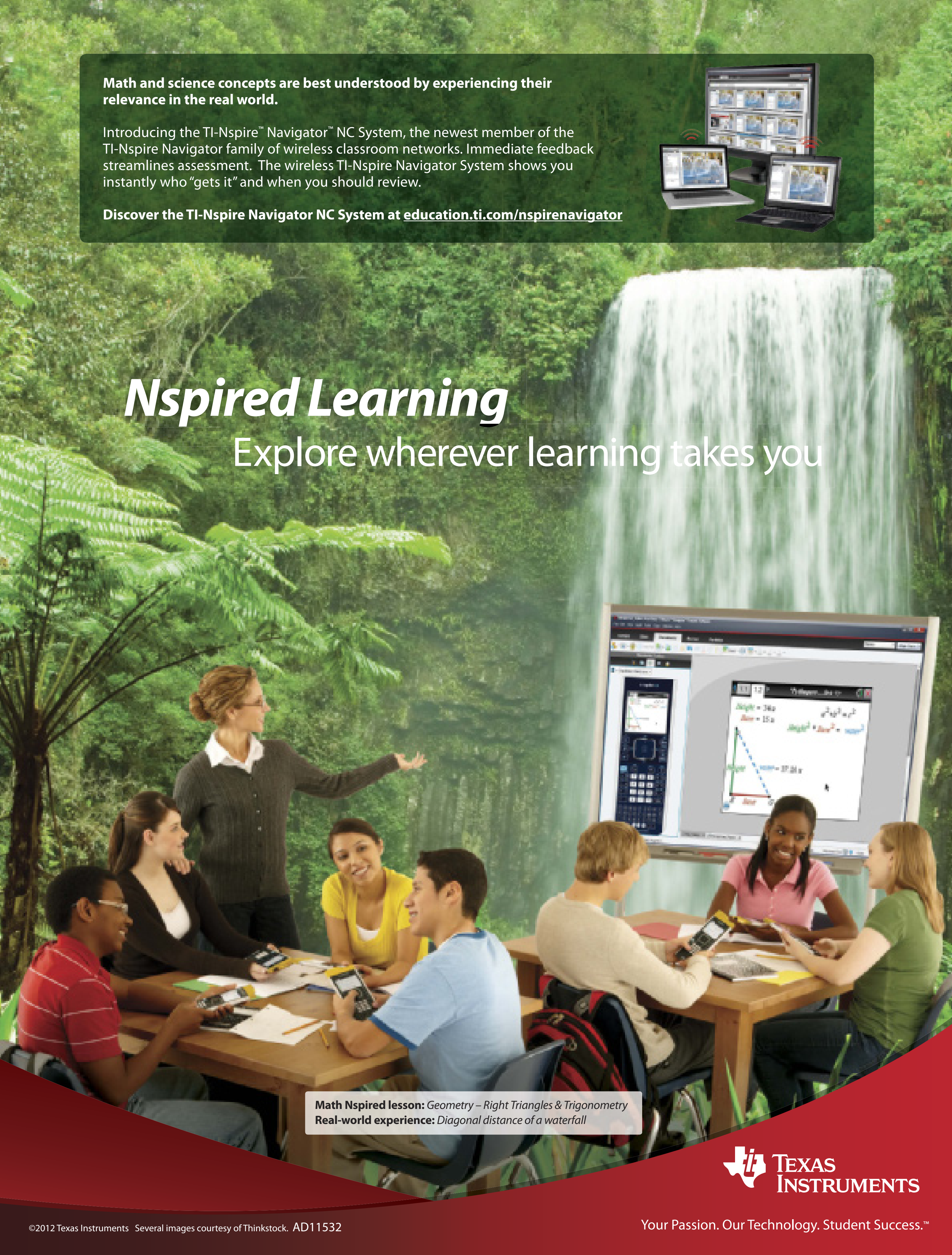
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Fraction Multiplication from a Korean Perspective

Ji-Won Son

Students should not only understand the meaning of fraction operations but also be able to explain why the procedures work (NCTM 2000). However, a report from the *Trends in International Mathematics and Science Study* (TIMSS) indicates that U.S. students do not perform as well as those in many other countries, including Japan, South Korea, and Singapore (Mullis et al. 2008). This difference can be seen by inspecting the multiple-choice word problem involving the multiplication of fractions shown in **figure 1**. Only 34 percent of U.S. eighth graders tested were able to correctly solve this word problem compared with 66 percent of Korean students tested.

