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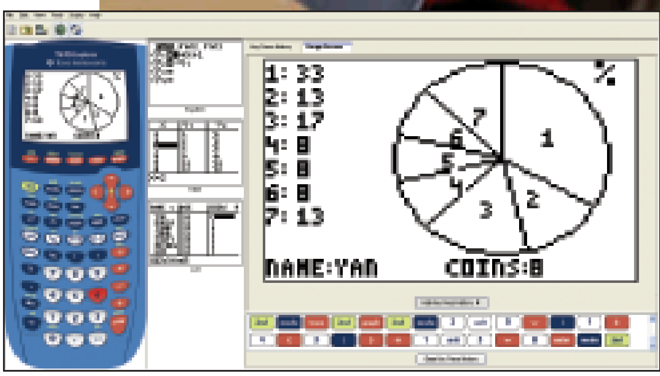

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Rachel Wing DiMatteo

A Model Approach to Problem Solving

Students who pursue careers in business, medicine, engineering, or architecture need robust mathematical backgrounds, with a particularly strong emphasis on problem solving. For example, when civil engineers are asked to determine the best placement for a cell phone tower, some of their thinking is related to school-based mathematical procedures and concepts. However, for the most part, they problem solve, using computer programs to construct mathematical models that relate the three-dimensional topography of the earth and the availability and cost of property for tower placement.

To provide a clear picture of where education has been and where it needs to go in terms of problem solving, three questions will be addressed:

1. What does it mean to be a good problem solver?
2. What does a good problem-solving activity look like?
3. What should teachers keep in mind as they bring problem-solving activities to the classroom?

WHAT *IS* A GOOD PROBLEM SOLVER?

At one time, the activity of mathematical problem solving was seen as a means to an end; context was simply intended to add motivation as students engaged in carefully crafted problems that afforded opportunities to practice new skills and techniques. Students were taught formulas and algorithms in decontextualized situations.

They practiced the newly mastered ideas while solving word problems using the learned procedures. Under this conceptualization of problem solving, being a good problem solver meant fluently and efficiently finding the answer to canned word problems.

The next theme in mathematical problem solving came in response to NCTM's *Agenda for Action*. During the 1980s, problem solving was seen as an important activity in and of itself, and emphasis was placed on students learning specific skills and strategies that would facilitate their problem solving in messy real-life situations. Pólya-type heuristics (such as work backward, identify the given, consider a similar problem, draw pictures, and guess and check) were championed as the keys to good problem solving (Pólya 1957). However, research eventually revealed that simply having more problem-solving heuristics did not make students better problem solvers (Lester and Kehle 2003).

Perhaps Pólya's heuristics are more valuable when used to guide students' reflections while looking back on how

Edited by **Debra Johanning**, debra.johanning@utoledo.edu. This department provides information from research that teachers can use in their classrooms. It is intended to empower teachers with research-based information in their discussions with peers, administrators, and parents. Send submissions by accessing **mtms.msubmit.net**.

they solved a complex problem (Lesh and Zawojewski 2007), since teaching students what to do did not necessarily make them experts when they were grappling with a difficult question.

The current perspective on problem solving recognizes that it is more about interpreting, describing, explaining, and modeling situations mathematically than about learning a set of strategies. For students to improve their problem-solving abilities, they need to gain experience constructing their own understandings of messy meaningful mathematical situations (Stein, Boaler, and Silver 2003).

One current conceptualization of problem solving that has drawn attention and interest is the model-building approach (Lesh and Zawojewski 2007). Students learn how to describe mathematical situations in the form of explicit conceptual systems called *models*. Problems that require students to construct models place a lot of emphasis on the process of interpreting a problem. Although students can draw on previously taught procedures as they work on these types of activities, simulations of real-life problems are not immediately understood because no prescribed strategy, procedure, or rule maps onto the situation. The process of interpreting usually involves several iterative cycles of creating, testing, and revising mathematical representations.

WHAT DOES GOOD PROBLEM SOLVING LOOK LIKE?

