Carryn Bellomo

The Golden Ratio: Real-Life Math

Edited by **Gwen Johnson**, gwendolyn.johnson@unt.edu, University of North Texas, Dallas, and **James Dogbey**, jdogbey@clemson.edu, Clemson University, South Carolina. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on NCTM's Content and Process Standards and the Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing **mtms.submit.net**.

The *golden ratio* is a beautiful example of the mathematics that is found in nature, art, and architecture. It goes by many other names, such as the *golden section*, the *golden proportion*, or the *golden number*. The golden ratio is approximately 1.618, an irrational mathematical number equal to

$$\varphi = \frac{1+\sqrt{5}}{2}.$$

Two quantities (A and B) are in the golden ratio if A divided by B (or B divided by A) is equal to φ (phi).

This activity reviews measurement, analyzes data collection, and explores division, which can all occur within the context of studying the golden ratio. Students also observe and use technology to plot data, determine whether a correlation exists, and identify the golden ratio.

Careful consideration has been paid to providing an enrichment activity that teachers can directly tie to existing curriculum. The images, which maintain and are examples of the golden ratio, were found in my environment.

THE NECESSARY PREREQUISITE SKILLS

This lesson was presented to three sixth-grade classes of about thirty students each; two of these classes were advanced. My students were already comfortable identifying and using ratios and had plotted ordered pairs on a coordinate axis in the context of connecting dots to make figures. In rare instances, they had been exposed to scatter plots to determine if a data set had a positive, a negative, or no correlation. In addition, students were comfortable using centimeter rulers to measure line segments to the nearest half centimeter.

LESSON HIGHLIGHTS

The lesson began with a brief discussion of *ratio*, its definition, and ways to express it; the golden ratio was then explored using the symbols B/A, which is approximately equal to 1.618. We had a brief discussion about the word *approximate* by reviewing terminating and repeating decimals. I asked students if they knew of any other nonterminating, nonrepeating decimal (hoping to elicit a discussion of pi, which many of my students already knew).

I showed students a picture of the Luxor Hotel, which is modeled after the pyramid at Giza. The hotel is familiar to my students because we are in Las Vegas. I then showed them a photo of the *actual* Grand Pyramid at Giza, and we discussed

The measurement of the Parthenon approximates a 1.67 ratio.

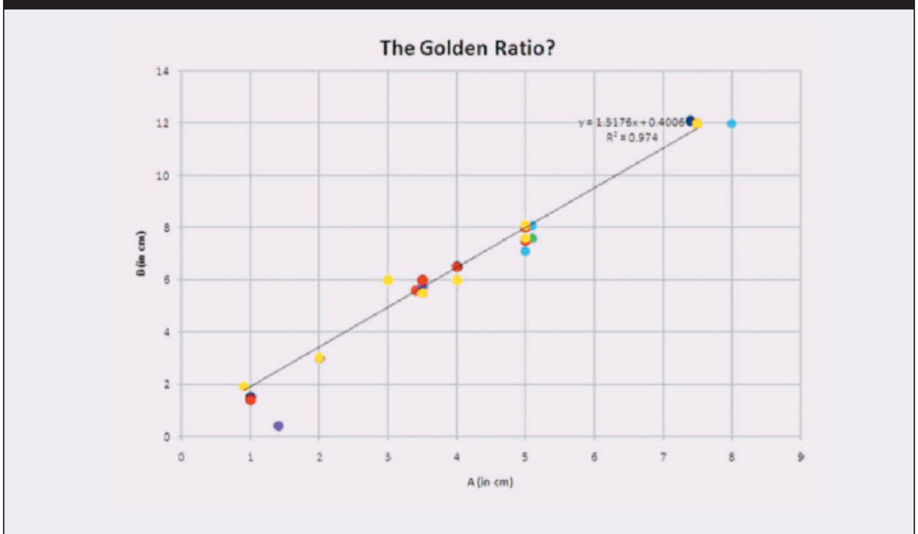
how it exhibited the golden ratio. The values used on the Grand Pyramid are approximately A = 377.9 feet (half the base) and B = 614.8 feet (the length of the face). Using a calculator, students found the ratio B/A, and I reaffirmed that the ratio is approximately 1.618. We discussed why their results might be off (unit conversion, erosion, round-off error, and so on).