Research shows that many U.S. students learn fraction operations through procedure-oriented, memory-based instruction, which attributes little meaning to such operations (Kilpatrick, Swafford, and Findell 2001). NCTM (2006) asserts that instruction should emphasize a balance between mathematical concepts and procedures for student learning. Teachers who provide the foundation for understanding fraction computation by building on students' understanding of whole-number operations can help students' reorganize fraction schemes (Mack 2001). When students have opportunities to compare and contrast multiple solution methods, students make greater gains in procedural flexibility (Rittle-Johnson and Star 2007).

Kilpatrick, Swafford, and Findell (2001) indicate that looking at curricular approaches used across countries can provide a better picture of what matters in instruction aimed at developing proficiency. Drawing from research that examined U.S. reform and Korean textbooks (Son and Senk 2010), this article will share how Korean textbooks address and develop meaning for fraction multiplication and what algorithms are accessed in accordance with these meanings.

The Korean approach embodies many of the ideas that research identifies as beneficial for developing both an understanding of and procedures for fraction multiplication. It is hoped that this international perspective specifically will increase U.S. teachers' experience and awareness as they strive to help students understand fraction multiplication. This overarching framework can give teachers information about how to approach instruction when developing operations in general.

KOREAN TEXTBOOKS

Mathematics study in Korea adheres to a nationwide curriculum developed by the Ministry of Education. Only one set of textbooks is available for teaching mathematics in grades 1 through 6.

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Multiplication of fractions is addressed in the first semester of grade 5. Unit 7 comprises eight lessons. The textbook analysis by Son and Senk (2010) revealed that Korean textbooks develop three different meanings of fraction multiplication as (1) repeated addition, (2) operator, and (3) finding a part of a part. Three different problem types are developed: whole number × fraction, fraction × whole number, and fraction × fraction. Throughout the unit, students use a three-step process:

1. Develop the meaning of the operation using real-life contexts.
2. Develop strategies for computing.
3. Practice strategies and strategy selection.

Fraction Multiplication as Repeated Addition

Korean textbooks first introduce fraction multiplication as *repeated addition* using the problem type *whole number × fraction.* The meaning of repeated addition is first studied in whole-number multiplication. Students learn that 3 × 4 means "3 groups of 4 objects" (i.e., adding 4 three times). The first factor (the multiplier) represents the number of groups, and the second factor (the multiplicand) tells the size in each group. Similarly, the first fraction multiplication lesson asks students to find 5 × 3/8 using repeated addition (i.e., adding 3/8 five times). **Figure 2** shows the introductory problem for the unit (Korean Ministry of Education 2002a, pp. 108–9, translated by the author), which includes a real-life situation involving pizzas. Activity 1 (see **fig. 2**) asks students to determine what fraction represents 5 × 3/8 by modeling repeated addition with area models. Students use fraction-circle pieces to represent 5 groups of 3/8 (see **fig. 3**).

In the second activity, students learn strategies for computing *whole number × fraction* problems. On the

Fig. 1 TIMSS reported this multiplication of fractions problem and results.

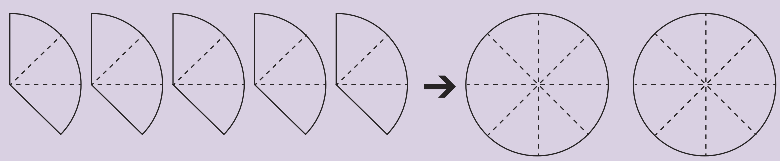
Dana makes a large batch of cranberry bread that is one and a half times the original recipe. If the original recipe requires 3/4 cup of sugar, how many cups of sugar are required for the bread Dana is making?

International average	27.4 percent
United States	34.2 percent
Korea	66.2 percent

Fig. 2 Korean students see this introductory problem for fraction multiplication.

