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Tennis Balls, Lines, and Geometric Transformations

Activities that use linear data as a model will help students understand linear functions. In this activity, students collect data and make connections to concepts used in geometry and measurement. Rather than teaching introductory algebra and geometry, this activity is intended to serve as a culminating task to reinforce material already learned.

The Algebra Standard for grades 6–8 states that students should—

- explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope; and
- use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships. (NCTM 2000, p. 222)

As students work though **activity sheet 1**, they will explore the behavior of transformations and algebraic relationships.

To begin, students are placed into groups of two to four and given the materials shown in the sidebar. They should soak a tennis ball in water, roll it across a large sheet of easel-sized grid paper, and draw a line just above or below the wet path that the ball makes before the paper dries.

Before they begin answering questions about their line, students must place axes on the grid paper. Most students place the axes so that the line has a positive slope, although any placement is possible. Students should draw their axes so that the *x*- and the *y*-intercepts are readily apparent.

LINEAR EXPLORATION

While exploring the line, students should answer questions about slope and intercepts using a variety of methods. After looking at the line on the grid paper, they will use visual estimation skills, formulas, a ruler, and the graphing calculator. They will then compare the answers found from all these methods. The questions on the activity sheet are straightforward, leading students from answers that are obtained visually to answers obtained algebraically and with technology. Students are also asked to compare answers found from other methods as they work. They will look at slopes and intercepts from varying viewpoints while using a hands-on approach to explore concepts previously learned.

CONNECTING ALGEBRA AND GEOMETRY

Transformations are introduced in the middle school curricula in our two states of Georgia (Georgia Department of Education) and Texas (Texas

Edited by **Gwen Johnson**, gjohnson@coedu.usf.edu, Secondary Education, University of South Florida, and **James Dogbey**, jdogbey@mail.usf.edu, Secondary Education, University of South Florida. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on the NCTM's Content and Process Standards and Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing **mtms.msubmit.net**.

How to Get Started

MATERIALS NEEDED FOR EACH GROUP

Tennis ball, bowl of water, easel grid paper, marker, graphing calculator, ruler, colored pencils or crayons, protractor

PROCEDURE

a. Place the grid paper on a long flat surface.
b. Cover the tennis ball with water until it is soaked, take it out of the bowl, and roll the ball across the grid paper.
c. Use the marker to trace just above (or below) the line made by the tennis ball.

Education Agency). The NCTM *Standards* also state that students should—

- describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- examine the congruence, similarity, and line or rotational symmetry of objects using transformations. (NCTM 2000, p. 232)

The next part of the activity promotes these connections. Students use their knowledge of algebra to develop concepts of transformational geometry. Translations, reflections, and rotations of their original line will be done physically *and* algebraically. While producing transformations physically, students should record their new images with colored pencils or crayons to provide a permanent physical representation of each transformation.

The easiest transformation for most students to understand is a vertical translation; it is placed first on **activity sheet 2**. Students should draw a new line that moves their original y-intercept up 3 units. The new line must be parallel to the original line or it is not a translation, which requires that distance be preserved. Students will create a table of values for both their original line and the new line to emphasize that although the x-values remain fixed, the y-values change with the vertical translation. The table should help students move to the symbolic notation of

$$(x, y) \rightarrow (x, y + 3).$$

Since distance is preserved, the slope will be the same. This can be shown algebraically using the slope formula and the original coordinates (x_1, y_1) and (x_2, y_2) along with the new coordinates $(x_1, y_1 + 3)$ and $(x_2, y_2 + 3)$:

