TAKE TIME FOR ACTION

ESTABLISHING BENCHM

BARBARA J. REYS, OK-KYEONG KIM, AND JENNIFER M. BAY

THE USE OF BENCHMARKS, SUCH AS 0, 1/2, and 1, for comparing the size of fractions is not emphasized very often in instruction. Comparing fractions with 0, 1/2, or 1 by comparing the numerator with the denominator or by mentally modeling the fraction is a powerful tool for gauging the size of fractions, making quick estimates, and judging the reasonableness of computed results (Bezuk and Bieck 1992). If students are asked to compare 5/8 with 4/9, the traditional technique is to find the common denominator, convert both fractions to equivalent forms using this common denominator, then compare numerators. However, this problem can be solved more efficiently by comparing each fraction with 1/2: 5/8 is larger than 1/2, since it is larger than 4/8; and 4/9 is less than 1/2, since 4/9 is less than 4.5/9, or 1/2.

Benchmarks are also useful for estimating the answers to computations with fractions. For example, 5/6 + 8/9 is less than 2, since each fraction is less than 1. Likewise, 3/8 of 520 is less than half of 520, since 3/8 is less than 1/2. Using benchmarks enables students to estimate and gives them a tool for judging the reasonableness of their answers.

Do your students use benchmarks to think about and solve problems involving fractions? Do they have efficient techniques for estimating with fractions? To explore these questions, we interviewed twenty students in one fifth-grade class (see **fig. 1**). You may presume, as we did, that middle school students who are computing with fractions—adding, subtracting, multiplying, or dividing—can make sense of such basic fraction ideas as visualizing a fractional amount. Before reading about our findings, discover how your middle school students conceptualize fractions by using the following interview. Collect information by selecting a few students and interviewing them or by using the set of questions as an open-ended, whole-class assessment.

Interview Questions

1. Think about the fraction 2/5. What can you tell me about it? Prepare a "report" for me about the fraction 2/5. (Ask students to use drawings, written explanations, and whatever they like to "impress you" with what they know about 2/5. After their initial responses, prompt them to continue to think about and write statements about the fraction 2/5 using some of the following questions: How large is it? What is it close to? What is it larger or smaller than?)
2. At a party, several large pizzas were ordered. Jake, John, and Josh each ate a different kind of pizza. Jake ate 1/3 of a pepperoni pizza, John ate 4/8 of a veggie pizza, and Josh ate 3/5 of a cheese pizza. Who ate the most pizza? Explain how you know.
3. Are the following sums larger or smaller than 1? Explain how you know. (Encourage students to use estimation.)

   a) 3/8 + 4/9
   b) 1/2 + 1/3

Fig. 1

BARBARA REYS, *cibr@showme.missouri.edu, is a faculty member in the College of Education at the University of Missouri—Columbia, Columbia, MO 65211.* OK-KYEONG KIM, *c722896@showme.missouri.edu, is a graduate student at the University of Missouri—Columbia. She taught elementary school in Korea.* JENNIFER BAY, *c498845@showme.missouri.edu, is a doctoral student at the University of Missouri. She taught middle school for nine years and is currently working with preservice and in-service teachers.*

Edited by MICHAELE CHAPPELL, *chappell@tempest.coedu.usf.edu, teaches at the University of South Florida—Tampa, Tampa, FL 33620-5650. Her interests include ethnomathematics, multicultural mathematics education, and alternative assessment.*

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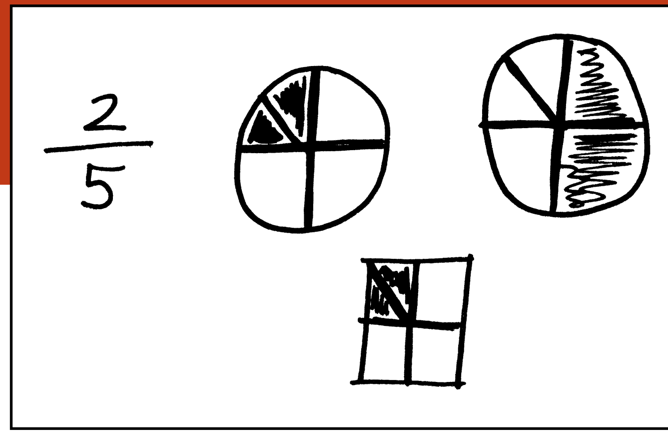

