

# Understanding the Value of Pi

At times, we must ask our students to accept certain content without giving them a full explanation, because understanding the underlying mathematics would require knowledge beyond their current level. One good example of such a concept is the irrational number pi.

In middle school, students are taught that pi is the ratio of the circumference of a circle to its diameter. They learn to use this number to calculate measurements involving a circle, such as area and circumference. However, it is likely that students do not have a full understanding of pi's value, because they have not seen how its infinite digits are derived. The problem with demonstrating the derivation of pi is that most methods require an understanding of mathematics that is beyond the middle school level.

Students are told that pi is approximately equal to 3.14. Since it is irrational, one cannot write the number in its entirety, which is why it is represented with the Greek letter  $\pi$ . A popular poster in many math classrooms shows the first 100 digits of pi. How can we show students at this level where these digits originated?

A variety of measurement activities lend support to the fact that the ratio of the circumference to the diameter

of a circle is approximately equal to 3.14. However, any measurement itself is an estimate, and the precise and infinite digits that comprise pi cannot be generated by measuring. Although measurement activities may help explain the value that we tell students is equal to pi, these tasks fall dismally short of demonstrating where the precise digits come from.

Our quest to discover a method for generating pi that would be understandable to middle school students led us to Archimedes. He was born in 287 BC and studied at the University of Alexandria under the successors of Euclid and perhaps Euclid himself. He took the concept of "equal to" and turned it into the concept of "as closely as desired." He used this concept to develop a method of calculating pi to any desired degree of accuracy (Beckmann 1971). In other words, before the invention of calculus, Archimedes could calculate pi to as many place values as he wanted. The activity for finding the circumference of a circle described in this article is based on his method.

## THE METHOD OF ARCHIMEDES

The Archimedean method of finding pi involves estimating the circumference of a circle by finding the perimeter of a regular polygon inscribed in

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Edited by **Gwen Johnson**, [gjohnson@co.edu.usf.edu](mailto:gjohnson@co.edu.usf.edu), University of South Florida, and **James Dogbey**, Clemson University, Clemson, South Carolina. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on NCTM's Content and Process Standards and Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing [mtms.msubmit.net](http://mtms.msubmit.net).

# Graphing Pi on the TI-73

Follow the steps below to program the calculation of pi on the TI-73 calculator. The program itself is in **figure 2**.

1. Press PRGM.
2. Highlight NEW.
3. Select Create New.
4. Press ENTER.
5. To enter a name for the program, press 2nd and MATH to access the TEXT function.
6. After typing a name (we call it PERIMETE), press ENTER.

A colon will appear. You are ready to enter the first program command. These commands must be entered by using the CATALOG.

7. Press 2nd.
8. Press PRGM to access the CATALOG.

The commands are alphabetical as you scroll down. You can quickly access symbols such as quotation marks and  $\pi$  by scrolling up. Press the first letter

of a command to jump to the first command starting with that letter.

9. Scroll down to Lbl.
10. Press ENTER.
11. Press 1.
12. Press ENTER.

Now you are ready for the second line.

13. Use CATALOG to enter the program commands, trig functions, and quotation marks.
14. Use the TEXT function to enter the text in between the quotes and variables such as N and D. The calculator keys can be used to enter parentheses, commas, and the arrow (STO).
15. After entering the last line, press 2nd and MODE to QUIT.
16. Return to the home screen.

The program name should appear when you press PRGM. The program can be sent to additional calculators using the Link application. To enter

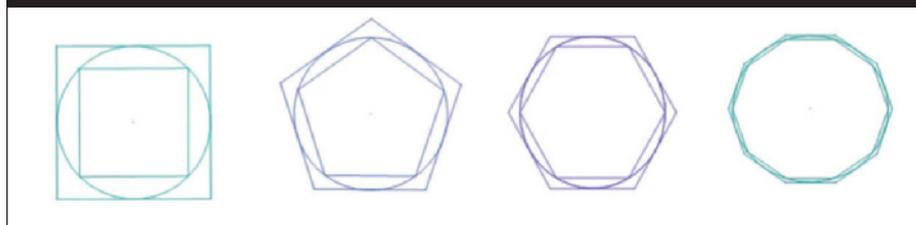
the program on the TI-83 or TI-84, a similar procedure can be followed. However there are some differences. For example, to enter text, you use the ALPHA function.