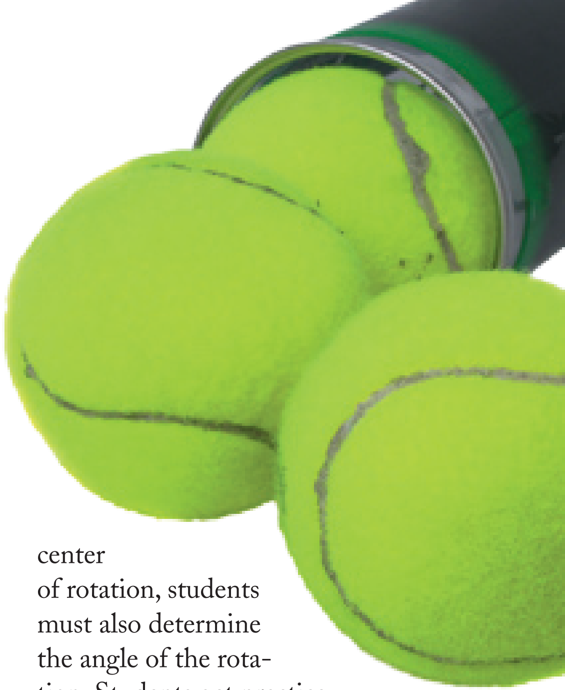
$$m_{original} = \frac{y_2 - y_1}{x_2 - x_1}$$

and

$$m_{new} = \frac{(y_2 + 3) - (y_1 + 3)}{x_2 - x_1} = m_{original}$$

It should be noted that a horizontal translation that creates a parallel line, although not performed here, will keep the y-values fixed while changing the x-values. This can be an additional extension if the teacher chooses.

Rotations are usually more difficult for students to understand. In this activity, students are told that the transformation they are using is a rotation and that it is their job to find the key elements needed. They must determine the center of rotation, which is where the two lines intersect, as well as the direction of the rotation. Different curricula specify the direction of a rotation as always clockwise or always counterclockwise. Students should use the standard that has been established in their classroom. In addition to the center of rotation, students must also determine the angle of the rotation. Students get practice using a protractor or angle ruler with this activity by measuring the angle of rotation.



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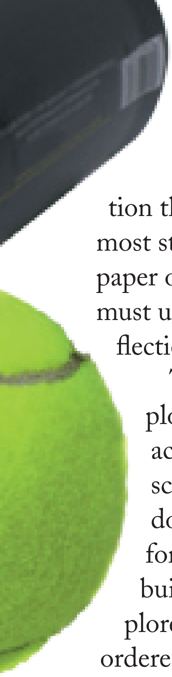
A reflection over the y-axis creates ordered pairs with the same y-coordinate, but the opposite x-coordinate. This provides the scenario for the third transformation that students will perform. The relationship between the ordered pairs will be discovered by students through building a table of values, similar to what was completed with the translation. The table will develop the relationship between the original and new ordered pairs. When students calculate slopes algebraically, they will find that the slopes have the same magnitude but opposite signs.

It should be noted that a reflection over the x-axis will keep the same x-coordinates, but the y-values will change signs. Again, the slopes will have the same magnitude but different signs. Teachers can use this portion as an extension if they choose. An additional extension would be to show that the line created from a y-axis reflection and the line created from an x-axis reflection will be parallel.

Students often encounter difficulty drawing the reflected line. Since they are not using tracing paper, it will be necessary for them to trace the original line by pressing down hard so that the imprint will go through the easel paper. This may require more instruc-



tion than usual by the teacher. Since most state tests do not allow tracing paper or reflection devices, students must understand the process of a reflection without using manipulatives.

The final transformation explored is a *dilation,* which is easily accomplished by changing the scale of the grid paper. Students do not need to draw a new line for this transformation, but will build a table of values and explore the relationships between the ordered pairs. In this case, both the x- and y-values are multiplied by 5, and the slope of the line does not change.

CONCLUSION

This multiple representation activity began as a simple way to explore lines. Through work with middle school students and teachers, it has grown into the unit outlined here. Teachers should be aware that each portion of the activity requires several class periods, either near the end of the term or incrementally during the year. The grid paper can be stored easily so that as new concepts are learned, students can go back and revisit their previous work and extend it to include the new mathematics. This activity provides a hands-on way to explore algebraic and geometric concepts and connect the two.

