Todd O. Moyer

Keeping All the Trains on the Tracks

Many eighth-grade and algebra 1 textbooks explore the formula *distance* = *rate* × *time*. Problems involving this formula can help students learn to write expressions, solve equations, and make judgments as to the reasonableness of the answers found.

While teaching the $d = rt$ formula over the years, I have found student interest to be lacking. I have also found that the textbook's approach to this formula involved procedural knowledge rather than conceptual understanding.

A typical textbook lesson begins by asking students to create a chart and fill in the blanks. Students often do not understand where the expressions or equations come from and may be unable to develop such equations in the future.

The first step in any instruction is getting the students' attention. The following method, which does just that, can be adapted to other problems, as well, to show students the logic behind setting up expressions, solving equations, and working with the solutions.

This activity uses tables of numbers, symbols, and a dynamic representation of the problem on the graphing calculator. A table of values, algebra, and a visual display of trains help students gain an understanding of the algebra involved and its purpose. Generating the values on the table leads directly to the algebraic

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Table 1 Students can construct a table to compare the two train trips.

Hour	Train 1	Train 2
0	0	0
1	70	0
2	140	0
3	210	85
4	280	170
5	350	255
6	420	340
7	490	425
8	560	510
9	630	595
10	700	680
11	770	765

representation for each train, which is then used with the graphing calculator to simulate the trains' movement.

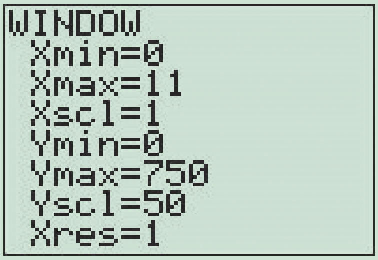
To begin, **activity sheet 1** presents the following scenario:

> Liza Dash lives in Philadelphia and has to get to Atlanta as fast as possible. Since Liza has a fear of flying, she plans to travel by train. One train leaves at 12:00 and travels an average of 70 mph. A second train leaves 2 hours later but averages 85 mph. Assuming that the distance between the two cities is about 750 miles, which train should she take to arrive in Atlanta as early as possible?

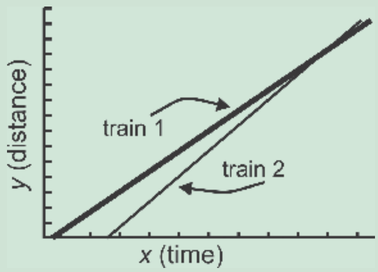
If your students have only a basic understanding of algebra, we recommend that they create a table showing the total distance traveled by each train per hour (see **table 1**). Ask them to look for patterns and generate equations for each train.

Students should be able to see that by adding 70 and 85 onto the previous distance, respectively, that each constant represents the slope of the

Fig. 1 Using the window settings in (a), students get a good sense of the relationship between the two trains from the graph in (b).



(a)



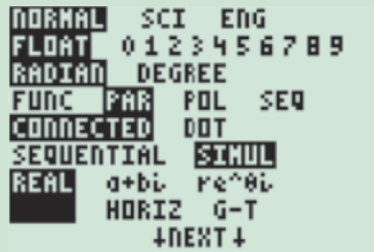
(b)

linear equation. Now introduce the graphing calculator into this problem. One could try using Y1 = $70x$ and Y2 = $85(x - 2)$. Using the numbers from **table 1**, set the window as shown in **figure 1a**.

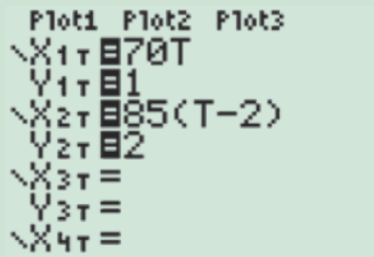
MANIPULATING CALCULATOR MODE FOR UNDERSTANDING

When students press the graph button, they watch the calculator graph the two lines on the same set of axis, in Sequential mode, which is the default of most calculators. The animation shows Y1, which is train 1, graphed first, followed by the graph of Y2. Changing the mode on the calculator from Sequential to Simultaneous (see **fig. 2a**) will change the animation that graphs the two lines. **Activity sheet 2** uses Simultaneous mode, where both Y1 and Y2 will be graphed at the same time, based on the value of *x*. When students press the graph button, they get a sense that the trains appear to start from two different locations along the bottom of the screen and are