Learn from real life:
There are five pizzas of which 3/8 of each pizza remains. How much pizza is there in total?

Activity 1. Find out what fraction represents 5 × 3/8.
- Use fraction circle pieces and represent 5 groups of 3/8.
- Color two circles to express 5 of 3/8 pizza.
- How much is $5\times\frac{3}{8}$?
- Why do you think so?



Activity 2. Figure out how to calculate 5 × 3/8.
- Add 3/8 five times.

$$5\times\frac{3}{8}=\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}=\frac{\square}{8}=\square$$

- How is $5\times\frac{3}{8}$ similar to or different from $\frac{5\times 3}{8}$?
- Why? Explain your reasoning.

Lesson learned from the activity:
Tell how to find out the product of a fraction by a whole number.

Fig. 3 Fraction-circle pieces represent 5 groups of 3/8.

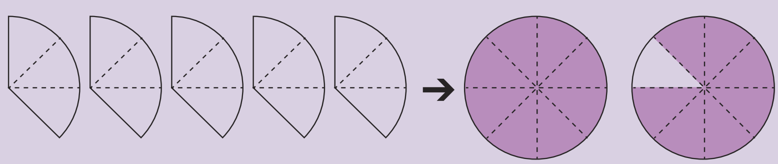
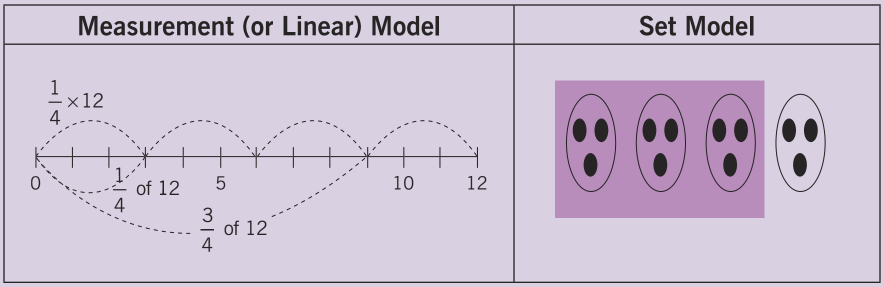


Fig. 4 Two models illustrate how and in what ways to partition wire.



basis of the first activity, students are expected to realize that 5 × 3/8 means "5 groups of 3/8" and express this thinking process with a numerical sentence modeling multiplication as repeated addition:

$$\begin{aligned} 5\times\frac{3}{8} &= \frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8} \\ &= \frac{15}{8} \\ &= 1\frac{7}{8} \end{aligned}$$

The third activity asks students to apply their understanding to several fraction multiplication problems of the same type. For example, with 3 × 3/4, students will model the problem by using fraction circles and by writing number sentences that make explicit the fact that multiplication is repeated addition. The goal of the final problem leads students to formalize the following algorithm:

$$\begin{aligned} &(\text{whole number})\times\frac{(\text{numerator})}{(\text{denominator})} \\ &= \frac{(\text{whole number})\times(\text{numerator})}{(\text{denominator})} \end{aligned}$$

Fraction Multiplication as Operator

Korean textbooks next address the problem type *fraction* × *whole number*. When the multiplier (the first factor) is a fraction and the second is a whole number (e.g., 3/4 × 12), students must modify it because the repeated-addition interpretation does not make sense. Korean textbooks address this form of fraction multiplication with the notion of "operators." The operator interpretation means "3/4 of 12"; this computation can be illustrated by drawing 12 units (or wholes) and taking 3/4 of that entire region. Similar to the development of multiplication as repeated addition, Korean textbooks develop the operator interpretation using the same three-step process:

1. Develop a real-life context.
2. Develop strategies for computing.
3. Practice the strategies.

Korean textbooks employ area models for the repeated-addition interpretation, whereas they use length models and set models for the operator interpretation (Son and Senk 2010).

In the first activity, students are asked to model the following wire problem using length models (i.e., number lines):

> Twelve meters of wire were bought to make a wire-sculptured animal with clay. If 3/4 of the wire is used, how many meters of the wire are used?

