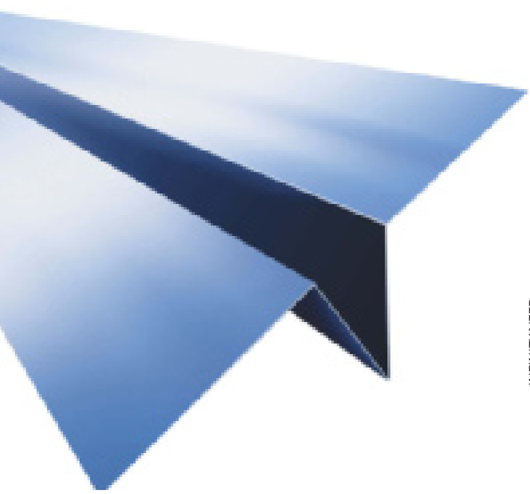
Cleared for Paper Airpla

Takeoff:
nes in Flight

Math and science, as well as mean, median, and mode, made smooth landings during a three-day data-collection unit.

Stacy L. Reeder



INFINITY/VEER

As middle school mathematics becomes more abstract, it is imperative for teachers to introduce concepts in ways that are interesting and meaningful to students. Since my students struggled at times to stay engaged in mathematics and seemed to have difficulty developing conceptual understanding, I looked for ways to create learning contexts that would be noteworthy and relevant. Working with data seemed to lend itself readily to contexts that were appealing to students. "Middle school students' curiosity about themselves, their peers, and their surroundings can motivate them to study statistics. The data to be gathered, organized, and studied should be interesting and relevant" (NCTM 1989, p. 105).

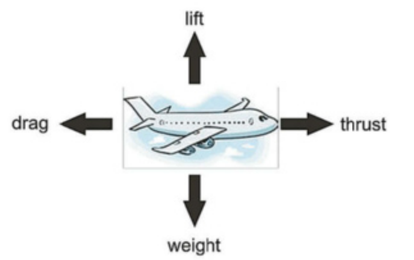
Because most textbook problems using data are devoid of a context that is meaningful to students, I never worried about leaving the textbook on the shelf and involving my students with data sets that they gathered themselves. To give my students additional experience with measures of central tendency and data collection, I developed an activity that involved making and flying paper airplanes. My eighth-grade students were thrilled with this idea and were immediately interested.

This lesson, which ultimately focused on measures of central tendency, also addressed measurement, scatter plots, and the four main forces of flight. The unit began with an engaging conversation about flying. The simple question, "How many of you have flown in an airplane?" resulted in interesting conversations among my students about the fear of flying, the thrill of flying, where they have flown, where they want to travel, and so on. We then discussed making and flying paper airplanes, a common adolescent activity. Students shared their ideas about how to fold a good paper airplane and how to make a "trick" paper airplane or one that will fly "really far." I then moved the discussion to main forces of flight.

SETTING THE STAGE

On the first day, I introduced students to the concepts of drag, lift, thrust, and gravity (see **fig. 1**) using information from the NASA Education website (http://www.nasa.gov/audience/foreducators/index.html). Once these concepts were presented, we referred to them as variables that affect flight. These terms allowed us to discuss how the use of the term *variable* in mathematics is similar to, and also somewhat different from, how it is often used in everyday communications and in science. This conversation gave students an opportunity to share their understanding of both independent and dependent variables and the connections they have made between those ideas in both science and mathematics. As part of this discussion, students were asked to identify other variables that they believed would affect the flying of a paper airplane. Their responses included these:

Fig. 1 This information, produced in the form of a poster, helped students visualize the four forces of flight.



Lift: An upward movement
Drag: A backward pull
Weight: A downward push
Thrust: A forward push

- "How you throw it will make a difference. How hard or light."
- "Things sticking up—like different designs with flaps."
- "What paper you use can make a difference if it is too light or too heavy."

After a list of variables was generated, students were divided into groups of three and worked to categorize each item on the list as one of the four types of forces. After working for a few minutes in their groups, students were pulled together to discuss the variables they had identified. Each variable was classified in terms of the four forces that affect flight. For example, "things sticking up" was classified as drag, whereas "what paper you use" was classified as weight and also as drag.

As the first day of this activity drew to a close, students were allowed to build a prototype for their paper airplane and test fly them in our classroom. They were asked to keep in mind our discussion of variables and forces as they constructed, tested, and modified their paper airplanes.