Fig. 2 Inaccurate diagrams of 2/5

Discussion of the Study

WE INTERVIEWED THE FIFTH-GRADE STUDENTS WHEN they had just finished a unit on fractions that included comparing and ordering fractions, formulating equivalent fractions, and using conventional paper-and-pencil computational algorithms for adding and subtracting fractions. The first interview question offers an open-ended opportunity to determine how students think about a particular fraction: Do they relate it to a physical or mental model? Do they compare it with another fraction or whole number? Do they relate it to a real-world example? The fraction 2/5 was chosen because it is a common fraction that is close to 1/2. Students were encouraged to make as many statements or representations of the fraction as they could. Most students responded with one or two ideas. Their responses fell into three main categories:

1. Described a mental image of an object (e.g., two of five slices of pizza) (11 students)
2. Drew a model of the fraction using a circle or set model (19 students)
3. Related the size of 2/5 to 1/2 (e.g., 2/5 is a fraction less than 1/2) (7 students)

Although these students had recently completed a six-week unit on fractions, the responses from seven of the twenty students revealed a basic misconception about fractions: that a whole need not be partitioned into equal-sized pieces. For example, three students drew a circle, divided it into fourths, then divided one of the fourths in half to form the "fifth" section. When asked if it made any difference which two sections of the pizza were shaded to represent 2/5, one student indicated that it did not matter whether two "large" sections or two "small" sections were shaded and was not troubled that the choice led to different representations of the fraction 2/5. See **figure 2.**

Among the twenty students interviewed, seven initially stated that 2/5 was less than, or close to, 1/2. With a prompt (e.g., Is 2/5 more or less than a half?) another six students correctly identified 2/5 as being less than 1/2. The students compared the numerator with the denominator, noting that 2 is less than 2.5; therefore, the fraction was less than 1/2. Some of these students naturally used 1/2 to think about the size of other fractions; others did so when prompted. The remaining seven students used only the common-denominator approach to make the comparison.

The second interview question asked students to compare the relative sizes of 1/3, 4/8, and 3/5. These fractions were chosen in part because they are less than, equal to, and greater than 1/2, respectively. We were curious to see whether students would use 1/2 as a benchmark to compare the fractions or whether they would use the common-denominator strategy taught previously. **Table 1** summarizes the strategies that students used *initially,* with no prompting, to compare the fractions.

Although benchmarks were not part of the instructional sequence that students had experienced, six students used 1/2 to compare the fractions, five of them successfully. For example, one student said, "Jake ate the most pizza because 4/8 is half, 1/3 is less than half, and 3/5 is over half." This benchmark strategy had a higher success rate than the other two strategies of drawing a picture and finding a common denominator. The use of a benchmark not only resulted in more correct answers, it was also the "easiest" strategy used, requiring little time or cognitive effort. By contrast, seven of the nine students relied on inaccurate pictures that they had drawn to determine the largest fraction. In fact, even the two students who produced a correct answer using a drawing strategy relied on inaccurate pictures.

Five of the twenty students converted the fractions to equivalent fractions with common denominators. After prompting, several of these students indicated that they could also compare each fraction with 1/2 to find the largest fraction. When asked why they had used the common-denominator strategy, several students indicated that finding a common denominator was "doing math" or "using math." Some students confessed that they had just discovered the benchmark strategy during the interview after the interviewers' prompt and were not sure whether it was the correct way to do it.

