Running by the Numbers

PAUL MILLIKEN

IN 490 B.C., THE MESSENGER PHEIDIPPEDES ran twenty-six miles to Athens carrying the news of Greek victory at the battle of Marathon. He delivered the news and dropped dead from the effort. Today, we celebrate that famous run with one of the most demanding events in human athletics, the marathon. Like Pheidippedes, the modern runner strives to complete the distance in as little time as possible. Unlike that early messenger, today's competitors undergo extensive training to ensure that they remain alive when they have finished the run. Kevin Smith uses mathematics to help runners prepare for marathons.

Marathon Dynamics, Inc., in Mississauga, Ontario, is Kevin Smith's company. He provides a variety of services for the running community, including hosting running clinics, conducting fitness and health presentations and seminars, managing and promoting local running events, coaching runners, and creating customized training plans for individual runners using software that he developed himself.

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"Basically my days are filled with crunching numbers," Kevin says, "involving calculations of distances, times, paces, and heart rates." He works with runners at all levels, from "recreational joggers to competitive athletes." Each training plan is customized "in order for an individual to improve running performance." Kevin brings an essential "appreciation and understanding of the relationships" among all the variables in his number-crunching.

Kevin says that he has "had to, at one time or another, apply the skills, formulae, and thought processes learned in" a whole range of mathematics disciplines, including "calculus, probability, algebra, and tons of more basic percentage and ex-

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"Math at Work" explores how mathematics is used in the workplace. Each article will highlight a particular career and the mathematics specific to that discipline. Readers are encouraged to submit manuscripts for this department by sending them to "Math at Work," MTMS, *NCTM, 1906 Association Drive, Reston, VA 20191-9988.*

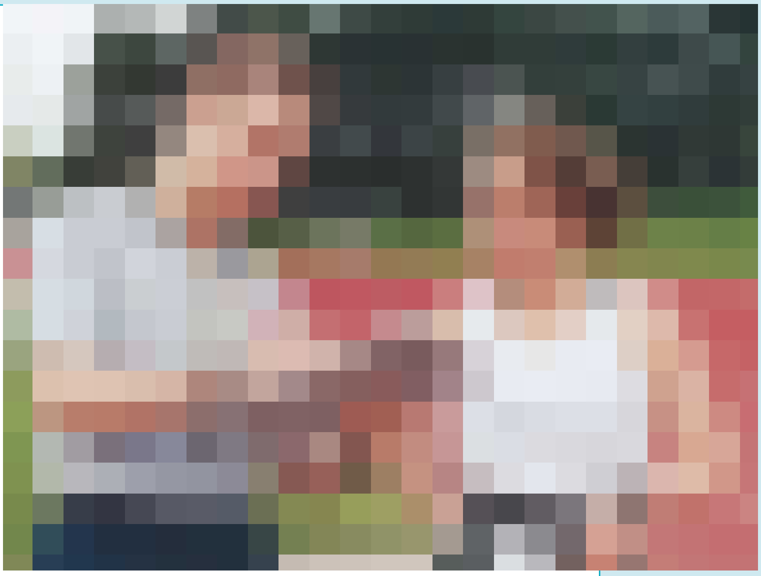
ponential calculation, and unit conversions (miles to kilometers, miles per hour to kilometers per hour, or meters per second, and so on).” All this work, on top of the accounting, record keeping, and financial management of operating his own company, means that Kevin is always doing mathematics on the job.

Does that mean that Kevin studied mathematics in college to prepare for his career? No. “I had no idea,” he says, “I would be using math in this way or to this extent. I created the job I do when we founded our company.” The mathematics-related courses that have been most useful to him are the accounting and economics courses that he took as part of his business administration degree at the University of Western Ontario. “As co-manager of the business, my responsibilities include most of the financial management duties of any small business, but the entrepreneurial spirit and desire will only get one so far.” He still has to do the bookkeeping.

“I suppose my attitude toward mathematics would have been a little different,” he admits, “had I known how it would all end up.” He always did well but looked on mathematics “as a chore. I was not a natural, so I had to work at it. If I had known how vitally essential a comfort level with numbers was going to be in my career, I might have had a little more pure motivation to excel in math.”

Instead of a specific interest in mathematics, a technology connection spurred Kevin’s career and got his company started. “I started to toy around with a simple Lotus spreadsheet idea I had six or seven years ago,” he explains, “about a way to help runners of vastly different experience and ability” plan their training. The resulting software program helped runners calculate “how frequently, how much, and how fast to train.” Kevin claims that he “had no idea it would turn into what our customized training software has become—a matrix of over sixty interrelated Microsoft Excel spreadsheets, each of which has hundreds of lines of code and formulae embedded in it. I created the job I do after university, so I had no preconceived notion of how math would be involved in my current career.”

