TAKE TIME FOR ACTION

# Investigating Students'

LYNDA R. WIEST

CAN STUDENTS ARTICULATE HOW THEY REASON about a spatial task? Can they integrate visual and analytic reasoning to make the best use of both types of thinking when working on a single task? Are "integrated" thinkers more successful than those who draw mainly on one side of their brain?

Spatial abilities are important in many school subjects and for many life tasks. *Principles and Standards for School Mathematics* recognizes the value of spatial reasoning: "Students' skills in visualizing and reasoning about spatial relationships are fundamental in geometry. . . . Experience with models of three-dimensional shapes and their two-dimensional 'nets' is useful in such visualization" (NCTM 2000, p. 237). Lord and Clausen-May (2002) point out that schools highly favor linguistic and numeric rather than spatial skills and thus favor left-brained thinkers. "The key for teachers is to engage both sides of the brain, for the more stimulated and connected the two brain hemispheres are, the greater the potential for learning" (Lord and Clausen-May 2002, p. 23).

## The Task: Sketching Nets

TO HELP ME GAIN INSIGHT INTO THESE QUESTIONS, I asked two groups of students to draw nets, two-dimensional "blueprints," for selected solids (three-dimensional shapes). The students included eight middle school girls (seven seventh graders and one eighth grader) from various northern Nevada schools and eight northern Nevada undergraduate (one) and graduate (seven) students in elementary and secondary education. You are invited to investigate *your* students' thinking on this task.

I began this spatial perception task by showing students a net for a cube (hexahedron) and a triangular pyramid (tetrahedron), in turn, and asking them to imagine what three-dimensional shape each would become when folded. After discussing students' thinking and reaching agreement, I told students that these two-dimensional plans for three-dimensional shapes are called *nets*. I then held up a cereal box and asked students to sketch its net. Students made freehand attempts and shared them with others.

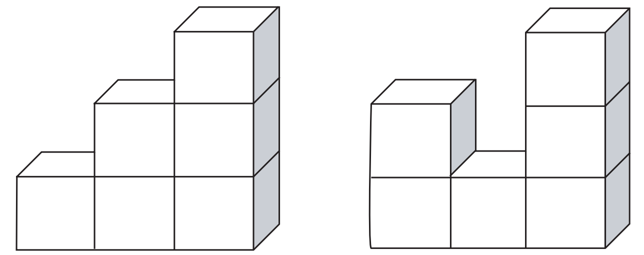

**Fig. 1 Cube structures used as models for creating nets**

I then asked students to sketch freehand the nets for two cube structures (interlocking cubes of one-two-three and two-one-three cubes high, as shown in **fig. 1**), as well as for a cylinder, a sphere, a cone, and a hemisphere. I also asked students to write a short explanation of the reasoning they used to create each net. The adults who completed this task were given one additional tip not given to the middle school students: to consider labeling parts of the cube structure net to ensure clarity about which sketched components represented which faces on the solid. During this activity, the solids were placed on tables in front of students so that the shapes could be examined visually and physically during this task.

## Student Performance

THE MIDDLE SCHOOL GIRLS AND THE ADULTS USED similar strategies to sketch nets for the given solids. However, they placed varying degrees of emphasis on these strategies. The most common strategy was to imagine unfolding the solid and, less frequently, folding the solid from a mentally constructed net. In some cases, this strategy was solely described as visualization. In others, the imagined unfolding/folding was visual-analytic by virtue of the systematical, detail-oriented approach that accompanied the mental picture. Examples of purely "visual" comments included these: "I thought of what it looked like before it was built up" (seventh grader) and " 'Banana peel.' I peel down from the top and visualize what each side, or peel, needs to

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*Prepared by* LYNDA WIEST, *wiest@unr.edu, a mathematics education professor in the Department of Educational Specialties, University of Nevada, Reno, NV 89557-0251*

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cover" (adult). These comments referred to the visual-analytic: "I laid down the sides [physically] and made imaginary cuts on each corner" (seventh grader) and, in reference to a cylinder, "I pictured unwrapping the rectangular piece after 'cutting off the lids'; picture a tuna can" (adult). The approaches deemed visual-analytic, which were more common among adults as well as more sophisticated, focused to a greater degree on specific aspects of the solids. These higher-level skills merged pertinent visualizing with more linguistically and/or numerically oriented reasoning, blending right- and left-brain thinking.

