GAI into the Common

Two recent sets of guidelines that intersect statistics and complement each other can be used to plot an orderly progression of study.

In education, it is common to set aside older curriculum documents when newer ones are released. In fact, some instructional leaders have encouraged the "out with the old, in with the new" process by asking teachers to turn in all copies of the older document (Balka, Hull, and Miles 2010). Doing so makes sense when the old curriculum document is incompatible with the new. However, when the two documents are complementary, a great deal can be lost by discarding the older curriculum.

At this writing, the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) themselves are new, influential, national curriculum guidelines. Although these standards will replace many current state requirements, some older curriculum documents are useful for helping to implement CCSSM and other middle school curricula.

The GAISE report (*Guidelines for Assessment and Instruction in Statistics Education Report: A Pre-K–12 Curriculum Framework*) (Franklin et al. 2007), for example, contains valuable insight about teaching and learning the statistical content prescribed in CCSSM. This document, endorsed by the American Statistical Association, provides a framework for students' learning progressions that aligns well with CCSSM. CCSSM places a large amount of statistics content in the middle grades, thus charging middle school teachers with the responsibility of delivering a major portion of students' statistics education. We will explain how GAISE (available online

Randall E. Groth and
Anna E. Bargagliotti

SEing Core of Statistics

at http://www.amstat.org/education/gaise/) is an important tool for practitioners who are working to implement CCSSM for statistics and probability in grades 6–8.

CONNECTING GAISE AND THE COMMON CORE

The pre-K–grade 12 GAISE guidelines were written to provide a vision of how data analysis and probability in *Principles and Standards for School Mathematics* (NCTM 2000) can be attained in the classroom. GAISE recommendations span three developmental levels—A, B, and C—corresponding roughly to elementary, middle, and high school, respectively (in lieu of grade bands). Essentially, level A competencies form the foundation for level B, and level B does the same for level C. If students do not study level A content in elementary school, they need to do so in middle school or high school before progressing to levels B and C. At each level, it is recommended that teachers engage students in these four processes of statistical investigation:

1. Formulate questions
2. Collect data
3. Analyze data
4. Interpret results

CCSSM contains ambitious expectations for statistics in grades 6–8. Middle-grades students are to develop an understanding of statistical variability, summarize and describe distributions, use random sampling to draw inferences about populations, draw informal comparative inferences about two populations, investigate chance processes, and investigate patterns of association in bivariate data. These learning expectations for statistics naturally fall under GAISE, thereby allowing practitioners to use GAISE as a roadmap to help implement CCSSM. In addition, GAISE is a compelling supplement to CCSSM because it offers these ideas not contained in CCSSM:

- Pedagogical approaches for statistics
- Meaningful statistical connections
- Developmental trajectories for students' statistical learning
- Enhancement of the curriculum prescribed by CCSSM

What follows in an exploration of each of these features.

PEDAGOGICAL APPROACHES FOR STATISTICS

The following rationale explains why CCSSM writers intentionally avoided prescribing pedagogy:

> The best understanding of what works in the classroom comes from the teachers who spend their days with students. That's why these standards will establish *what* students need to learn, but they will not dictate how teachers should teach. Instead, schools and teachers will decide how best to help students reach the standards. (CCSSI 2011, n.p.)

The lack of pedagogical recommendations in CCSSM means that teachers and schools are responsible for seeking, testing, and refining teaching approaches. This responsibility can be quite challenging in the case of middle school statistics, where much of the content prescribed in CCSSM is new to existing curricula.

GAISE recommends the overarching pedagogical approach of teaching through the process of statistical investigation. For example, one sample activity asks students to investigate which types of music are preferred by students at their grade level (level B, pp. 38–40). As students conduct surveys to address the question, they begin to recognize individual-to-individual variability (CCSSM 6.SP.1, p. 45), since not all respondents answer the survey in the same way. Students also begin to develop strategies for describing data with an appropriate measure of center (CCSSM 6.SP.2, p. 45), not focused solely on the mean. Since data on favorite type of music are categorical, the mode is a useful tool for summarizing students' preferences. Percentages and proportions of students preferring each type of music are also described as ways to help summarize the data.