REFERENCES

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activity sheet 1

Name ________________________________

ROLLING THE TENNIS BALL

- Place the grid paper on a long flat surface.
- Cover the tennis ball with water until it is soaked, take it out of the bowl, and roll the ball across the grid paper.
- Use the marker to trace just above (or below) the line made by the tennis ball.

1. Draw an *x*- and *y*-axis on the grid paper so that the line intersects both axes.

2. List at least 6 ordered pairs from the graph.

x-value						
y-value						

3. *By looking only* at the graph on your grid paper, *locate* the *x*- and *y*-intercepts.

4. *By looking only* at the graph on your grid paper, *estimate* the slope of your line. Do not do any computations.

5. Use algebra to calculate the linear equation using 2 points that you think best represent the line. Show your work below.

6. Use your equation from question 5 to calculate the slope and intercepts. Show your work below.

7. How well do the slope and intercepts you located visually in questions 3 and 4 match the slope and intercepts you computed in question 6? Explain the consistency or inconsistency in your values.

8. Using your graphing calculator, take the 6 ordered pairs from question 2 and calculate the linear regression equation. Write the equation below.

Name ______________________________

9. Compare and contrast the equation of your line from question 5 with the equation of the line from the calculator in question 8.

10. Use the calculator to determine the *x*- and *y*-intercepts. How do they compare with your visual answers (from questions 3 and 4) and algebraic answers (from question 6)? Explain the consistency or inconsistency in the values.

11. Locate two points on your line on the grid paper. Using a ruler, measure the—

a. distance on the line between two points. __________

b. midpoint between two points. ____________

c. slope between two points (measure the rise and run). ______________

12. Use your two points and formulas from algebra to find the—

a. distance between two points. ____________

b. midpoint between two points. ____________

13. Compare and contrast your distance and midpoint answers from questions 11 and 12. Also compare and contrast your slope answers from questions 11 and 6.

14. Create a real-world scenario that could be modeled with your line. Explain what the slope and the *y*-intercept of your line means in terms of your scenario.

15. Use a pencil to draw a vertical line anywhere on the grid paper, and calculate the slope using two points on your vertical line. What is the equation of the line?

16. Use a pencil to draw a horizontal line anywhere on the grid paper, and calculate the slope using two points on your horizontal line. What is the equation of the line?

activity sheet 2

Name ______________________________

GEOMETRY CONNECTIONS

1. On the same large graph, use a colored pencil or crayon to draw a new line so that the new *y*-intercept is 3 units above your original *y*-intercept and is parallel to your original line.

2. Write the equation of your new line.

3. Using the same 6 *x*-values that you used previously, fill in the table below.

Original *x*-value						
Original *y*-value						
New *x*-value						
New *y*-value						

4. What geometric transformation did you use to get your new line?

5. What conclusions can you draw regarding the line created from a vertical translation of 3 units and—

a. ordered pairs?

b. slopes?

6. Using a different colored pencil, draw a new line that intersects your original line. The slope of this new line must have the opposite sign of the slope of your original line. You should be able to get this new line by performing a rotation. What is the center of your rotation, and in what direction did you move?

7. Measure the angle between your two lines with a protractor. What is the equation of your new line?

8. Fold your paper along the *y*-axis. Use a different colored pencil to draw the new line created by your paper fold. What is the equation of your new line?

Name ______________________________

9. Using the same 6 *x*-values that you used previously, fill in the table below.

Original *x*-value						
Original *y*-value						
New *x*-value						
New *y*-value						

10. What geometric transformation did you use to get your new line?

11. What conclusions can you draw regarding the line created from your transformation and—

a. ordered pairs?

b. slopes?

12. Instead of each square representing 1 inch, suppose each square represents 5 inches. What is your new equation?

13. Using the same 6 *x*-values that you used previously, fill in the table below.

Original *x*-value						
Original *y*-value						
New *x*-value						
New *y*-value						

14. What conclusions can you draw regarding the line created from your transformation and—

a. ordered pairs?

b. slopes?

15. What geometric transformation did you use?