First, students find 1/4 of 12 on a number line by partitioning 12 into 4 groups, finding 3 meters in each group. Next, they determine how many are in the 3 groups as indicated by the numerator 3 in 3/4 of 12. This interpretation and solution process require students to recognize two ideas. First, 1/4 of 12 is equivalent to 12 ÷ 4 because 1/4 × 12 is the quotient when 12 is divided into four groups on the number line. Second, 3/4 is three times as large as 1/4; therefore, 3/4 × 12 is equivalent to 3 × (1/4 × 12). The problem is also explored using set models. **Figure 4** shows the two models used with the wire problem.

In the second activity, students learn strategies for computing *fraction* × *whole number* problems by referring to the representations in **figure 4**. Students are asked to record their thinking processes using symbols by completing the following problem, resulting in 12 ÷ 4 × 3:

$$\begin{aligned} \frac{3}{4}\times 12 &= \left(\square\times\frac{1}{4}\right)\times 12 \\ &= \square\times\frac{12}{4} \\ &= \frac{\square\times\cancel{12}^{3}}{\cancel{4}^{1}} = \square \end{aligned}$$

In the third activity, students are required, through solving several similar problems (e.g., 5/12 × 18), to formalize *fraction* × *whole number* multiplication in this way:

$$\begin{aligned} &\frac{(\text{numerator})}{(\text{denominator})}\times(\text{whole number}) \\ &= \frac{(\text{numerator})\times(\text{whole number})}{(\text{denominator})} \end{aligned}$$

Unlike many U.S. reform textbooks, Korean students are asked to evaluate different reduction methods (Son and Senk 2010). The three reduction methods are illustrated in **figure 5**.

Fraction Multiplication as Taking a Part of a Part

Finally, Korean textbooks expand the meaning of multiplication using the interpretation *finding part of a fractional part* in a problem situation involving a vegetable garden (see **fig. 6**):

Fig. 5 Korean textbooks contain additional strategies for multiplying fractions.

$$\frac{5}{12}\times 18 = \frac{5\times 18}{12} = \frac{\overset{15}{\cancel{90}}}{\underset{2}{\cancel{12}}} = \frac{15}{2} = 7\frac{1}{2}$$

$$\frac{5}{12}\times 18 = \frac{5\times \overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}} = \frac{15}{2} = 7\frac{1}{2}$$

$$\frac{5}{\underset{2}{\cancel{12}}}\times \overset{3}{\cancel{18}} = \frac{15}{2} = 7\frac{1}{2}$$

> The Joohos planted vegetables in 3/4 of their garden. They used 5/7 of the vegetable space for radishes. How much of the total garden is occupied by radish plants?

According to Son and Senk (2010), this question sets up the typical meaning of fraction multiplication addressed and developed in U.S. textbooks.

Korean textbooks use an area model to initially develop *fraction* × *fraction* multiplication. Students are asked to shade area models (i.e., a rectangle), coloring 3/4 after segmenting the rectangle vertically into 4 equal pieces and then shading 5/7 after segmenting the rectangle horizontally into sevenths. The accompanying teacher's manual suggests that while students are completing this coloring activity, teachers lead them to focus on how the denominators relate to how the grid is partitioned and how the numerator affects the solution to the problem, as directed in **figure 6**.

In the second activity, students learn strategies for computing *fraction* × *fraction* problems by referring to the representations in **figure 6**. Students then formalize the procedure for multiplying fractions, as follows:

$$\frac{5}{7}\times\frac{3}{4} = \frac{5\times\square}{7\times\square} = \square$$

As with previous problems, Korean textbooks provide several *fraction* × *fraction* multiplication problems so that students formalize the fraction multiplication algorithm shown above.