FLYING THE UNIQUE PLANES

The second day, our activities began when I asked the students to reflect on

Flying Paper Airplanes and Collecting Data

1. Go into the hallway in groups of three or four and fly your paper airplane.

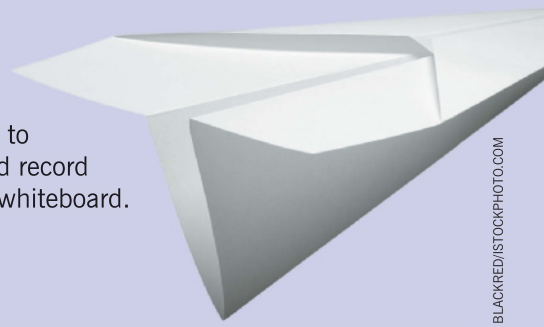
2. When all the paper airplanes in your group have flown, take two measurements, in inches:

   - The first measurement: How far down the landing strip your plane flew.
   - The second measurement: How far off of the landing strip your plane landed (measure in a straight line perpendicular from the landing strip).

3. Since we are interested in the flight accuracy of your paper airplane, the distance away from the landing strip should be subtracted from the distance down the landing strip. Please record your flight data, and leave your paper airplane where it landed in the hallway.

4. You have permission to fly your paper airplane a second time if you have a negative number for your flight data. This could occur if you subtract your two measures and the distance off the landing strip is greater than the distance down the landing strip. If your plane lands behind you or flies a short distance down the landing strip and veers off quickly to the right or left, a negative number could also result.

5. Once you have collected your flight data, return to the classroom and record your data on the whiteboard.



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our discussions from the previous day and their prototype paper airplanes. I asked a few students to summarize what we had discussed the day before and asked a few others to share what they learned from working with their prototype. One student had talked with an older brother who could, in his estimation, build "really great paper airplanes" so he learned from him how to construct a good plane.

Following this discussion, students were offered three kinds of paper—notebook paper, copier paper, and construction paper—with which to fold a paper airplane for the day's activities. These paper airplanes were called Unique Planes. During construction, students were allowed to go into the hallway to see where we would be flying the paper airplanes so that we could discuss the area to be used once all planes were completed. One strip of masking tape ran down the center and most of the length of the hallway, and another strip of masking tape ran perpendicular to the longer piece, across the width of the hallway. The masking tape formed our landing strip. Also, since our hallway had several doors along one wall and mostly windows along the other wall, all doors and windows were closed to control for gusts of wind or drafts.

When all students had completed the construction of their paper airplanes and had visited the hallway to see the landing strip, they were given measuring sticks, paper, and specific instructions on gathering data (see the **sidebar**). After collecting data in the hallway, students were asked to leave their paper airplanes on the floor where they landed. All data collection in the hallway took about thirty minutes for my class of twenty-two students.

Flight data that could potentially result in negative numbers (paper airplanes inadvertently flying backward, for example) provided an opportunity

Using the hallway as a landing strip, the planes collectively provided a physical representation of the class's data.

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to discuss the fact that distance cannot be negative, which is an important idea for middle school students. Often when introduced to negative integers, students think that some distances (i.e., the depth of a hole in the ground or a downhill design) will result in negative numbers. Although these measurements are often represented by negative numbers, students struggle with understanding that representing length as –5 feet to show its relative distance from 0 is not the same as computing a five-foot-deep hole or traveling five feet down a hill.

EXAMINING THE DATA FOR THE UNIQUE PLANES

Once all the paper airplanes had been flown and data collected and reported,

Fig. 2 This activity sheet helped students structure their data collection and analysis.

Flights—Unique Planes

Record the flight data
(in inches)

Flight	Tally	Frequency

Total of flight data in inches: __________
Number of flights: __________

Mean: __________
Median: ________
Mode: __________
Range: __________

Fig. 3 Students were asked to think about the main forces that affect flight and to answer this question: "By folding the same plane and using the same paper, could we control for more variables?"

- The thrust will still be different even if we build the same plane with the same paper because it is not possible for all of us to throw the planes exactly the same way.

- The drag and the weight should be controlled for because we are all making the same exact folds with nothing sticking up like the 'trick planes,' and we are all using the same paper so the weight will be exactly the same.

- The lift also might be better controlled for with the plane but will still be different because depending on how you throw the plane will affect the lift.

the class returned to the hallway to examine the landing strip where the paper airplanes were scattered on the floor. The students looked at the landing strip and the paper airplanes as a physical representation of our data and were asked to agree on three words or phrases that would best describe the Unique Plane data. "Spread everywhere," "scattered," and "dispersed" were chosen. Students were asked to keep this hallway image in mind as we returned to the classroom.