**Figures 2** and **3** show sample student nets where more visual versus more integrated visual-analytic approaches were used. In general, the adults' greater use of, and refinement of, their visual-analytic approaches indicates that more advanced experience with both shapes and systematic reasoning is an advantage for this type of task. Of course, the extra suggestion given to the adults regarding the use of a labeling system may have encouraged this approach to a greater degree. Nevertheless, the adults clearly showed advanced thinking. Note bottom right in **figure 3** that the middle school girl's visual-analytic approach, typical of that of her peers, was more limited in that she failed to include several faces of the structure.

Adults tended to use verbal reasoning and counting methods to ensure greater success in accounting for all faces in an appropriately connected relationship (see Tony's example in **fig. 3**). Janet's example in **figure 3** shows visual-analytic thinking in that she considered each key part of the figure separately in creating her drawing. Further, she considered how those parts should relate to each other by noting that the circular base may be too small. Similarly, a seventh grader detached the one removable base from the cylinder in front of her and placed it adjacent to the standing tube to help her better "see" the individual parts comprising it.

More adults than middle school students physically traced faces of their cube structures in creating their nets. At first, this seemed surprising. However, it seems conducive to a more systematic approach. The most problematic shape for all participants seemed to be the sphere. Some either did not attempt it or ran out of time to do so. One girl drew a circle for the sphere's net, and two adults said it probably could not be done.