The label (Lbl) command works with Goto to create a loop. The display (Disp) command shows what will actually be displayed on the calculator screen. The user will first enter the number of sides, N, for the polygon desired, and next will enter the diameter, D. The formula is used to determine the perimeter of the inscribed polygon; this perimeter is displayed on the screen. The formula shown in the program uses radians rather than degrees, so it is important to set the calculator mode to radians, or simply change  $\pi$  to 180 and use the degree mode. Next, the formula to calculate the circumscribed perimeter is used, and this perimeter is displayed. The program automatically loops back to the beginning so that the number of sides for another polygon can be entered.

the circle and the perimeter of a regular polygon of the same number of sides circumscribed about the circle. The circumference of the circle will fall between the perimeters of the polygons. If the diameter of the circle is one unit, then the circumference is pi units.

By increasing the number of sides in the regular polygon, the interval between the two perimeters narrows, and the perimeters approach the circumference of the circle from both sides (see **fig. 1**). The circumference, or  $\pi$ , can be calculated to any desired place value. After Archimedes laid the groundwork, the number of decimal places to which pi could be calculated was entirely due to computational ability, perseverance, and computer programming (Beckmann 1971).

**Fig. 1** These shapes illustrate how pi can be measured.



## DEVELOPING A GRAPHING CALCULATOR PROGRAM

By generalizing the procedure for finding the perimeter of a polygon with  $n$  sides, a program can be written that enables students to determine these perimeters using a graphing calculator. Using a calculator eliminates the drudgery of tedious calculations, perhaps using mathematics that they are not familiar with, and enables

students to focus on the goal of finding increasingly precise values for pi. The program, shown in **figure 2**, was written for the TI-73 graphing calculator but also works with TI-83 and TI-84 calculators. (See the **sidebar** for detailed instructions on entering the program into a TI-73.) This information should be entered into the graphing calculator before presenting this activity to students.