Students were given **activity sheet 1**, a warm-up lesson that involved measuring pyramid sides and recording and plotting the results. They completed it in ten to fifteen minutes, which was more time than I expected. They needed to be coached on plotting the point (1.5, 2.5), because the axis that was given for A did not have 1.5 as a value (1.4 and 1.6 were on the graph, however). We then reviewed question 4 together, and students were asked: Do these values approximate the golden ratio? Some measuring discrepancies and issues of rounding occurred, which led to a great discussion about measurement techniques and round-off error.

Groups of four to five students then worked on **activity sheet 2**, which contained photographs with marked lengths of A and B for students to measure. Each length was designed to be rounded to the nearest half centimeter. Some students were able to measure precisely. Since we had not discussed rounding our measurements in any detail before this point in the year, there was prudence in scaling properly. Since I wanted the photographs to be in color, I printed a class set of the images and put them in sheet protectors to be used for each class.

Students used **activity sheet 3** to record their data and plot their points. To save time plotting points by hand, students entered the data on a laptop. I already had the plot color coded, so each team could identify its data. The scatter plot for one class is shown in **figure 1**. This plot will obviously differ with each class, depending on how well students collected the data.

Fig. 1 The color-coded data for each class will differ, depending on rounding error.



The class discussed the "spread" of the data, and whether the points should be close together, on top of one another, and why. We also discussed the positive correlation of the data. I mentioned the idea of the slope of the line, used the spreadsheet program to find the best-fit line, and displayed it on the graph (the correlation coefficient may also be found automatically, depending on your software program). The slope of the line in this example was 1.5176; we discussed how this number approximates the golden ratio. I did try to go into some detail about the correlation coefficient but decided that I should save that discussion for more accelerated students. One class had a correlation coefficient of 99 percent (recall that a perfect positive correlation is 100 percent).

To wrap up the activity, I presented students with other photographs of the golden ratio and golden spiral. (See the **sidebar** on p. 440.)

STUDENT WORK AND REACTIONS

The activity was a success in each class. Because there were enough photographs per group, each student was able to measure, record, and divide at least once, and all were excited about collecting data. The activity took about fifty minutes.

5 Places to Find the Golden Ratio

An Internet search will provide information about depictions of the golden ratio, which can be found in—

1. Leonardo da Vinci's *Vitruvian Man;*

2. flowers, in particular, petal and center distances (the photograph below is in a 2.4 unit:1.5 unit ratio);

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3. works of art, such as *Holy Family* by Michelangelo, *Crucifixion* by Raphael, and *Self Portrait* by Rembrandt;

4. architecture, including the pyramids, the Parthenon, and the Acropolis;

5. fingerprints, sea shells, hurricanes, pinecones, and galaxies.

I was expecting a vast difference between skill levels in the classes and was surprised by the similar dialogue that occurred. The accelerated class did not have any advantage in terms of understanding the concepts or vocabulary, but the accuracy of the data varied.

LESSON HIGHLIGHTS

This activity was a great lesson for students at all skill levels. Because they worked in groups and checked one another's work, the students who needed guidance received it from peers.

The photographs given to students were carefully scaled to be values close to the nearest half centimeter. This helped students perfect their measuring techniques without testing their ability to measure precisely. Using centimeters also helped when entering the data (1 1/2 cm = 1.5 cm), as opposed to the more cumbersome conversion using inches.

In one class, I allowed each group to record its own data. The regular class needed incentive to move more quickly, so I used data entering as an incentive: The group that finished first could enter their data themselves; the others just turned in their tables. In each class, students needed prompts and help with correlation.

CONCLUSION

I am always inspired when I see students who are excited about exploring mathematical concepts, stretching their minds, and making connections. This lesson does all that. In addition, it brings the real world directly into the classroom by relating art and mathematics. There is no better way to teach.