Mathematics teachers need not bear sole responsibility for helping students attain the learning goals in CCSSM if activities in science classes address them in a complementary manner.

After carrying out the investigation, students can discuss whether the survey results would vary for different populations (CCSSM 7.SP.1, p. 50). As middle school students engage in the investigation and its follow-up, they simultaneously attain learning goals set forth in CCSSM and other curriculum frameworks.

As students expand the scope of their statistical investigations from surveying students in one class to surveying larger populations, random sampling can be introduced as a tool for drawing trustworthy inferences. One of the goals in the seventh-grade CCSSM (7.SP.2) is that students are to "Use data from a random sample to draw inferences about a population with an unknown characteristic of interest" (CCSSM 2010, p. 50). Appreciating the power of random sampling is a nontrivial matter. According to Rubin, Bruce, and Tenney (1991), students often make two types of mistakes when working with random samples:

1. They believe that a random sample should be perfectly representative of the population.
2. They have little trust in random samples because the samples do not contain the entire population.

Students need to reach middle ground between these two extremes. In so doing, they will recognize that the characteristics of random samples will likely vary somewhat from the population but still can be used as a basis for inference.

According to CCSSM, students are to develop sound intuitions about inference from random samples as they "generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions" (CCSSM 2010, p. 50). The music survey example from GAISE sets the stage for this type of activity. Groups of students can choose random samples from their classroom or school population and administer a music preference survey. Once complete, it is possible to compare the samples with one another.

Suppose, for instance, fifteen pairs of students from a class each selected twenty-five students from the entire school population by drawing names from a hat. The students in each random sample could be asked, "What is your favorite type of music?" This question could be accompanied by this set of choices: country, rock, classical, pop, blues, jazz, and other. When the surveys have been completed, each pair of students could compute the proportion of their sample preferring country music. Data from the entire class could then be pooled by having each pair plot the proportion they computed on a dot plot (see **fig. 1**).

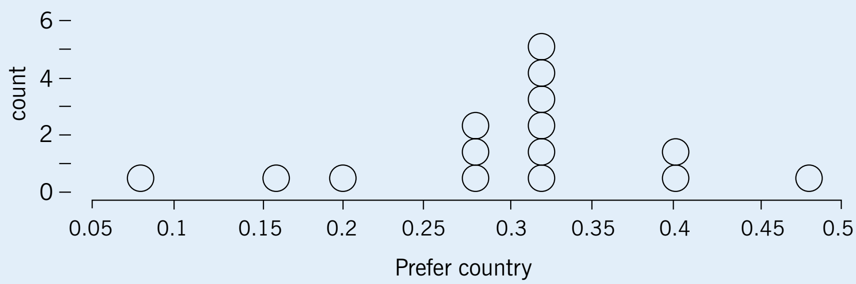
In most cases, the proportions will

vary from one another yet tend to cluster around the actual population proportion. In **figure 1**, for example, the sample proportions tend to cluster around 0.3. The clustering behavior helps illustrate that moderately sized random samples may not capture the population proportion exactly, but are not likely to grossly misrepresent the population. Given a display such as that shown in **figure 1**, students can be asked to form reasonable estimates of the actual proportion of students in the entire school who prefer country music.

The music survey activity also helps lay the groundwork for the study of sampling distributions and hypothesis testing. Level C in the GAISE document (pp. 71–75) provides suggestions about how to extend the activity to address such concepts.

Fig. 1 A dot-plot representation of a random sample can help students generalize about the greater population.

What information does the dot plot contain?



Pairs of students surveyed a random sample of 25 of their peers and placed a dot above the proportion of people in their sampling who prefer country music. In this example, one pair of students found that 20 percent of their sample preferred country music, whereas two pairs found that 40 percent of their sample preferred country music.