**Fig. 2** These graphing calculator commands should be completed before students cut into this pi activity.

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: Lbl 1
: Disp "ENTER NUMBER"
: Disp "OF SIDES"
: Input N
: Disp "ENTER DIAMETER"
: Input D
: ND(sin (π/N)) → P
: Disp "THE INSCRIBED"
: Disp "PERIMETER IS ", P
: ND(tan (π/N)) → P
: Disp "THE CIRCUMSCRIBED"
: Disp "PERIMETER IS", P
: Goto 1

```

*Note:* For calculators using “degrees” mode rather than “radians,”  $\pi$  should be replaced by 180 in this program.

We used trigonometry in the formulas for our program although Archimedes computed the perimeters using other methods. For more information on how Archimedes calculated the perimeters, see Beckmann (1971). (See **fig. 3** for a derivation of the formulas.)

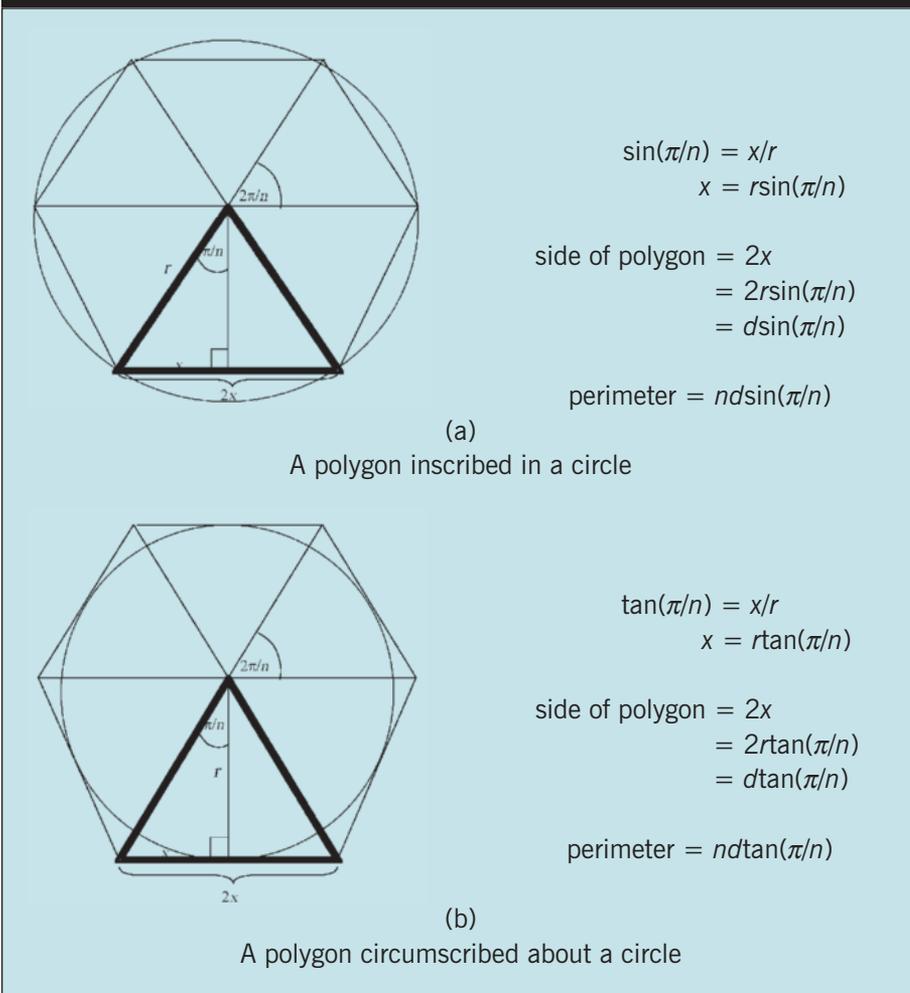
## THE CLASSROOM ACTIVITY

**Activity sheet 1**, “Finding the Circumference of a Circle,” is a self-guided student investigation that culminates with the formula for the circumference of a circle. To successfully complete this activity on their own, students must have prior knowledge and experience in applying the Pythagorean theorem. We recommend that students work in groups or pairs. Each group of students will need—

1. **activity sheet 1**;
2. a graphing calculator (with the program shown in **fig. 2**); and
3. red, blue, and green markers.

Students trace the circumference and perimeters of the figure using

**Fig. 3** The formulas for the perimeters of regular polygons are derived in this way.



specified colors to ensure that they understand the vocabulary introduced in the opening paragraphs. With a quick visual check, the teacher can determine if *perimeter*, *circumference*, *inscribed*, and *circumscribed* are understood. Students also define a regular polygon.

Students then find the perimeters of the circumscribed and inscribed squares if the diameter of the circle is 1 unit. The circumscribed square is very simple, and students should have no problem determining this perimeter to be 4 units. Finding the perimeter of the inscribed square requires applying the Pythagorean theorem. This perimeter is 2.8 units when rounded to the nearest tenth. Students should conclude that the circle has a circumference between 2.8 units and 4 units.

**Activity sheet 2**, “Graphing and Charting a Circle,” introduces students to the calculator program to find perimeters of inscribed and circumscribed polygons. The example of a decagon is shown to illustrate that the interval containing the circumference gets much smaller as the number of sides of the polygon increases. Using a diameter of 1 unit, students determine this interval to be between 3.09017 units and 3.24920 units when rounded to the nearest hundred thousandth. Rounded to the nearest whole number would make the circumference of the circle 3 units. Students use the calculator program to determine the inscribed and circumscribed perimeters of polygons with increasing numbers of sides, always using a diameter of

1 unit. The numbers of sides were carefully chosen to illustrate the value of the circumference with increasing precision.

With a decagon, the circumference equals 3 units when rounded to the nearest whole number. With a 25-gon the value of the circumference is precise to the tenths place, with a value of 3.1 units. If we increase the number of sides to 100, the circumference is 3.14 units rounded to the hundredths place. A 200-gon gives a value of 3.141 and a 2500-gon gives a value of 3.14159.