The third question focused on the addition of fractions. Although students had learned the standard algorithm, they had not discussed estimation with fractions. In both problems, each addend was less than 1/2, so if students used the benchmark of 1/2, they could see that the sum was less than 1. In the first item, 3/8 + 4/9, the fractions were a bit awkward so that finding an exact answer using a common-denominator algorithm would have been cumbersome. ☞

TABLE 1
Students' Initial Strategies and Outcomes for Comparing the Fractions 1/3, 3/5, and 4/8

Strategies	Correct	Incorrect	Total
Drew a picture of each fraction and compared pictures.	2	7	9
Found a common denominator (two denominators at a time).	3	2	5
Used the benchmark 1/2 to compare fractions.	5	1	6
Total	**10**	**10**	**20**

Only three of the twenty students gave an estimate of "around 1" to the first problem. Each student explained the strategy as comparing each fraction with 1/2. The remaining students gave no indication of thinking about the relative size of 3/8 or 4/9. In fact, without being allowed to find the exact answer, seventeen of these twenty students, or 85 percent, could not devise a reasonable estimate.

The second item, 1/2 + 1/3, was placed last to see whether students would apply strategies that they had learned on the first item. In fact, twelve students found a reasonable answer. Seven students acknowledged that since 1/3 is less than 1/2, then the sum must add to less than 1. The remaining eight students were unable to produce a reasonable estimate—they applied incorrect algorithms, guessed, or gave up.

Implications for Teaching and Learning

TEACHERS CAN LEARN A GREAT DEAL ABOUT STUDENTS' understandings of fractions by using just a few questions. Students who retain basic misconceptions about not needing equal-sized pieces to represent partitioned wholes will have difficulty using benchmark strategies, such as comparing fractions with 1/2.

Teachers may decide that some students need more experience visualizing fractions. Have students shade fractional parts of various shapes, have them locate a fraction on a number line with endpoints of 0 and 1, or have them separate tiles to illustrate a factional amount. These types of activities provide opportunities for students to visualize what fractions look like as area and linear models and as quantities. All these representations are important for students to fully conceptualize fractions. Although these activities may resemble work done in earlier grades, such remediation is crucial for students who do not know that fractional parts need to be of equal size before they move on to complex procedures with fractions. Engaging these students regularly in modeling fractions can improve their fundamental understanding.

To begin to use benchmarks as a natural and instinctive process, students benefit from intentional modeling and prompting by the teacher. Teachers can help students recognize and use the power of benchmarks by thinking out loud in various situations. For example, if seventeen out of a class of twenty-eight students are riding the bus, the teacher might represent that situation as the fraction 17/28 and mention that "over half the class is riding the bus." Before computing $11/12 \times 48$, the teacher might say, "If 11/12 is multiplied by 48, then the product should be almost 48." Modeling and encouraging the use of benchmarks legitimizes the benchmarking process in the minds of students.

Students' responses to the estimation items clearly point out some misconceptions that students have about fractions and about the process of estimation. Estimating fraction sums is simpler than finding an exact answer if conceptual understanding is in place. This conceptual foundation includes understanding the size of fractions, particularly how they relate to such benchmarks as 0, 1/2, and 1; having mental models to call on to visualize fractions (e.g., a pie representation or number-line model); and understanding the meaning and relationship of the numerator and denominator (e.g., what does the denominator tell me about the fraction?).

As important as teaching students how to estimate is teaching them that estimation is a valued and useful process. Outside their mathematics class, students will estimate far more than they will find exact answers. Posing a few problems each day is an effective way to increase students' ability to work with fractions and to communicate the importance of operating flexibly with fractions.

Teachers are quite aware that middle school students struggle with fractions and that many display poor number sense when operating with fractions. We hope that this discussion will prompt you to assess your students' understanding and use of benchmarks and to make benchmarks an integral part of the middle school fraction curriculum.

Reference

Bezuk, Nadine S., and Marilyn Bieck. "Current Research on Rational Numbers and Common Fractions: Summary and Implications for Teachers." In *Research Ideas for the Classroom: Middle Grades Mathematics,* edited by Douglas T. Owens, 118–36. New York: Macmillan Publishing Co., 1992.