palette of problems

David Rock and Mary K. Porter

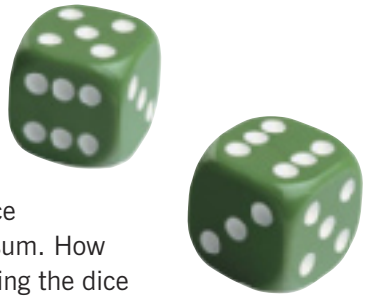
1. Find the smallest positive integer such that the product of this integer and 180 is a perfect square.

2. The diameter of circle *A* is the radius of circle *B*. What is the ratio of the area of circle *B* to the area of circle *A*? Your answer should be a simplified fraction.

3. Twin brothers each have two favorite sweatshirts. After doing laundry, their father cannot tell which sweatshirt goes to which brother. If their dad randomly gives each brother two of the four sweatshirts, what is the probability that each brother will get his correct two sweatshirts?

4. Lilia fills a cup with hot chocolate. She drinks 1/2 cup and then pours coffee into the cup until it is full. She stirs the new mixture and drinks 1/2 cup and then fills the cup again with coffee. Lilia continues this pattern until she has consumed 2 full cups. When she next fills the cup with coffee, what percentage of the original hot chocolate remains in her cup?

5. Begin with 8 times the number of days in February in a leap year. Multiply that result by the smallest prime number. From this result, subtract the number of pounds in a ton. Finally, to this last result, add the number of yards in 2 miles. What is your final answer?



6. Whitney rolls 2 standard six-sided dice and records the sum of the 2 numbers showing. Then she rolls the dice again and records the second sum. How many times must she keep rolling the dice to ensure that she will repeat at least one sum from a previous roll? (*Hint:* The answer is not 2.)

7. Find the sum of the digits of the result of 33333333334^2.

8. Bert draws an octagon that has 8 interior angles, 5 of which sum to 845 degrees. Two of the remaining 3 angles are supplementary, and 2 of the remaining 3 angles are complementary. Find the angle measures of the remaining 3 angles.

9. Caden found a belt on a clearance rack. The original price was $18. On clearance, everything was 25% off the original price. He also has a coupon for an additional 25% off his purchase. If Caden has $10, does he have enough money to buy the belt?

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Problems 10, 11, 13, and 14 were submitted by **Joel Amidon**, University of Mississippi, Oxford.

10. Sarah and James drove from point *A* to point *B* along different routes. Sarah's route was 9 miles, and she traveled at an average speed of 45 miles per hour (mph). James's route was 2 miles longer. Sarah and James arrived at the same time. Sarah claimed that James must have broken the speed limit of 55 mph, but James insisted that he did not. Who is correct?

11. I am a three-digit number less than 499. The sum of my digits is a prime number less than 15. The units digit is less than the hundreds digit, and the tens digit is less than the units digit. If none of my digits is a 0, what number, or numbers, can I be?

For problem 12, use these results of @ as an operator.

$$1 \text{ @ } 6 = 1$$
$$2 \text{ @ } 4 = 16$$
$$3 \text{ @ } 5 = 243$$
$$4 \text{ @ } 7 = 16{,}384$$

12. Find the value of 5 @ 9.

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13. Ahtim and Ruba use incandescent bulbs, which cost 97¢ each, and compact florescent lamp (CFL) bulbs, which cost $3.17 each. When the CFL bulb burned out, Ahtim remarked, "We could have bought 4 CFL bulbs for the cost of all the incandescent bulbs." Ruba replied, "One more incandescent bulb, and we could have bought 5 CFL bulbs." How many incandescent bulbs did they use during the life of 1 CFL bulb?

14. Cassidy has a bag that contains 120 pieces of candy. Cassidy shares her candy by giving Katelyn 3/4 of 2/3 of her candy. Katelyn shares her candy with Kyle by giving Kyle 3/4 of 2/3 of her candy. If 2/3 of Kyle's candies are red, how many of his pieces are red?

15. What is the units digit of 3^{722}?

(*Answers on page 446*)

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