This activity concludes by introducing the symbol,  $\pi$ , to represent the value of the circumference of a circle with a diameter of 1 unit. This decimal number will never terminate, and we can get closer and closer to the circle by continuing to increase the number of sides in the polygons. However, the polygon will never actually become a circle.

### CONCLUSION

The Archimedean method can be used to calculate the circumference of a circle to as many place values as desired. However, calculators have a finite number of display positions. This limitation restricts the number of place values that we can determine for pi with this technology. Nevertheless, this activity provides a method of generating the digits of pi that is understandable to students at the middle school level.

We have used this activity successfully with preservice elementary teachers. Although they are very familiar with using pi, they are surprised to see the number appear as they find the perimeters of the polygons and come increasingly closer to the circumference of the circle. *Principles and Standards for School Mathematics* states that “Learning mathematics with understanding is essential” (NCTM 2000, p. 20)

and that “Technology enhances mathematics learning” (NCTM 2000, p. 25). As educators, we must find ways to help students learn with understanding even when such understanding appears to be beyond their reach. Technology can often bridge that gap.

### SOLUTIONS

#### Activity Sheet 1

4. A *regular* polygon is a polygon in which all sides are the same length and all angles are the same measure.
6. 4
8. Approximately 2.8
9. The circumference is between 2.8 and 4 units.

#### Activity Sheet 2

2. If we rounded to the nearest whole number, the circumference of this circle would be 3 units.
- 3.

No. of Sides	Inscribed Perimeter	Circumscribed Perimeter
4	2.82843	4
10	3.09017	3.24920
25	3.13333	3.15823
100	3.14108	3.14263
200	3.14146	3.14185
2500	3.14159	3.14159

4. If we rounded to the nearest hundred thousandth, the circumference of this circle would be 3.14159; if we needed more accuracy, we could use a polygon with even more sides. For most practical purposes, the circumference measured to the nearest hundredth of a unit is close enough.

5.

Diameter	Inscribed Circumference (to the Nearest Ten Thousandth)
1	3.14160
2	6.28319
3	9.42477
4	12.56636
5	15.70681
$d$	$\pi \cdot d$

### REFERENCES

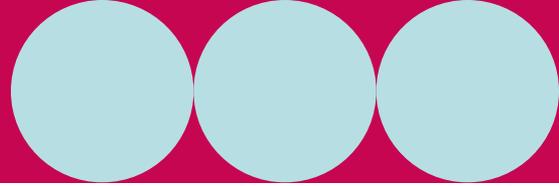
- Beckmann, Petr. *A History of  $\pi$* . New York: St. Martin's Press, 1971.
- National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

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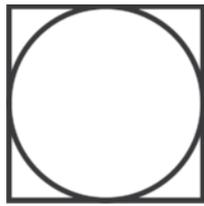


Name \_\_\_\_\_

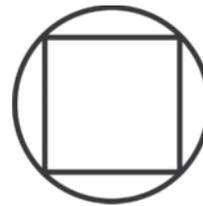
## FINDING THE CIRCUMFERENCE OF A CIRCLE

The circumference of a circle can be estimated by finding the perimeter of a regular polygon inscribed inside the circle and the perimeter of a regular polygon circumscribed outside the circle. The circumference of the circle will fall in between these two perimeters.

Consider the squares inscribed inside and circumscribed about the circle, as shown below. The circumference of this circle ( $C$ ) is greater than the perimeter of the inner square and less than the perimeter of the outer square.



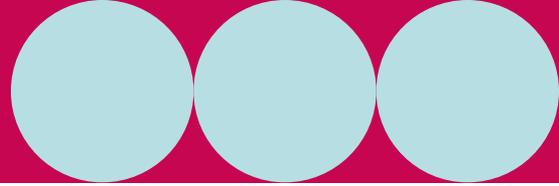
A square circumscribed about a circle with a diameter of 1 unit