After referencing the class data on the whiteboard, students were asked what they thought we should do first to begin to understand the data set. Many indicated that we should order the data from least to greatest to find the range. We discussed whether we needed to order all the data to determine the range and decided that ordering was unnecessary. We could simply identify the largest and smallest number and determine the range with those two data points. However, the class agreed that ordering the data would be important for general understanding, so they ordered the numbers and checked their work with one another to ensure that they were correct. The ordered data were transferred to an Excel® spreadsheet and projected so that all students could refer to them easily for the remainder of the project.

How best to represent the data in a tally chart was the next task. Given that most of the generated data had few, if any, duplicate entries, students immediately suggested that we needed to "clump" the data to create a tally chart because, in their words, "It would seem silly to have thirty different numbers all with one tally mark next to each one." We ultimately determined that we would create intervals within our data—from 0–10, 11–20, 21–30, and so on—for our tally chart. While students worked in pairs to complete this task, I moved around the room to monitor progress and ask questions. Students were given an activity sheet (see **fig. 2**) to help complete this task.

The tally chart was recorded on a large piece of paper and posted in the front of the room so that all students could refer to the same chart during our discussion. To help them develop a conceptual understanding of *mean* beyond a simple calculation, they were asked to think about what the mean of our data set might be. They responded that it might be in one of the groups that had the most tallies. To stretch their thinking, I asked if the mean would be found in the bottom interval if the group 0–10 had the most tally marks. This question resulted in a rich discussion on the notion that "it depended on the rest of the data set."

Students were asked if anyone's paper airplane flew into the end wall of the hallway ("Yes!") and to think about what might happen to the mean of the data set if the wall constraint was removed. For example, what would happen if the hallway was extended. Students agreed that the mean would go up. When asked what would happen to the mean if the two side walls of the hallway were removed, for example, if we had been in the gym and there were no side barriers, many thought that the mean would go up; others contended it would be pulled down. After some discussion and examples of paper airplanes being able to fly farther away from the landing strip without the side walls, we agreed that we could not predict what would happen to the mean if we removed the boundaries on both sides of the strip.

Students used the remainder of day 2 working in pairs with calculators to complete the activity sheet and compute the mean, median, and mode (see **fig. 3**).

FLYING THE ROCKET PLANE

On day 3, each group of students shared their calculated results for the mean, median, and mode. Once we agreed on our findings, we discussed whose flight was above and below the mean. I reminded them that our data did not represent how far a paper airplane had flown but rather what we had defined as "accurate." Following this discussion, I told students that we would be repeating the paper airplane activity with one important difference: We would all be constructing the same version—NASA's Rocket Plane, considered the "best paper airplane"—and using the same paper. Before folding the Rocket Plane, I asked students to think about the main forces that affect flight and to consider if, by folding the same paper airplane and using the same paper, we could control for more variables. They answered yes. A few of their comments are in **figure 3**.

After considering what variables would be impacted by all students folding the same paper airplane, we discussed how measures of center might look for the data we would be collecting with the Rocket Planes. Students conjectured that the mean would be higher for this set of data because they were convinced that the paper airplanes would fly farther and in a straight line. The students also agreed that the mode and median would also likely be higher than the mode and median from the previous day's data for the same reasons. After this discussion, we folded the Rocket Planes.

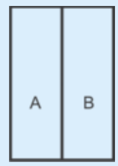
I projected a copy of the paper-folding directions (see **fig. 4**), and we worked as a group to fold the paper airplanes. In addition, I modeled each fold, step by step, in front of the class so that we could complete each fold together. This type of folding activity is quite challenging for some students because many do not have a well-developed spatial sense and it can be frustrating. The students then followed the same process for flying paper airplanes, collecting data, and recording the data on the classroom whiteboard.

EXAMINING THE ROCKET PLANE DATA

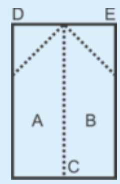
Once all paper airplanes were flown and the data were collected, the entire class returned to the hallway just as before to examine the landing strip where the paper airplanes were

Fig. 4 These instructions were given to students so that planes would be folded consistently before flight data were collected.

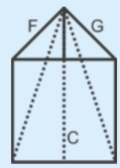
Step 1: Fold the paper in half the long way (hot-dog style). Open the sheet so you see a side A, a side B, and a center line.



