Teacher to Teacher

ORDERED-PAIR RELATIONS— a Performance Assessment

ROBERT M. ALLEN

ASSESSMENT GOALS MAY INCLUDE having students demonstrate factual knowledge, apply skills, integrate previously learned skills with new concepts, and synthesize concepts from various disciplines. The assessment strategy described here seeks to help students integrate new concepts with previously learned topics from different strands and eventually apply the results of the problems to future situations.

My student teacher and I chose to explore the study of ordered pairs in greater detail to allow students to demonstrate their own creativity, apply previously learned computer skills, evaluate their own work, and draw connections to other mathematical strands. The initial activity on relations is a fairly typical teacher-directed lesson from the textbook. We introduced students to terminology, including *relation, domain, range, mapping,* and *inverse.* After assigning appropriate practice problems, we introduced students to their ordered-pair-relations project.

The assessment directions (see **fig. 1**) ask students to construct an original drawing composed of con-

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nected ordered pairs, record the pairs in a table, and map them from x to y **(fig. 2).** Using their computer-spreadsheet knowledge, students enter the ordered pairs in a spreadsheet and graph the results, including the inverse and the negation of the relation (see **fig. 3**). They copy the graphs, paste them onto a new page, and produce printouts of both the spreadsheet and the graphs. A questionnaire prompts students to reflect on their results and record their findings (see **fig. 4**). The questions allow discourse in both algebraic and geometric concepts, which range from using factual information, communicating the resulting image to the reader, and predicting a new situation given a set of conditions.

Worksheet 1

Ordered-Pair Relations

Directions:

1. On grid paper, construct with a minimum of fifteen points an original drawing composed entirely of connected lines.
2. Record in a table the outline of the picture as coordinate pairs; label them in the order you drew them from start to finish.
3. Map your points from x to y.

Fig. 1 The first worksheet

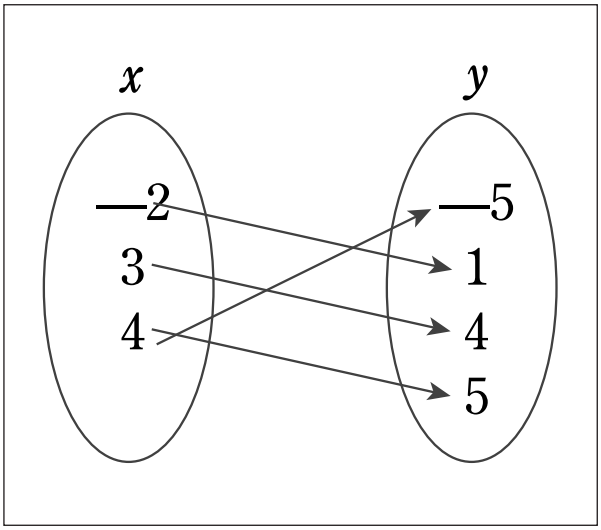

Fig. 2 A mapping example

Worksheet 2

Computer Lab Assignment

Directions:

1. Enter the coordinate pairs in a spreadsheet under the headings Point No., *x,* and *y.*

For example:

Point No.	*x*	*y*
1	3	4
2	–2	1
3	4	–5
4	4	5

2. Graph the points, using the Make Chart function. Label the graph with your name and the picture's name.
3. Copy and paste the chart to a new page.
4. Repeat steps 1 and 2, but use the inverse of the set of ordered pairs. Label your graph "inverse." Copy and paste the picture to the same page as in step 3.
5. Repeat the process one more time, using the negative of each ordered pair. Label the picture "negative."
6. Complete the questionnaire, and submit the following pages:

   - Original drawing, list of coordinate pairs, and mapping
   - Spreadsheet of points
   - Printouts of three graphs
   - Questionnaire

Fig. 3 The second worksheet

Worksheet 3

Follow-up Questions

1. When you plotted the inverse relation, how did it change the graph?
2. Where is the line across which the picture was reflected? Indicate it on the appropriate graph.
3. When you plotted the negative relation, how did the graph change?
4. Where is the line across which the picture was reflected? Indicate it on the appropriate graph.
5. Describe what would happen if you inverted and negated your original coordinate pairs—how would the image be reflected? Where would the line of reflection be drawn?

Fig. 4 A questionnaire to help students reflect on their work

Outcomes and Assessment

THE EXPECTED OUTCOME IS FOR STUDENTS TO apply concepts and skills to situations requiring mathematics. For this activity, they created an artistic design on a coordinate plane and transformed the image. See **figure 5** for examples of students' work. The state of Arizona has developed performance assessments with accompanying scoring rubrics, and my school district has issued assessment guidelines. In our classroom, we used a 4-point scale based on the state and district models to evaluate our students' work (see **figs. 6** and **7**).