A square inscribed inside a circle with a diameter of 1 unit

1. Use a red marker, and trace the circumference of the circles shown above.
2. Use a blue marker, and trace the perimeter of the inscribed square.
3. Use a green marker, and trace the perimeter of the circumscribed square.
4. What is a “regular” polygon?
5. Draw a diameter vertically through the center of the circle shown inside the circumscribed square.
6. If the diameter of this circle is 1 unit, what is the perimeter of the circumscribed square? Show your work, or explain how you determined the perimeter.
7. Draw a diagonal of the inscribed square (this will also be a diameter of the circle).
8. If the diameter of this circle is 1 unit, what is the perimeter of the inscribed square? (Round your answer to the nearest hundred thousandth, and show your work.)
9. The circumference of a circle that has a diameter of 1 unit is between \_\_\_\_\_ units and \_\_\_\_\_ units.

# activity sheet 2



Name \_\_\_\_\_

## GRAPHING AND CHARTING A CIRCLE

If we draw more sides on a polygon, we can narrow the interval that the circumference of the circle lies in between. The circle pictured below is circumscribed and inscribed by decagons. Look how closely the decagons squeeze the circle.

A decagon has \_\_\_\_\_ sides.



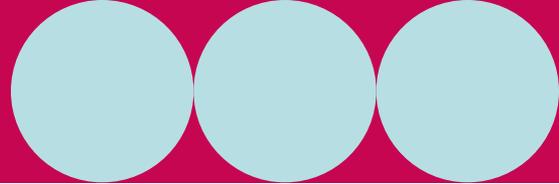
1. Use the program "PERIMETE" on the graphing calculator to find the perimeter of the inscribed and circumscribed decagons. Remember that the number of sides is 10 and to use a diameter of 1 unit.

To run the program:

- a. Press PRGM.
  - b. Highlight PERIMETE.
  - c. Press ENTER.
  - d. Press ENTER again.
2. From your results using the PERIMETE program, complete this statement (round to the nearest hundred thousandth):  
The circumference of a circle that has a diameter of 1 unit is between \_\_\_\_\_ units and \_\_\_\_\_ units.
  3. Using the calculator program, complete the chart below. Use a diameter of 1 unit, and round to the nearest hundred thousandth.

Number of Sides	Inscribed Perimeter	Circumscribed Perimeter
4		
10		
25		
100		
200		
2500		

# activity sheet 2 (continued)



Name \_\_\_\_\_

4. a. What do you notice about the perimeters as the number of sides of the polygon increases?

b. What is the circumference of a circle with a diameter of 1 unit to the nearest hundredth?

If we continued to use polygons with more and more sides, the perimeters of the inscribed and circumscribed polygons would get closer and closer to being the same, but they would never be equal. In other words, the circumference of the circle is a decimal number that never ends and never repeats, but we can find as many digits as we want. This is called an *irrational number*. Because the number cannot be written down completely, we use the Greek letter  $\pi$  to represent it.

5. The same process of inscribing and circumscribing regular polygons can be applied to circles with diameters other than 1. Use the calculator program, and enter a polygon with 1500 sides to complete the chart below. Notice that you will be using different diameters. (Round to the nearest ten thousandth.)

Diameter	Inscribed Circumference (to the Nearest Ten Thousandth)
1	
2	
3	
4	
5	
$d$	

6. Look for a pattern, and write an expression to represent the circumference of a circle with a diameter of  $d$  units.

7. Use pi ( $\pi$ ) to represent the circumference of a circle with a diameter of 1 unit, and write a formula for finding the circumference of a circle:

$C =$  \_\_\_\_\_