That said, it can be hard to use class time on an enrichment activity when the required curriculum and state tests are priorities. One way to complete this activity but decrease the class time is to assign components of the lesson for homework. First, take a few minutes of class time to review **activity sheet 1**. Then give students the images and a blank recording table to take home. The next day, when students turn in their tables, their data could be entered into a spreadsheet for display and discussion during class.

This lesson took time to develop. Finding images and taking photographs that displayed the golden ratio proved to be time-consuming. In addition, ensuring that the golden ratio was maintained and simultaneously scaling measurements to the nearest half inch proved to be difficult tasks. Although many examples are online, copyright issues prohibit their use for publication. The **activity sheets** and the lesson described have been prepared in a format that fits nicely into a sixth-grade curriculum.

Note: For images whose measurements are rounded to the nearest tenth of an inch, as described in this article, visit the author's page at http://faculty.unlv.edu/bellomo/Grants-Proj/GoldenRatio.pdf.

BIBLIOGRAPHY

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Carryn Bellomo, carryn.bellomo@unlv.edu, currently teaches at the University of Nevada, Las Vegas. She is interested in mathematics education at the secondary level.

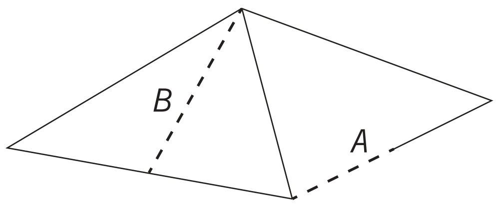
Solutions to **activity sheets 1**, **2**, and **3** are appended online to "Mathematical Explorations" at **www.nctm.org/mtms**.

activity sheet 1

Name ______________________________

A PYRAMID RATIO

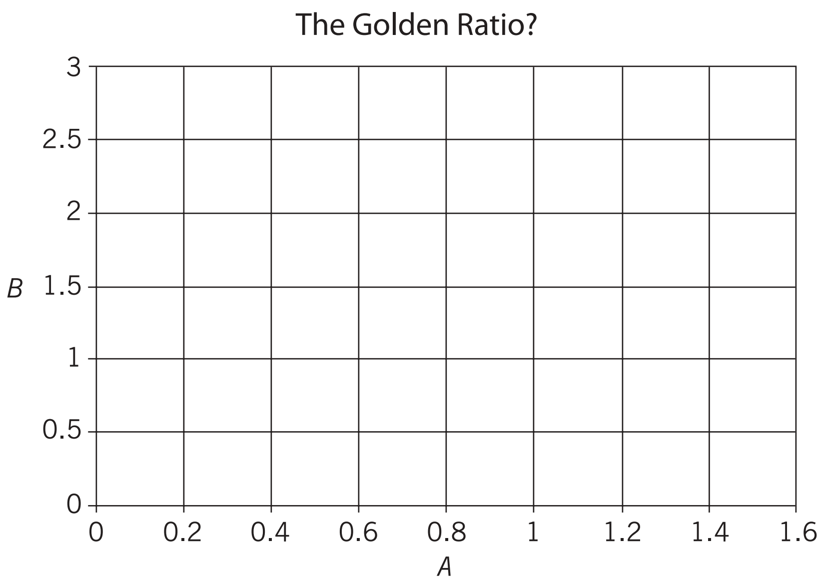
1. Measure the line segments for *A* and *B*, in centimeters.