MEANINGFUL STATISTICAL CONNECTIONS

The process of statistical investigation outlined in GAISE helps connect statistics done in the classroom to the broader discipline. Rather than producing graphical displays and summary statistics for their own sake, students' work can be closer to the heart of statisticians' work when these activities are performed in the context of investigations. Graphical displays then become tools for revealing patterns and trends in data. Summary statistics become useful measures of center and spread for describing and comparing distributions. Statistical tools, in general, become a means to the end of answering questions during statistical investigations rather than ends in themselves. As students are encouraged to delve into the process of statistical investigation, they can begin to adopt the same modes of inquiry and patterns of thinking used to address authentic problems in practice.

The process of statistical investigation also helps connect statistics to other disciplines. The sciences, in particular, frequently use statistics to address questions of interest. The GAISE process of statistical investigation complements scientific inquiry, which was described in the *National Science Education Standards* (NRC 1996). In particular, the four components of statistical investigation in the GAISE framework (i.e., formulate questions, collect data, analyze data, and interpret results) are implicit in the *National Science Education Standards* characterization of scientific inquiry. Therefore, using the GAISE process of statistical investigation as an overarching pedagogy can parallel and build on what students do in their science classes, as well. In turn, this process helps open the door for communication between mathematics and science teachers about how their classes might jointly address CCSSM for statistics and probability. Mathematics teachers need not bear sole responsibility for helping students attain the learning goals in the CCSSM document if activities in science classes address them in a complementary manner.

