APRIL

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

1
In the addition problem

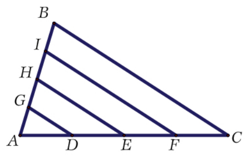
$$AH + HA = DAD,$$

each different letter in the two-digit and three-digit numbers represents a different digit. Find the number represented by *DAD*.

2
The lengths of the sides of $\triangle ABC$ are $AB = 17$, $BC = 24$, and $AC = 20$. If the lengths $AD = DE = EF = FC$ and

$$\overline{GD} \parallel \overline{HE} \parallel \overline{IF} \parallel \overline{BC},$$

find *IF*.



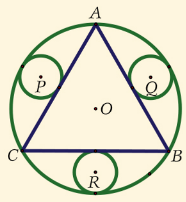
3
Two sides of a parallelogram measure 6 cm and 7 cm. Find the lengths of the diagonals if both are integers.

4
All nine entries of the mystical multiplication square are positive integers, but they may not all be different. The product for each row, column, and the two main diagonals is constant. If no two rows or columns contain the same three numbers, find the missing numbers that result in the least product.

	48	
		3
12		

5
The average length of a toothpick is 5 cm. If a dozen toothpicks are arranged in a straight line with a 2 cm space between them, what is the total expected length of the lineup of toothpicks?

6
Equilateral $\triangle ABC$ is inscribed in circle *O*, and circles *P*, *Q*, and *R* are tangent to both the triangle and circle *O*. Find the ratio of the area of $\triangle ABC$ to the sum of the areas of the three small circles.



7
Several cylinders are of different sizes, but for each one the surface area and the volume have the same numerical value. If the radius and height are both integers, what is the largest volume possible?

8
Prove that in a parallelogram the sum of the squares of the diagonals equals the sum of the squares of the four sides.

9
The nonparallel sides of a trapezoid are 13 and 20, a third side is 9, and the trapezoid's height is 12. Find the possible lengths for the fourth side of the trapezoid.

10
Find the sum of the squares of the lengths of the diagonals for each of the two trapezoids in the solution to problem 9.

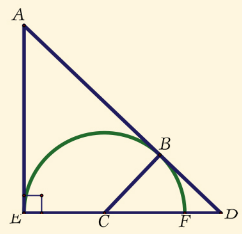
11
For the trapezoids of the problem for April 10, the sum of the squares of the diagonals is less than the sum of the squares of the four sides. Explain how the differences are related.

12
The numbers 1, 2, 3, … 10 are written on ten index cards, one number per card. You shuffle the cards and, without looking at them, randomly select one card and throw it away. If you randomly choose a second card, what is the probability that it will have a prime number written on it?

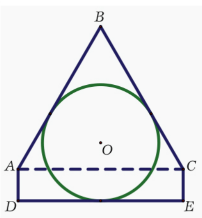
13
Two concentric circles are drawn so that the smaller circle trisects a chord of the larger circle. The length of the chord equals the sum of the lengths of the two radii. Find the ratio of the smaller radius to the larger radius.

14
How many three-digit primes have all three digits as different primes?

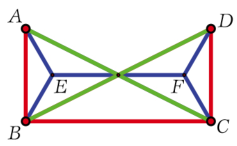
15
Semicircle $\widehat{FBE}$ of circle *C* is tangent to the hypotenuse *AD* of right triangle *AED* at *B*. If $CE = 60$ and $BD = 63$, find the area of $\triangle AED$.



16
Equilateral triangle *ABC* sits on side $\overline{AC}$ of rectangle *ACED*. Circle *O* is tangent to two sides of the triangle and to the base of the rectangle. The length of the radius of the circle is $8\sqrt{3}/3$, and $DE = 12$. Find the area of rectangle *ACED*.



17
Four computers in a room are arranged at the vertices *A*, *B*, *C*, and *D* of a 6 ft. × 10 ft. rectangle. They are all to be connected with wire. Which of the three possible arrangements shown as red, green, or blue wire in the diagrams requires the least amount of wire?

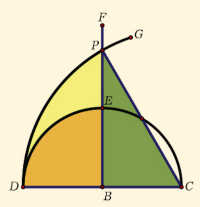


18
Given $\triangle ADC$ with $AD < AC < DC$, find the probability that the triangle is acute (i.e., that all three angles are acute).

19
If $9^x - 9^{x-1} = 24$, find $(6x)^x$.

20
The sum of the positive divisors of 120 is 360. Find the sum of the reciprocals of all the positive divisors of 120.

21
The yellow region in the diagram is bounded by circles centered at *B* and *C* and the line *FB*. Assume that $DB = 1$. If a circle is inscribed in the yellow region (internally tangent to $\widehat{PD}$, externally tangent to $\widehat{ED}$ and tangent to $\overline{PE}$), find its radius.



22
Two congruent squares overlap to form a regular octagon and an equilateral hexadecagon (16 sides). Find the ratio of the area of the hexadecagon to the area of the octagon.

23
In circle *O* the length of the radius is *x*, and in circle *P* the length of the radius is *y*. The common internal tangent is $\overline{RQ}$, and the line of centers is $\overline{OP}$. Prove the following:

$$RQ^2 + (x + y)^2 = OP^2$$

24
In $\triangle ABC$, the bisector of $\angle A$ intersects $\overline{BC}$ at *D*. If $AB = 8$, $AC = 15$, and $m\angle A = 60°$, find *BD*.

25
The angle bisector of $\angle A$ in $\triangle ABC$ divides $\overline{BC}$ into two segments of lengths 4 and 6. If the length of the angle bisector is 8, compute the product $AB \cdot AC$.

26
Simplify the following fraction:

$$\frac{(1+2+3+\cdots+19)\cdot 20}{(21+22+23+\cdots+39)\cdot 40}$$

27
Two congruent equilateral triangles overlap to form a regular hexagon and an equilateral dodecagon (12 sides). Find the ratio of the area of the dodecagon to the area of the hexagon.

28
The circumference of a large circle is 2 cm greater than the circumference of a small circle. How much bigger is the radius of the large circle than the radius of the small circle?

29
Prove the general statement introduced numerically in problem 26. Show that for all integers *n*,

$$\frac{(1+2+3+\cdots+(2n-1))\cdot 2n}{((2n+1)+(2n+2)+(2n+3)+\cdots+(4n-1))\cdot(4n)} = \frac{1}{6}.$$

30
The number 2015 can be written as the sum of two or more consecutive positive integers in seven different ways. Find the total of the smallest integers from each of the seven ways.