2. Record the data in the table. Find *B*/*A*.

	A	*B*	*B*/*A*
The Great Pyramid			

3. Plot the point on the coordinate axis below.



4. Does this value approximate the golden ratio?

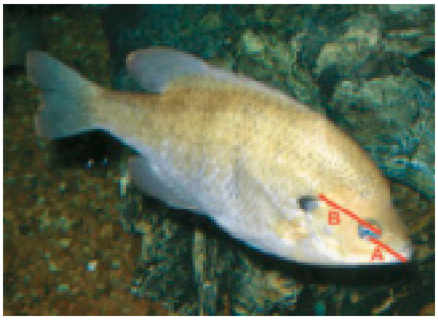
from the March 2011 issue of mathematics teaching in the MIDDLE SCHOOL

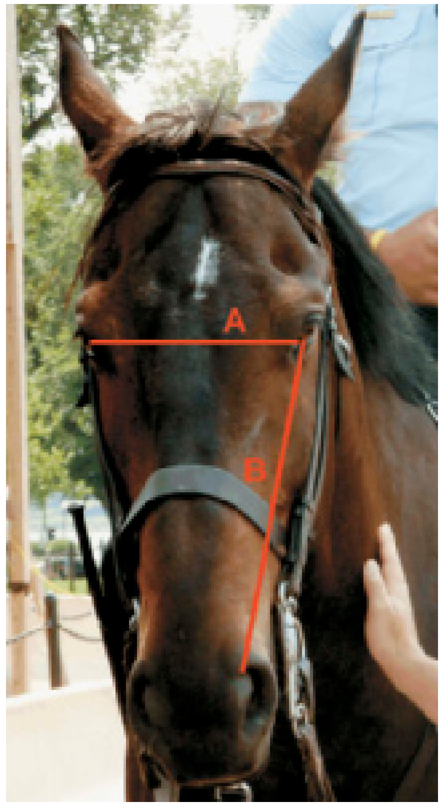
activity sheet 2

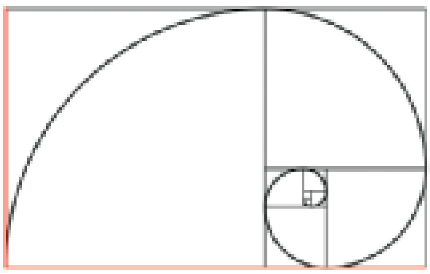
Name ______________________________

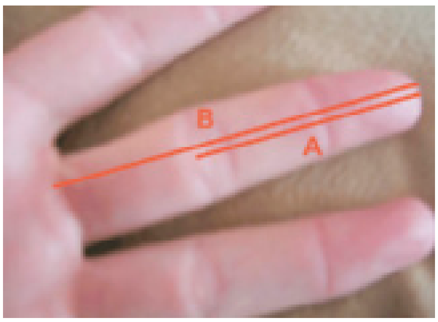
MEASURE THE IMAGES

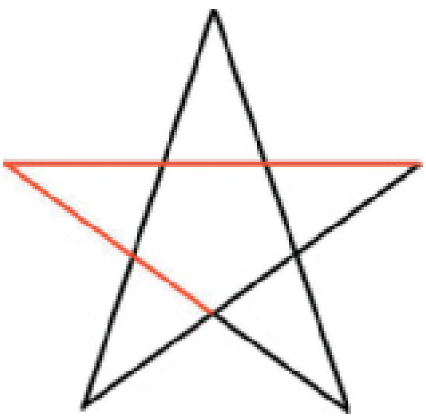
For each image below, measure the two lengths and record your results on the next page, titled “Table and Plot.”