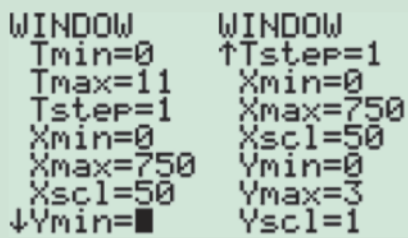
Fig. 2 These screen shots demonstrate the settings on a TI-83/84 calculator for Simultaneous as well as Parametric equations.



(a)
Pressing the MODE button



(b)
Pressing the Y= key, after setting the mode to Parametric



(c)
The window settings for the parametric equations

headed toward the upper-right corner (see **fig. 1b**). (Train 1 is represented by the thicker graph.)

Students might be confused as to the problem and its representation. Although both trains leave from the station in Philadelphia, that does not appear to be the case. The speed of the trains is truly reflected in the slopes of the two lines, but it does not show that train 2 is actually traveling faster than train 1. Changing the mode from Function to Parametric will resolve these two concerns.

Parametric mode (see **fig. 2a**) allows both trains to travel across the screen by using an independent variable, t, to represent time. The variable x represents the distance traveled by each train, and y denotes the track number. Train 1 can be represented as $70t$, where t represents the number of hours passed since 12:00. Train 2 can be represented as $85(t - 2)$, since it left two hours after train 1. (See **fig. 2b–2c** for help with calculator settings.)

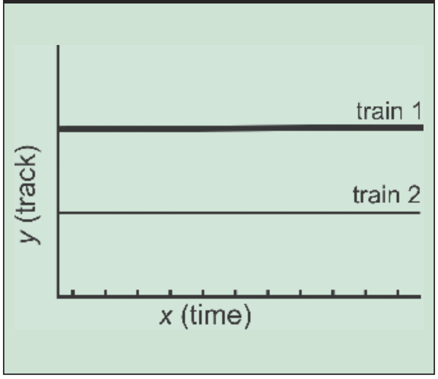
TRAINS MOVING ACROSS THE SCREEN

We have now reached a discussion point. From the table, it becomes obvious that each train arrives at some point before the eleventh hour. Is it possible that train 2 travels 750 miles before train 1 does? If the average rates for each train are given, is that the speed at which they travel? A student can also observe that the difference between the two trains' distance is approaching zero. At some point soon, train 2 will catch train 1. Does that matter? We are interested in which train reaches Atlanta (750 miles) first, not where the second train to depart catches up.

The window settings for the parametric equations are seen in **figure 2c**. Ask the class to determine these window settings with the following questions:

- Since T represents the time in hours passed, what should be its minimum value? Its maximum value? (*Answer: 0 to 11. Also, the Tstep will be changed to start at T = 1.*)
- What should be the minimum X and Y values? (*Answer: The values of X should range from 0 to 750 with a scale of 50.*)
- What should the Y-range be to see the two trains, set as 1 and 2? (*Answer: Y should be 0 to 3 with a scale of 1 so that the values of 1 and 2 can be seen.*)

Fig. 3 For students, watching the animated construction of the parametric graph as the two lines/trains move across the screen, rather than observing the resulting graph, is invaluable.



Have students determine which train will reach the right side of the calculator first. Train 1 will be the lower train; train 2, the upper train. The animation on the screen, rather than the graph produced, is valuable to students when they press the Graph key. With Tstep = 1, both trains will travel across the screen very quickly, possibly too fast to determine a "winner." Lower the Tstep until you are satisfied that students can visually tell which train they think will arrive first (this will likely occur at Tstep = .1).

As a last check, students should work through the algebra to determine without a doubt which train arrives first. Taking student suggestions may yield setting both X_{1T} and X_{2T} equal to 750 to find the time that each train arrives. Some students, noting the patterns on the original table, may wish to solve $X_{1T} = X_{2T}$ (i.e., $70T = 85(T - 2)$) to find when the two trains have traveled the same distance, reasoning that train 1 would reach the 750-mile mark first if the solution yields a distance greater than 750 miles.

