skills and strategies are not learned overnight or individually. Rather they grow over time and complement one another.

The Common Core Standards, I believe, are consistent with Seeley's recommendation. The CCSSM's expectations for deeper understanding, coherence, number sense, and fluency call for increased attention to mental math instruction. The Partnership for Assessment of Readiness for College and Careers (PARCC) and the Smarter Balanced Assessment Consortium are the consortia writing the new assessments based on the CCSS. (The new exams will be given in more than thirty-five states in 2014–2015.) PARCC and Smarter Balanced have similar calculator policies: The assessments for grade 6 through high school will be divided into calculator and noncalculator sessions. This division finally sends the right message to students! The National Mathematics Advisory Panel (2008) concurred. One of its principal messages was that "use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing . . . the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts" (p. 11).

I have found that when junior high and senior high students overuse calculators, their mental math skills atrophy and their number sense plateaus or actually decreases. Doing calculations in one's head not only keeps the basic arithmetic facts fresh but also can enhance understanding of mathematical concepts. The strategies that I teach (examples are given below) are not advanced strategies or tricks but rather strategies that leverage mathematical properties such as the distributive property, factors of numbers, percentage, and inverses.

Helping students gain mental math proficiency is actually rather complex. (The extremes of "you-may-use-calculator-type-XYZ-in-this-class" and "no-calculators-are-allowed-in-this-class" are shortsighted and send the wrong message to students.) The following five keys for developing mental math skills and strategies are not learned overnight or individually. Rather they grow over time and complement one another.

## KEY 1: USEFULNESS

The first key to equipping students with mental math skills is to help them see the importance and usefulness of mental math and see mental math as one of three calculation methods. The following is my list of the top six reasons why mental math is important and useful (a poster of these six reasons hangs in my classroom):

6. Mental math methods are usually faster than technology—*if* you have an efficient strategy.
5. Mental math is useful in everyday life, when technology (and paper) may not be available or appropriate. We discuss examples such as working in a concession stand, finding sale prices at the sale rack, comparison shopping, and so on.
4. Mental math is useful for checking or estimating an answer obtained from a calculator (or other technology). Did the fast food restaurant ring up the total bill and change correctly? Is the answer to the homework problem reasonable?
3. Being able to do some math mentally leads to fluency and confidence. As with people who are fluent in a foreign language, who can carry on a conversation without stopping to look up words in a dictionary or taking long pauses to compose sentences, those fluent with numbers can engage in the flow of the process of problem solving and move through many calculations mentally—rather than disengaging to manipulate a keypad. *Fluency happens in the mind.* No matter what the activity, confidence helps students be more successful.
2. It will be on the test. Tests in my classes are two-part tests, containing a noncalculator portion and a calculator portion. (The PARCC and Smarter Balanced assessments in 2014–15 will have calculator and noncalculator portions.)
1. Mental math methods help students understand mathematics. Mental math methods add connections in the brain that make homework easier and can help make new concepts easier to learn. *Understanding occurs in the mind.*

Understanding is all about connections. Van de Walle, Karp, and Bay-Williams (2013) very nicely state, "Understanding can be defined as a measure of the quality and quantity of connections that an idea has with existing ideas" (p. 23). Van de Walle and colleagues describe numerous benefits of relational understanding, which lead to better recall of ideas, less frustration, more success, and higher achievement. Students' appreciation of the usefulness and benefits of mental math grows gradually over time.

## KEY 2: STRATEGIES

The second key is teaching strategies. Before taking my class, many of my students do not know, for example, how to multiply 20 by 30 by multiplying 2 and 3 and appending two zeros. Each semester I teach fifteen mental math strategies (details follow). I focus on natural strategies leveraging basic mathematical properties such as the commutative, associative, and distributive properties. For example, I teach the compatible-numbers strategy—

$$4 \cdot 1.7 \cdot 25 = (4 \cdot 25) \cdot 1.7 = 100 \cdot 1.7 = 170$$

—and the pull-out-a-factor-of-2 strategy—

$$35 \cdot 18 = 35 \cdot 2 \cdot 9 = 70 \cdot 9 = 630.$$

Some of my math students already have figured out many of these strategies on their own; however, many of the strategies are unknown and new to them. The average and below-average students are often pleased (and even surprised) to find out that there are some easy shortcuts to answers.