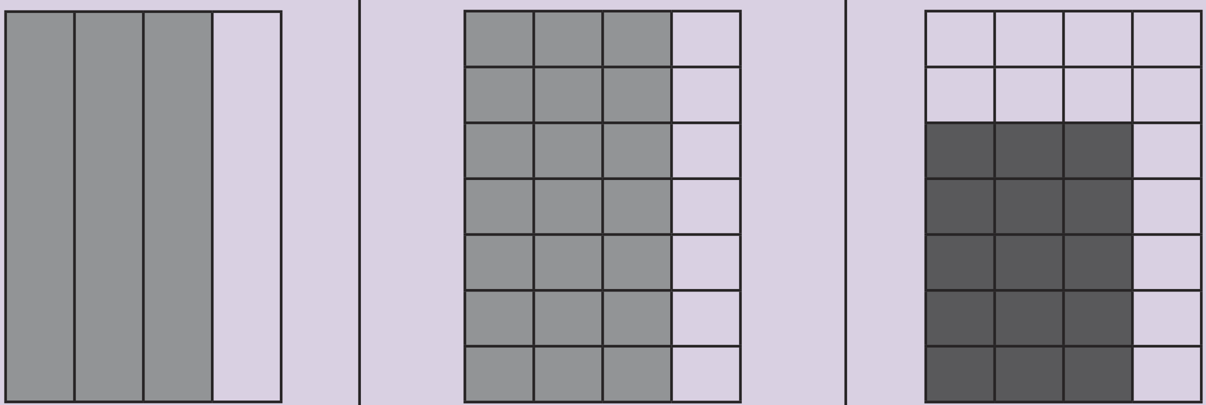
WHAT CAN WE LEARN FROM THE KOREAN PERSPECTIVE?

The description of the development of fraction multiplication in Korea reveals some noteworthy features to consider when designing fraction multiplication instruction. A better understanding of the Korean perspective reveals four points for teachers to consider in supporting students' understanding of fraction multiplication. These perspectives have implications when designing instructional sequences for other operations as well.

Fig. 6 These steps develop the algorithm for fraction × fraction problems.

$$\frac{5}{7} \times \frac{3}{4} \text{ means "finding } \frac{5}{7} \text{ of } \frac{3}{4} \text{ of a whole"}$$

Step 1: Make 3/4 as the vertical dimension.	Step 2: Cut the rectangle horizontally into 7 rows.	Step 3: Take 5/7 of 3/4 of a whole.



- How many small rectangles are in a big rectangle (a whole)?
  7 rows and 4 columns, so 7 × 4 = 28
- How many small rectangles are in the double-shaded parts?
  5 rows and 3 columns, so 5 × 3 = 15

1. Develop multiple meanings for fraction multiplication.
Examine your curriculum materials. What meanings are developed? Any operation has multiple meanings. By exploring these multiple meanings, students develop a deeper understanding of what an operation entails and when it is appropriate to use. When instruction focuses only on developing procedures, the meaning associated with them (i.e., when to use a procedure) remains underdeveloped. Explore various instructional resources to become familiar with the meanings for various operations.

2. Use multiple representational models in a systemic way.
The more ways students are given to think about mathematical ideas, the better they will understand them. Korean textbooks direct students to develop three representational models (area/circle or rectangle, discrete/set, and measurement/number line) in accordance with the different meanings and types of fraction multiplication. For any operation, it is important to use appropriate models to draw out and highlight concepts that are part of a deep understanding of that operation.

3. Develop multiple computational strategies.
Many U.S. textbooks focus on developing a single procedure for an operation. Developing multiple strategies for solving problems and discussing why they are useful lead to deeper understanding. Students' informal methods should be shared, discussed, and compared with the more formal strategies that are developed.

4. Develop conceptual understanding and procedural fluency simultaneously.
Korean textbooks develop conceptual understanding and procedural fluency simultaneously using the three-step method. Each type of fraction multiplication involves—

1. developing the meaning of the operation using a real-life context;
2. developing strategies for computing; and
3. using and applying strategies.

This framework can be used when developing any operation.

It is hoped that this critical reflection on how Korean textbooks approach fraction multiplication can give teachers additional ideas for designing meaningful leaning opportunities for students. Teachers and districts are encouraged to look deeply at the various approaches used across curricula when designing instruction.

When we engage in conversations about what content is most worth teaching, the most appropriate timeline for introducing the content, and how one might teach it, we grow professionally.

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