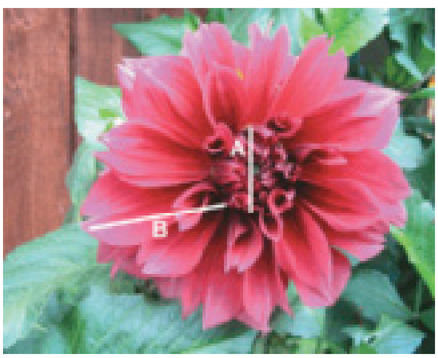


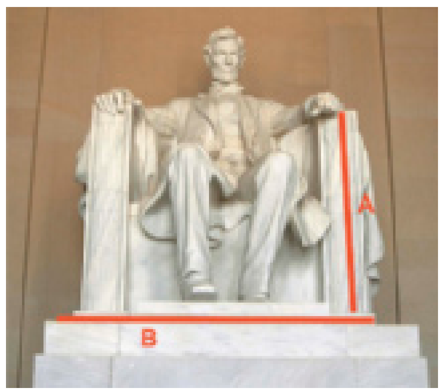












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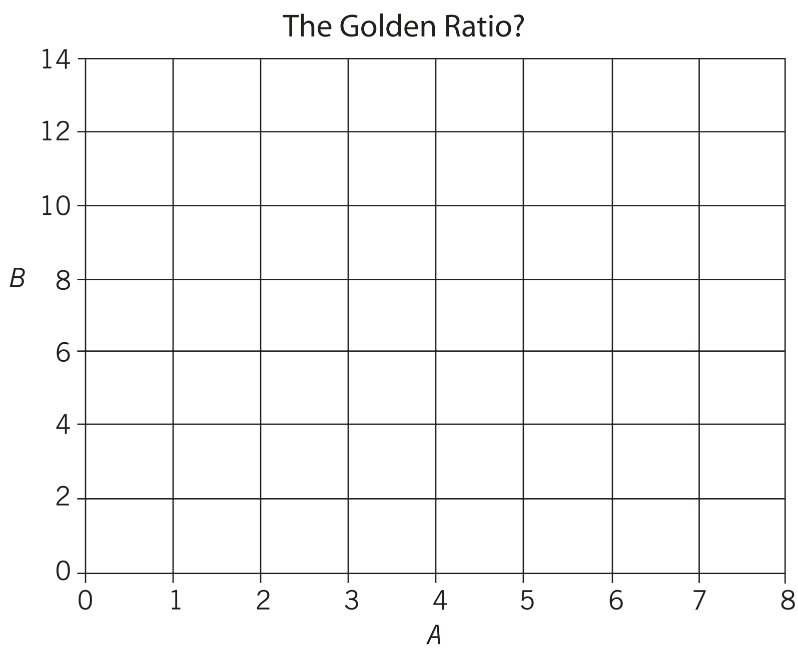
activity sheet 3

Name ______________________

TABLE AND PLOT

	A	*B*	*B/A*
Fish			
Spiral			
Star			
Arm			
Finger			
Flower			
Horse			
Lincoln Memorial			



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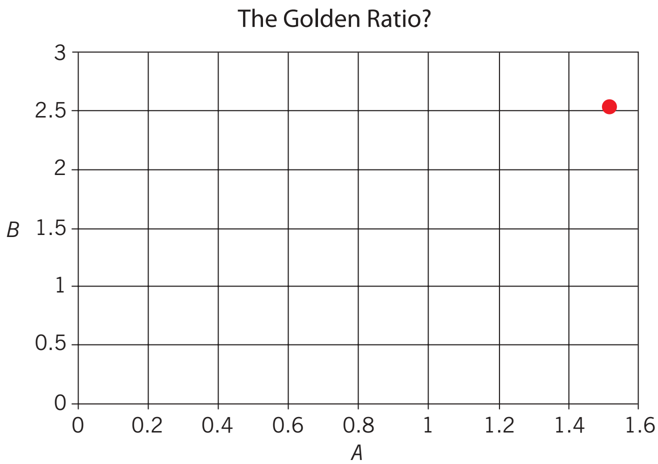
SOLUTIONS

Activity Sheet 1

2. A = 1.5 cm, B = 2.5 cm, A/B = 1.67

3.



4. Answers may vary. But yes, 1.67 approximates the golden ratio.

Activity Sheet 3

Note. The image sizes may vary if students are not using the images from http://faculty.unlv.edu/bellomo/GrantsProj/GoldenRatio.pdf, but the each ratio should approximate the golden ratio.

	A	B	B/A
Fish	1	1.5	1.50
Spiral	7.5	12	1.60
Star	5	8	1.60
Arm	5	7.5	1.50
Finger	4	6.5	1.63
Flower	2	3	1.50
Horse	3.5	5.5	1.57
Lincoln Memorial	3.5	6	1.71