As an extension, a slightly different problem can be posed:

> What if, by accident, the second train was leaving Atlanta for Philadelphia on the same track? How much time is there to prevent a collision? At what point on the track would the trains collide?

Change Y_{2T} to be equal to 1 in the Y= screen so that both trains are on the same track. What adjustment must be made to Y_{1T}? By watching the calculator, can students determine the time that the two trains will collide?

Although this activity was not conducted as a strict experimental study, some conclusions can be drawn, especially in light of these reactions:

1. The interest level in the lesson was one of the highest of that school year thus far.
2. In the two classes that used the calculator method, the teachers also noticed a higher level of interest and participation.
3. Students from the traditional classes consistently questioned why they had not used the calculator method first.
4. The teachers commented that the traditional classes could be described as "business as usual."

The two teachers noted the difference in the students' motivation. They found that students enjoyed the calculator approach and that opportunities for mathematical discourse presented themselves much more readily when the calculators were used.

BIBLIOGRAPHY

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activity sheet 1

Name ______________________________

GRAPHING TRAIN TRAVEL

Liza Dash lives in Philadelphia and has to get to Atlanta as fast as possible. Since Liza has a fear of flying, she plans to travel by train. One train leaves at 12:00 noon and travels an average of 70 mph. A second train leaves 2 hours later but averages 85 mph. Assuming that the distance between the two cities is about 750 miles, which train should she take to arrive in Atlanta as early as possible?

1. If you average 70 mph, how many hours will it take to travel at least 750 miles?

2. Fill in the table with the distance traveled by each train at the end of each hour.

Time	Hours	Train 1	Train 2
12:00 a.m.	0		
1:00 p.m.	1		
2:00 p.m.	2		
3:00 p.m.	3		
4:00 p.m.	4		
5:00 p.m.	5		
6:00 p.m.	6		
7:00 p.m.	7		
8:00 p.m.	8		
9:00 p.m.	9		
10:00 p.m.	10		
11:00 p.m.	11		

3. What is the pattern in the distance traveled by train 1? Write an expression that represents y, the distance traveled, in terms of x, the number of hours since the train left the station.

4. What is the pattern in the distance traveled by train 2? Write an expression that represents y, the distance traveled, in terms of x, the number of hours since the train left the station.

5. Using your graphing calculator and Y1 and Y2 to represent each train, graph the movement of each train from Philadelphia to Atlanta.

6. Sketch the graph and describe what it represents.

7. Describe the window settings you have chosen, and explain how they relate to the problem.

activity sheet 2

Name ______________________________

ANIMATING TRAINS

To see another representation of the movement of the trains, follow these steps, and relate what you see back to the original question.

1. Press the MODE button to get the screen below:

- Change from “FUNC” to “PAR” for parametric mode
- Change “SEQUENTIAL” to “SIMUL” for simultaneous graphing.

2. Press the Y= button to enter your equations (the calculator may use T instead of *x*):

- Enter your expression for train 1 into X_{1T}.
- Enter your expression for train 2 into X_{2T}.
- Let $Y_{1T} = 1$, and $Y_{2T} = 2$. (These are the tracks for each train.)

3. Press the WINDOW button:

- Since T represents the hours passed, its minimum value (Tmin) will be ______________.
- The maximum value (Tmax) will be ________________.
- Set Tstep = 1.

4. X measures the distance traveled:

- Its minimum value (Xmin) will be _________________.
- Its maximum value (Xmax) will be________________ .
- Choose Xscl = 50.

5. Y represents the track number:

- Set Ymin to 0, and Ymax to 3, so that the two tracks, 1 and 2, are easily visible.
- Choose Yscl = 1.

6. Press the GRAPH button. Which train appears to get to the far right of the screen first, train 1 or train 2?___________

7. Change the value of the Tstep so that the trains travel more slowly across the screen. Which train appears to be traveling 750 miles the fastest?________________

8. Confirm your guess by solving an appropriate equation or equations. Which train will arrive in Atlanta first?__________