Step 2: Fold corners D and E down to meet center line C.

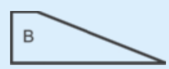


Step 3: Fold sides F and G to meet center line C.

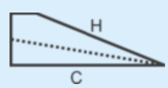


Your plane should look like this:

Step 4: Fold sides A and B together along center line C.



Step 5: Fold wing B in half, side H to line C. Then fold wing A with side G to line C.



Step 6: This is the finished plane.

Fig. 5 Journal entries contained these thoughts on the paper airplane project.

The main thing I learned this week is how all of the data work together. When we talked about "what if one plane flew a lot farther" and how that would make the mean go up or if one didn't go far it might make the mean go down.

I really like the activities we did these past 3 days. Even though we learned what mean, median, and mode were before and how to find them, I feel like I understand them better now. I also liked seeing all the airplanes on the floor all together so we could see the data together or apart and also see the outliers.

scattered on the floor. The students were asked to agree on three word phrases that best described the data represented on the hallway floor. The data were described as "less scattered," "more clustered together," and "more clumped together." Their descriptions of the Rocket Plane data were chosen in comparison with the Unique Plane data. Students were told to keep these words and differences in mind when we returned to the classroom and considered the numerical data.

The analysis of Rocket Plane data did not require as much time for students to complete as the same work the previous day, so we had time at the end of class to reflect on their conjectures about the mean, median, and mode. At the project's conclusion, students were asked to respond to a journal prompt: Discuss in a few sentences what was the most important thing, concept, or idea you feel you better understand after having completed this activity. Please discuss something other than how to fold a "really cool" paper airplane. (See **fig. 5**'s journal responses.)

DISCUSSION

This unit, involving both mathematics and science, allowed my students to experience data collection and data analysis in a way that was meaningful to them. Further, the students were engaged from the beginning to the end of this three-day unit because it involved something they could relate to and were interested in. The fact that we were flying paper airplanes in the hallway and then sitting on the ground measuring distances made the activity all the more entertaining.

Collecting the data in this manner provided a unique opportunity to view the landed paper airplanes as a physical representation of the data in a way that other lessons cannot. This aspect and their descriptors of "spread everywhere," "scattered," and "dispersed" seemed to help students understand the concept of data as either scattered or clustered in a way that I hoped they would retain over time.

REFERENCES

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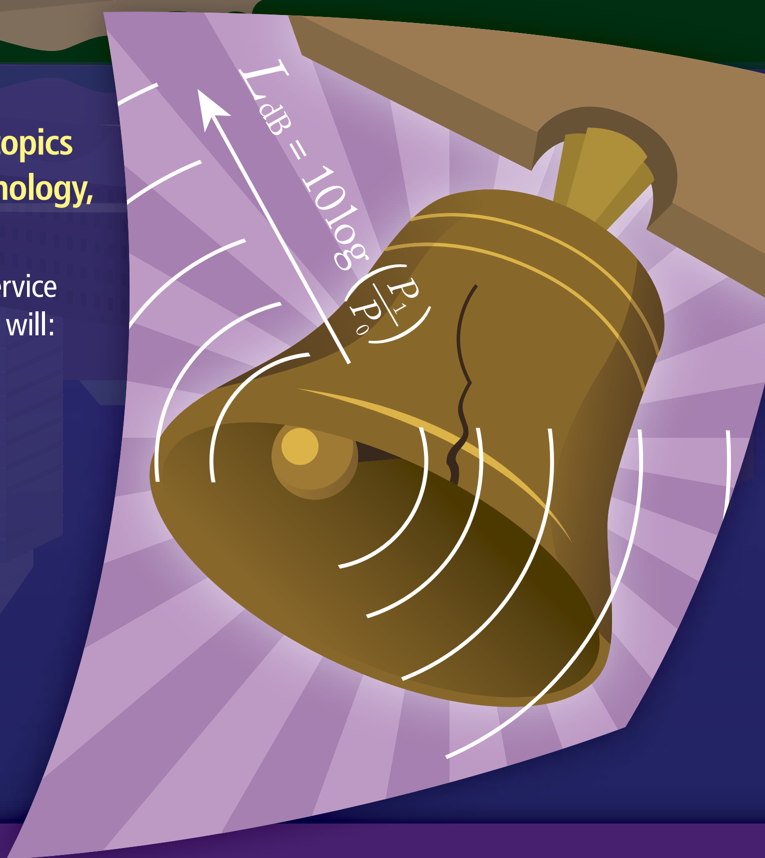

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