In general, this task was difficult and challenging for

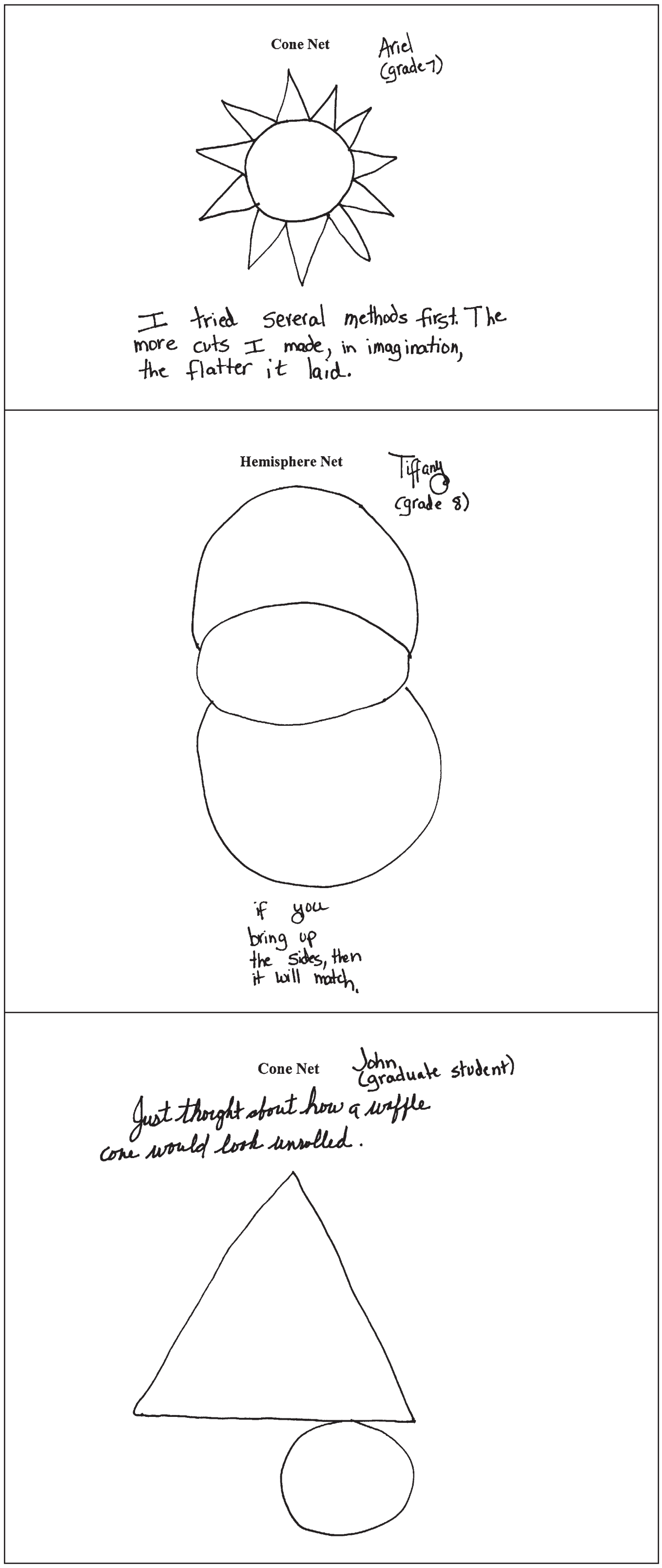


**Fig. 2 Three students' nets, with "visual" explanations**

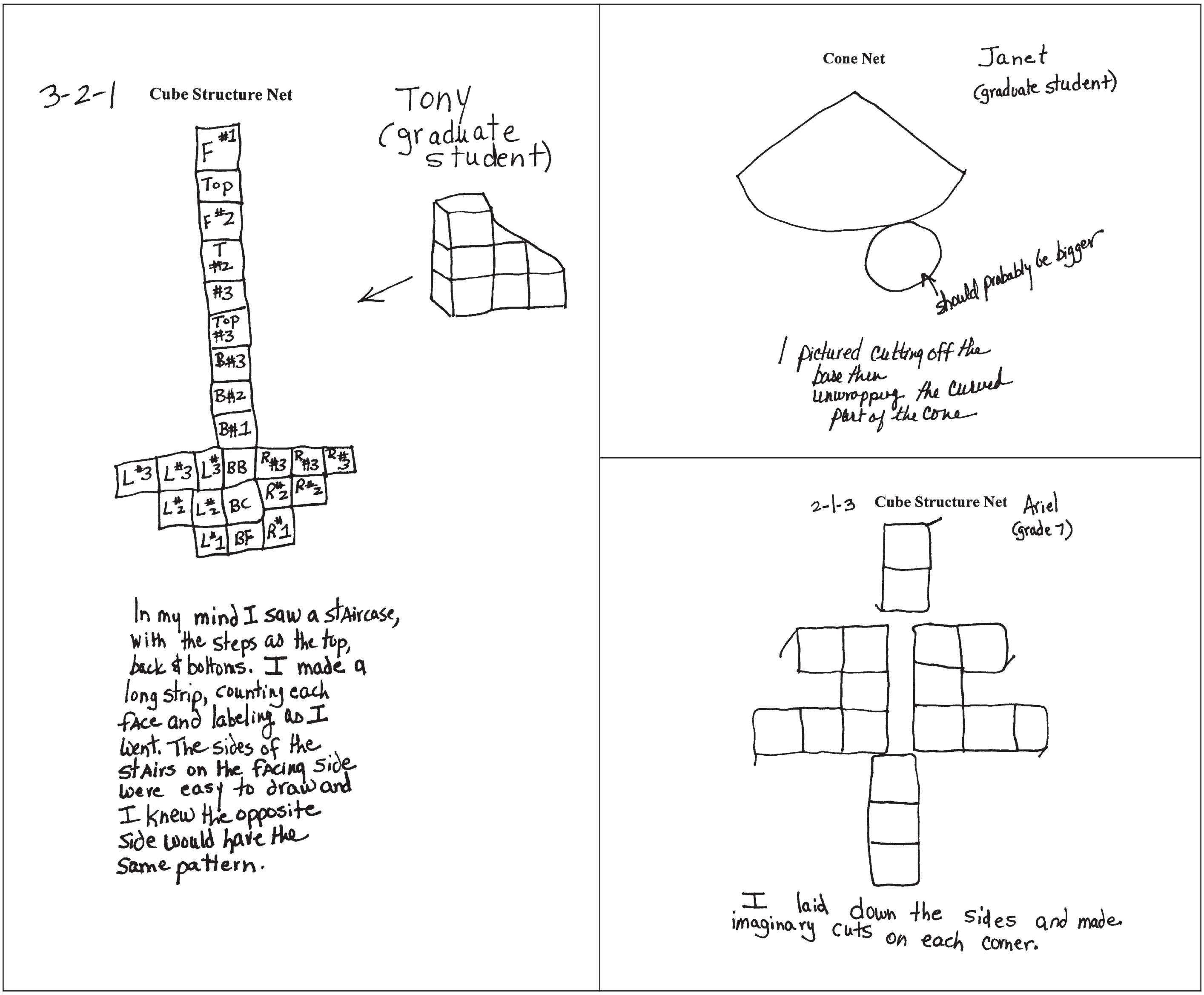


**Fig. 3 Three students' nets, with "visual-analytic" explanations**

participants of both age levels. On one hand, polygonal sides with countable squares made the polyhedra (cube structures) simpler in some ways to engage, especially in a visual-analytic manner. On the other hand, the fact that the cube structures were not as nicely packaged as, say, rectangular prisms raised the difficulty level significantly in terms of accounting for all faces and relating them to one another appropriately. Ariel, a seventh grader whose work appears in **figures 2** and **3,** was the most skilled of the middle school students on this task. Thus, her work is included to represent the two categories used here (visual, visual-analytic), but not the performance quality of her classmates. Also, students at both age levels who worked on this activity were not necessarily either solely visual or visual-analytic. They sometimes used different approaches for different solids. It should be noted that classifying the approaches was limited to students' written explanations, which may or may not accurately reflect their actual strategies.

## Implications for Teaching and Learning

INDIVIDUALS OF ALL AGES LACK MUCH-NEEDED SKILL in spatially oriented tasks, especially those solved analytically. The activity described in this article is one way to assess students' abilities in this arena. Teachers may build from students' knowledge by giving them more experience with tasks such as the net-sketching activity presented here. Spatial experience is an important foundation for such tasks. It is important for students to work with nets in two directions: predicting the solid formed by a given net and subsequently folding it to test it, where possible, as well as drawing nets for solids within view and sometimes from memory after having had sufficient experience with a variety of solids.

In addition to gaining spatial experience, students can explore and share promising techniques that aid performance, verbalizing them in detailed mathematical language and evaluating their methods. This can encourage greater use of analytic thinking (e.g., keeping records of sketched faces) with visual methods. Teachers can also share their own ideas and provide appropriate prompts when students might benefit by expanding their individual and collective strategy banks.

An important addition to this type of task is discussing its usefulness in mathematics and the real world. This activity relates to understanding shapes' properties and finding the surface area of solids, and it helps students understand the limitations of flat maps that represent spherical objects. Ariel, the seventh grader who performed better than the other middle school students on this task, said that she had used this skill in wood shop: "I was making a box and had to picture what it would look like all laid out [before building it]." Other students at both levels brainstormed the importance of this skill for architects, construction workers, people who design various types of containers, and so forth.

Working with solids and their nets in ways that encourage both right- and left-brain thinking is a worthwhile endeavor for middle school students and beyond. As one such exploration, you are invited and encouraged to take time to use this activity to understand and assess your students' thinking about nets.

## References


Lord, Thomas R., and Tandi Clausen-May. "Giving Spatial Perception Our Full Attention: Are We Reaching Spatial Learners?" *Science and Children* 39 (February 2002): 22–25.

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