## KEY 3: PRACTICE

The third key is to provide students with practice in strategy selection and use—and the practice should be within a context. Each time I teach a strategy, I first model the strategy, explaining when and how to use the strategy by doing examples; then provide practice problems on the new strategy learned; and finally provide mixed practice, in which the students are given problems for which they must select (and apply) a mental math strategy from the strategies learned so far in the course. The practice problems given are usually in a real-world or mathematical context. "If a car gets 20 mpg, how far can it go on 30 gallons?" or "Solve: $x/35 = 18$" would be typical practice problems. In this way, students are practicing mental math and the math skills that we want them to practice as well as seeing the relevance of the mental math strategies.

Brain-based learning tells us that humans crave meaning and relevance (this is principle 3 of the brain-mind principles of natural learning stated by Renate Caine and Geoffrey Caine in *Natural Learning for a Connected World: Education, Technology, and the Human Brain* [2011]) and that practice makes a difference for understanding and recall. Willis (2013) stated, "Multisensory instruction, practice, and review promote memory storage in multiple regions of the cortex" (p. 4).

## KEY 4: DECISION

The fourth key for developing student competence with mental math is helping students make good decisions regarding which calculation method to use. This determination is not as simple as it may sound. Sometimes people think that the size of the numbers determines whether to use mental math or not. Not true. Finding 30 percent of 2 million can be done in one's head, but finding 28 percent of 1.7 is more appropriate for a calculator.

I am very much in favor of using calculators and other technology appropriately in the classroom. The cogitated use of mental math strategies relates to three of the CCSSM Standards for Mathematical Practice: "Use appropriate tools strategically" (SMP 5); "Look for and make use of structure" (SMP 7); and "Look for and express regularity in repeated reasoning (SMP 8) (CCSSI 2010, pp. 7–8). In the classroom setting, the teacher can and should determine, according to such factors as instructional objectives, which calculations should be done with a calculator and which ones should be done mentally.

Ultimately, however, we want our students, to make good decisions in and outside the classroom as to which calculation method to use. As teachers, we can model this decision-making process in the classroom. Often, a math problem comes down to an equation such as, for example, $x - 98 = 53$. One of the most important questions I ask in class is, "Do we have a mental math strategy for this?" In this example we do—compensation, as $x = 53 + 98 = 53 + 100 - 2 = 153 - 1 = 151$. If we do not have a strategy, then we use a calculator (or work it out on paper). We base our decision on the particular numbers and the strategies we have at our disposal. As the course progresses, more and more of the calculations can be done mentally, as we learn more strategies.

## KEY 5: MINDSET

The fifth key is to develop a mindset and expectation of mental math. It will take considerable time for students to learn to use mental math strategies and make informed decisions about when to use mental math, especially if they have overused calculators in their previous course. The goal is to have students expect to use mental math on homework, quizzes, and tests—and of their own accord—and believe in the power of mental math. To this end, teachers should model the mindset; encourage, praise, and reward the use of mental math; and have no-calculator assessments. Having no-calculator and calculator-needed assessments clearly sends

## MENTAL MATH STRATEGY: TO DIVIDE BY 4, DIVIDE BY 2 TWICE

*When to use this strategy:* Use this strategy when you need to divide a number by 4.
*How to use this strategy:* Divide by 2 and then divide again by 2.
(Dividing a number by 2 is the same as taking half of it.)
Examples:

$\frac{288}{4} = \frac{144}{2} = 72$ $\quad\frac{18}{4} = \frac{9}{2} = 4.5$ $\quad 108 \div 4 = 54 \div 2 = 27$

| Use this (new) strategy on the following problems: | Use any strategy you know on the following problems: |
|---|---|
| 1. Evaluate: $\frac{14}{4}$ | 5. Solve: $\frac{x}{4} = 17$ |
| 2. Central Grade School has 88 third graders. The principal wants to have four classes with an equal number of children in each class. How many third graders should be in each class? | 6. In the first five games, Dustin scored 31, 28, 32, 33, and 28 points. About how many points has Dustin scored so far this year? |
| 3. Evaluate: $\frac{68x^3}{4x}$ | 7. Find the total deposit if you deposit the following checks: \$6.80 + \$1.50 + \$6.50 + \$2.20. |
| 4. $n$ = number of tickets sold<br>$p$ = profit (\$)<br>$p = 4n - 100$<br>If the profit is \$184, find the number of tickets sold. | 8. Matt brought \$92 to spend on a four-day road trip. What is the maximum amount that Matt should spend each day to have an equal amount of money every day? |

**Fig. 1** A sample worksheet presents a strategy to practice.

the message that there is a time and place for calculators and a time and place for mental math.