Since the mid 1990s, a number of curricula embodying the ideas of teaching math through problem solving have been developed. In these curricula, students engage with complex tasks, often in real-world contexts, and the tasks promote deep conceptual understanding. Research has shown that teachers with problem-oriented instructional practices produce students who have a greater conceptual understanding than those

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who follow more traditional curricula, and that these gains have not come at the expense of students' basic math skills (Huntley et al. 2000; Stein and Lane 1996).

One type of problem-solving (model-eliciting) task that has been used and studied extensively with middle-grades students is illustrated with the Big Foot activity (Lesh and Harel 2003). This task asks students to develop a model that can be shared with other students and reused for situations with different data.

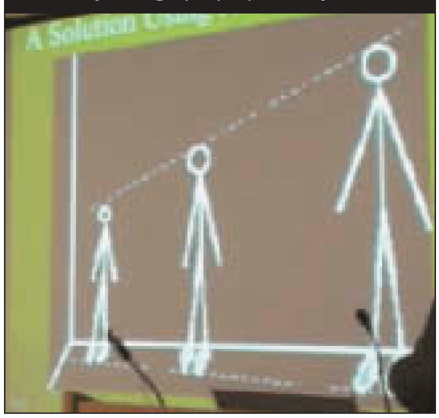
For the Big Foot activity, students develop a procedure (i.e., a model) that allows police detectives to predict a person's height from measuring a shoe imprint in soil. The procedure needs to be available for use by any police officer at various crime scenes. Students are given footprint outlines and told that they can measure the feet, the strides between feet, and anything else they deem important.

Most students and groups cannot immediately produce a solution, so they cycle through a process of proposing, testing, and revising models. At first, many students think additively about this problem, pursuing the idea that the difference in the lengths of the footprints corresponds exactly to the difference in the heights of the individuals (i.e., if two feet differ by two inches in length, the heights of the two individuals will differ by two inches as well). Students typically explore this idea, trying to verify it with existing data or their own footprints and heights. Before long, the additive perspective is abandoned.

Students then experiment with ways to strengthen their idea, which can sometimes mean brainstorming a completely new model. For one group of students, in particular, the next move was to ask their classmates to line up against the wall and order themselves from shortest to tallest. The group constructed a concrete graph of sorts and noticed that shoe size and height had a proportional relationship. The graph they created is depicted in **figure 1**.

When students were given a large footprint, they determined the owner's height by placing the footprint in its appropriate place along the foot portion of the graph and drawing a vertical line to the height portion of the graph. The large footprint was Shaquille O'Neal's. One student commented that Shaq's feet were huge; another student said that they were not too large for Shaq, a rather insightful comment alluding to a proportional relationship. After experimenting for a while, this student noticed that Shaq's height was six times his shoe length. Then he tested this model for his own shoe length

Fig. 1 Students note that shoe size and height have a proportional relationship, as shown by this graph prepared by students.



and height, and it held; he, too, was six times his shoe length. This example demonstrates students' modeling cycles. Their first way of describing something is rarely their last.

Such model-eliciting activities allow teachers and students to see how concepts and strategies develop and allow classmates to challenge a group's model. For example, a teacher or classmate might find fault with the proportional-reasoning model that the group found. The teacher might suggest that she has had the same size foot since she was 12 years old but her height has increased 6 inches since then. With this new information, the group could return to the cycle of revising and testing their model.

HOW DO YOU BRING PROBLEM SOLVING TO THE CLASSROOM?

Problem-solving activities can give students experience dealing with complicated, real-life situations. The following pointers are for teachers who wish to implement these activities in their classrooms:

1. *Maintain the level of difficulty.* Researchers found that it can be challenging for teachers to maintain task complexity and that numerous support factors prove helpful (Stein, Boaler, and Silver 2003). Teachers should—

   (a) select activities that build on students' previous knowledge,
   (b) scaffold students' thinking, and
   (c) persist in asking students to engage in sense making throughout the problem-solving process.

For example, to keep the cognitive demand high in the Big Foot exercise, students can be asked to explain how their models address the problem. As students listen to classmates sharing their models, they can be asked to compare and contrast their own models.