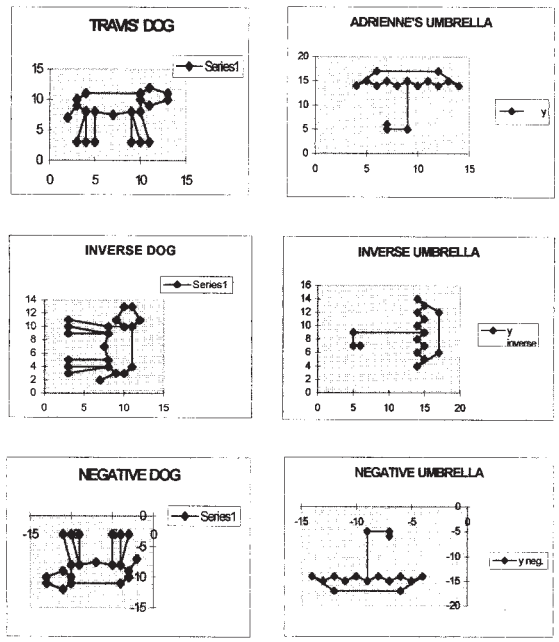

Fig. 5 Examples of students' work

Mathematics-Rubric Guidelines

Points

4 Answers exhibit the following:

a) An effective solution
b) Complete understanding of the problem
c) Addressing of all points relevant to the solution
d) Logical reasoning, valid conclusions
e) Effective and clear communication through writing and diagrams
f) Adequate and correct computations, set-up, or both

3 Answers exhibit the following:

a) Understanding of the problem
b) Adequate communication through writing or diagrams
c) Reasonable conclusions in general
d) Possible minor flaws in—

(1) reasoning
(2) computation
(3) addressing some aspect of the problem

2 Answers exhibit the following:

a) Gaps in understanding or execution
b) Faulty reasoning, weak conclusions
c) Failure to address all aspects of the problem
d) Unclear communication in writing or diagrams

1 Answers exhibit the following:

a) Some effort beyond restating or copying data
b) Little understanding of the problem
c) Major flaws in reasoning that lead to invalid conclusions
d) Lack of understanding of the relevant mathematics procedures and concepts

0 Answers exhibit the following:

a) No understanding of the problem
b) No effort displayed

Fig. 6 The school district's guidelines for the scoring rubric

Scoring rubric for worksheet 1

A score of 4 reflects the following:

- An original drawing, with a minimum of fifteen points, on grid paper composed entirely of connected lines
- A table listing the points associated with the drawing in order of being drawn, with the first and last ordered pairs being the same
- A mapping of the points from x to y

Scoring rubric for worksheet 2

A score of 4 reflects the following:

- A spreadsheet document containing lists of the original points, the inverse set, and the negation set
- Printouts of three graphs labeled according to instructions

Scoring rubric for worksheet 3

A score of 4 reflects the following:

- Responses to the questionnaire include descriptions of the new drawings generated by the inverse and negation sets of ordered pairs.
- Because of varying backgrounds in geometry, students may vary greatly in their responses. The instructor would look for a student response and drawing, which approximate a line through $y = x$ for the inverse relation.
- Descriptions of the negation could include flipping or reflecting across the x-axis if the drawing was symmetrical and centered at (0, 0) or across the y-axis for nonsymmetrical images or those located in one particular quadrant.
- Question 5 requires a fairly high level of abstract thinking from an eighth-grade student. The teacher should use discretion in scoring and look for responses that attempt to predict a transformation concerning a translation; a reflection of the image, relative to its original position, that would combine some of the foregoing responses; or both.

For each response earning fewer than 4 points, the generic rubric will be applied to determine the score of the worksheet.

Fig. 7 Scoring rubrics for the three worksheets

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Computer skills are important in this project, and each student had completed a unit on spreadsheets earlier in the year. We have a Macintosh computer lab with ClarisWorks software for classroom use. A small network in the mathematics class accommodates students who were absent during lab time and who need more time to complete the assignment.

We believe that this assessment encourages student creativity and independent thinking, incorporates technology, encourages connections among concepts, and contributes to a spiraling of topics and skills. As a performance assessment, we sought to have students apply their skills to an artistic situation, measure their ability to work with their mathematical tools, and self-evaluate their own results. Our teaching roles varied from direct instructor to facilitator and guide to evaluator of outcomes. During the process, we were encouraged by the students' enthusiasm and the resulting classroom tone, and we were impressed with the students' finished products.