## HOW TO TEACH THE STRATEGIES

I teach fifteen mental math strategies each semester, one per week. One day per week I teach a strategy (typically, Mental Math Monday), using a half-page handout that I pass out at the beginning of class. **Figure 1** shows a sample handout. The handout describes when and how to use the mental math strategy and provides examples. I briefly describe the strategy, show an example or two, and give the students work time in class to do the exercises. I circulate around the classroom and give hints and suggestions, as needed. Practice problems in real-world and mathematical contexts are given. Students practice the new strategy learned (key 2, on the left side) and are also given mixed practice in which they must select and use an appropriate strategy (key 3, on the right side of the handout). We often discuss the answers and the strategies used or put a student's answers on an Elmo document camera. This activity takes five to ten minutes, once a week.

**Figure 2** shows a list of ten strategies from my mental math curriculum giving examples but with few or no explanations. The letters in brackets indicate the operations for which the strategy works (A stands for addition, S for subtraction, M for multiplication, and D for division). Two more sample worksheet handouts are provided in the online component of this article (**www.nctm.org/mt068**). Additional strategies and associated worksheets are available from the author at http://wiu.edu/users/mfjro1/wiu/mentalmath/CourseSpecific/front.html.

I hope that you and your students will also find these keys for teaching mental mathematics strategies useful. As is clear from Seeley, brain-based

### TEN MENTAL MATH STRATEGIES

1. Compatible numbers for addition [A]
   *Example:*
   $25 + 8 + 6 + 12 + 25 = (25 + 25) + (8 + 12) + 6$
   $= 50 + 20 + 6 = 76$

2. Break apart [A, S, M]
   *Examples:*
   $3 \cdot 42 = 3 \cdot 40 + 3 \cdot 2 = 120 + 6 = 126$
   $215 - 83 = (210 - 80) + (5 - 3) = 130 + 2 = 132$

3. To divide by 5, double the number and divide by 10 [D]
   *Example:* $28 \div 5 = 56 \div 10 = 5.6$

4. Compensation [A, M]
   *Examples:*
   $65 + 38 = 65 + 40 - 2 = 105 - 2 = 103$
   $38 \cdot 5 = 40 \cdot 5 - 2 \cdot 5 = 200 - 10 = 190$

5. Use doubling when 2 is a factor [M]
   *Example:* $6 \cdot 35 = 3 \cdot (2 \cdot 35) = 3 \cdot 70 = 210$

6. To multiply decimals, drop the decimal point and put it back in later [M]
   *Example:* $0.5 \cdot 0.3 = 0.15$

7. Equal additions technique for subtraction [S]
   *Example:* $74 - 28 = 76 - 30 = 46$

8. To divide by 4, divide by 2 twice [D]
   *Example:* $68 \div 4 = 68 \div 2 \div 2 = 34 \div 2 = 17$

9. To multiply by a unit fraction, divide [M]
   *Example:* $(1/5) \cdot 35 = 35 \div 5 = 7$

10. To multiply a fraction times a number, divide and then multiply [M]
    *Example:* $(3/5) \cdot 35 = 35 \div 5 \cdot 3 = 7 \cdot 3 = 21$

**Fig. 2** Operations appropriate for each strategy are indicated in brackets: A for addition, S for subtraction, M for multiplication, and D for division.

learning research, the CCSSM, and the PARCC and Smarter Balanced calculator policies, we need to increase mental math instruction in secondary school mathematics and teacher education programs. Using the five keys can help students learn mental math strategies and skills and, as a result, improve their overall achievement.

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### Promoting student learning

- What assessment strategies foster opportunities for students to reflect on their own or their peers' mathematical strengths and weaknesses?
- How do students benefit from assessment that highlights mathematical connections?

### Respecting and responding to diversity

- What assessment strategies have you tried that honor student diversity?
- How do you negotiate the tension between the diversity of your students' experiences and the goal of being fair?

### Driving instructional planning

- How do you use formative and summative assessments in your process of instructional planning?
- How do assessment results guide your next steps?
- Which assessments have been effective and why?

### Creating alternative assessments

- What alternative assessments have you developed related to specific mathematical content?
- How do you use new technologies to assess student thinking?
- What did you learn about your students and their mathematical progress from these assessments that you would not otherwise have discovered?

### Teaching in a context of external assessments

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