The process of interpreting usually involves several iterative cycles of creating, testing, and revising mathematical representations.

2. *Ask students to generate solutions that can be shared and generalized.* Many problem-solving activities describe a particular situation. If students are required to build solutions that must be shared and reused, it encourages them to generalize.

3. *Encourage students to use a variety of media to express their solutions.* Middle-grades students often include spoken language, written symbols, graphs and graphics, and experience-based metaphors. In school and beyond, students will need to use and understand problem-solving solutions expressed in a variety of representational media. Giving them ample experience in school will serve them well to develop their fluency and their ability to use, interpret, and translate among a variety of media.

4. *Require students to problem solve with their peers.* Classmates can give one another feedback and work together to test and revise their thinking until all concerns are satisfied. Working with peers also pushes students to express their thinking in a form that others can understand, fostering the representational fluency referred to previously. For example, when groups present their models to one another, as done with the Big Foot activity, they should be required to make sense of different ways of thinking or reasoning about the problem situation. In so doing, they will gain representation fluency as they listen, explain, and converse about their understanding of their classmates' presentations. The social context of within-group and across-group discussion provides an opportunity to use a variety of media and to enhance students' representational fluency.

5. *Assess student progress on problem-solving activities differently than simply checking an answer in a story problem.* Take advantage of the fact that the types of problem-solving activities promoted in recent reform curricula make students' progression of thinking much more visible than with traditional story problems. The solutions that students generate often reveal the relationships, quantities, and patterns that are familiar to students. Their solutions usually evolve over iterative cycles. Therefore, teachers should consider how student-generated assumptions and constraints evolve and change, rather than merely checking if an answer is correct.

SUMMARY

Previous perspectives on problem solving relied heavily on prescribed procedures and strategies. However, research has repeatedly indicated that teaching students mathematical procedures and problem-solving strategies has not made them better problem solvers (Lester and Kehle 2003). The current conceptualization of problem solving places the emphasis on understanding and representing realistic situations mathematically rather on executing learned techniques. Presenting messy real-life questions and encouraging students to find a way to bring mathematical order and structure to the situations will lead to deeper conceptual understanding. Activities that require models are one way to bring this type of problem solving to students.

REFERENCES

Huntley, Mary Ann, Chris L. Rasmussen, Roberto S. Villarubi, Jaruwan Sangtong, and James T. Fey. "Effects of Standards-Based Mathematics Education: A Study of the Core-Plus Mathematics Project Algebra and Functions Strand." *Journal for Research in Mathematics Education* 31 (2000): 328–61.

Lesh, Richard, and Guershon Harel. "Problem Solving, Modeling, and Local Conceptual Development." *Mathematical Thinking and Learning* 5 (2003): 157–90.

Lesh, Richard, and Judi Zawojewski. "Problem Solving and Modeling." In *Second Handbook of Research on Mathematics Teaching and Learning,* edited by Frank K. Lester Jr., pp. 763–804. Reston, VA: National Council of Teachers of Mathematics, 2007.

Lester, F. K., and P. E. Kehle. "From Problem Solving to Modeling: The Evolution of Thinking about Research on Complex Mathematical Activity." In *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning and Teaching,* edited by Richard Lesh and Helen Doerr, pp. 501–18. Mahwah, NJ: Lawrence Erlbaum and Associates, 2003.

Pólya, George. *How to Solve It*. 2nd ed. Princeton, NJ: Princeton University Press, 1957.

Stein, Mary Kay, and Suzanne Lane. "Instructional Tasks and the Development of Student Capacity to Think and Reason: An Analysis of the Relationship between Teaching and Learning in a Reform Mathematics Project." *Educational Research and Evaluation: An International Journal on Theory and Practice* 2 (1996): 50–80.

Stein, Mary Kay, Jo Boaler, and Edward A. Silver. "Teaching Mathematics through Problem Solving: Research Perspectives." In *Teaching Mathematics through Problem Solving: Grades 6–12,* edited by Harold Schoen and Randall Charles, pp. 245–56. Reston, VA: National Council of Teachers of Mathematics, 2003.

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