Mathematics can save your life only if you know how to apply it. The first marathoner, Pheidippedes, did not know how to pace himself and died as a result. With help from Kevin Smith and some number-crunching through his customized-training-plan software, the famous messenger might have lived to deliver more news.



Teacher Notes

BEGIN WORK ON THE ACTIVITY SHEET ON PAGE 265 by having students measure their heart rates. In pairs, have them take each other’s pulses by having one person count and the other time the beats per minute. Record the beats per minute for each student. Graph the results, and look for trends. Have the students exert themselves by running in place, for one minute, and measure the rates again. Compare the results. This exercise prepares the students for question 1.

For the other questions, make sure to review conversion strategies. For example, when converting from kilometers per hour to meters per second, students must change units of both distance *and* time.

When calculating the amount of running done in one year for question 2, remember that every fifth day is a rest day.

See **figure 1** for one student's solution to the activity sheet. ▲

1. **a)** Max.: 226 – 14 = 212
Target: $0.8 \times 212 = 169.6 \approx 170$

b) Max.: 220 – 23 = 197
Target: $0.8 \times 197 = 157.6 \approx 158$

c) Max.: 226 – 35 = 191
Target: $0.8 \times 191 = 152.8 \approx 153$

d) Max.: 220 – 49 = 171
Target: $0.8 \times 171 = 136.8 \approx 137$

e) Max.: 226 – 13 = 213
Target: $0.8 \times 213 = 170.4 \approx 170$

2. Total distance ran: 5 km + 7.5 km + 10 km + 5 km = 27.5 km

$$\therefore \frac{27.5\text{ km}}{8\text{ km / hr.}} = 3.44\text{ hrs.}$$

Number of 5-day cycles within a year: 365 days ÷ 5 days = 73 5-day cycles.

∴ The total amount of time the runner spends on training in one year = 3.44 hrs. × 73 5-day cycles = 251.12 hrs.

3. Total distance: 7.5 km + 8.3 km + 6.8 km = 22.6 km
Total time: 37 min. + 41 min. + 32 min. = 110 min.

$$\frac{110\text{ min.}}{60\text{ min.}} = 1.83\text{ hrs.}$$

$$\begin{aligned}\therefore \text{Average pace}(\text{km / hr.}) &= \frac{22.6\text{ km}}{1.83\text{ hrs.}}\\ &= 12.35\text{ km / hr.}\end{aligned}$$

Average pace (m/sec): 12.35 km × 1 000 = 12 350 m
1 hour = 3 600 sec.
then 12 350 m/3 600 sec. = 3.43 m/s
∴ Average pace (m/sec.): 3.43 m/sec.

4. $$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{5\text{ km}}{12.35\text{ km / hr.}} = 0.405\text{ hrs.} = 24.30\text{ min.}$$
$$= 24\text{ min. } 18\text{ sec.}$$

5. Total distance: 7.5 km + 8.3 km + 6.8 km = 22.6 km

$$\therefore \text{Total distance (miles)} = \frac{22.6\text{ km}}{1.6\text{ km}} = 14.125\text{ miles}$$

Amount of 3-day cycles in a year: $\frac{365}{3} = 121.67$

∴ Total amount of miles ran in training in one year:
14.125 miles × 121.67 = 1718.59 miles

Fig. 1 One student's solution to the activity sheet

Running by the Numbers Activity Sheet

NAME ______________________________

1. An athlete's maximum exertion heart rate is calculated by subtracting his or her age from a fixed number: 220 for males and 226 for females. For example, a 24-year-old female runner has a maximum exertion heart rate of 202 beats per minute, or 226 – 24 = 202. The target performance heart rate is 80% of maximum. Calculate the target performance heart rate for the following runners:

a. 14-year-old female
b. 23-year-old male
c. 35-year-old female
d. 49-year-old male
e. yourself

2. A runner follows the training schedule in the table below.

DAY 1	DAY 2	DAY 3	DAY 4	DAY 5
5 km	7.5 km	10 km	5 km	Rest

If the runner maintains an average rate of 8 km/hr., how much time does he spend training in one year?

3. A runner trained for three days in a row. On day 1, she ran 7.5 km in 37 minutes. On day 2, she ran 8.3 km in 41 minutes. On day 3, she ran 6.8 km in 32 minutes. What was her average pace expressed in km/hr. and in m/sec.?

4. The record for the Math-at-Work 5-km Mini-Marathon is 38 minutes and 12 seconds. What is the likely finishing time for the runner in question 3?

5. One mile is approximately 1.6 km. How many miles does the runner in question 3 run in training in one year?

From the December 2000 issue of **Mathematics Teaching in the Middle School**