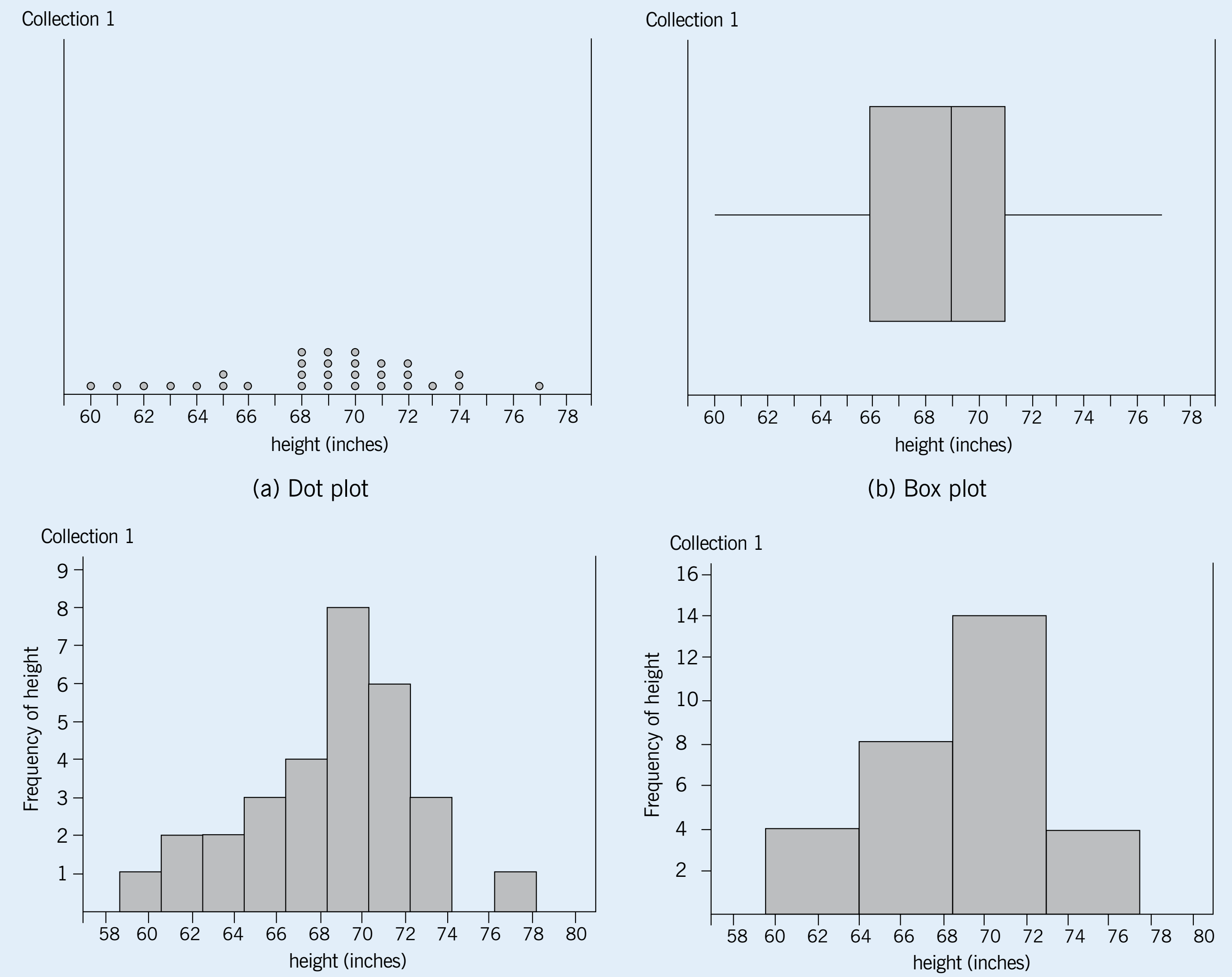
TRAJECTORIES FOR STATISTICAL LEARNING

CCSSM and GAISE provide a set of desired learning outcomes for students. GAISE, however, also provides a developmental perspective on students' attainment of the outcomes. Such a perspective can help teachers "unpack" the expectations in CCSSM and construct coherent sequences of learning experiences for students. To illustrate this point, two standards from CCSSM are discussed below (CCSSM 6.SP.4, 6.SP.5, p. 45), along with implications from GAISE about helping students attain the given standards.

CCSSM 6.SP.4. states that sixth graders should "display numerical data in plots on a number line, including dot plots, histograms, and box plots" (CCSSM, p. 45). GAISE also recommends that students learn to use these data displays. It also acknowledges that histograms and box plots are conceptually more difficult to interpret than dot plots (level B, pp. 44–48).

To illustrate the differences in interpreting dot plots, histograms, and

Fig. 2 A data set, comprising heights of thirty college students, is represented in four different ways, and each pose different challenges to students.



(a) Dot plot

(b) Box plot

(c) Histogram 1

(d) Histogram 2

box plots, consider the graphs in **figure 2**. They all represent the same set of data for heights of thirty students in a college classroom. Even though all four graphs represent the same set of data, they pose different interpretation challenges. The dot plot simply displays each individual height. The box plot, however, displays only the minimum height (the first quartile), the median height (the third quartile), and the maximum height. The two histograms show only the frequencies for select intervals. In addition, the histograms provide different views of the data, depending on the intervals, or "bin widths," chosen.

GAISE recommends that students use dot plots to compare data sets before dealing with box plots and histograms. Although all three types of displays are listed in the same standard in CCSSM, it would not necessarily be appropriate to deal with all three in an introductory lesson. Middle school teachers implementing CCSSM can rely on GAISE for the order in which to introduce these displays.

CCSSM 6.SP.5 states that sixth graders should summarize numerical data sets by using the median and/or mean. In attaining this goal, care must be taken that students do not develop a purely procedural understanding of the median and mean. That is, students need to understand that the two can be used to represent the center of a data set and are not just algorithms to be computed blindly and without purpose. GAISE has a great deal to say about learning progressions for the median and mean.

The intricacies of learning about measures of center described in GAISE suggest that work with the mean and median must occur over several lessons, even though both are mentioned in a single standard in CCSSM. When first working with the median, GAISE recommends that students be given data sets with odd numbers of values (level A, p. 30). This scenario allows them to focus on the median as a location of center before becoming concerned with algorithms for determining the median when an even number of data values exist.

When first working with the mean, GAISE recommends that students work with "fair share" problems (level A, p. 30). One such problem involves gathering eight pencils of different lengths, laying them end to end, and determining how long each section would be if all the pencils were glued together and cut into eight equal sections (see **fig. 3**). This demonstration forms a basis for learning the add-and-divide algorithm for the mean with understanding, since students physically add all lengths together and divide by the number of objects in the set. As students continue to study the mean, they will encounter opportunities to interpret it as the balance point for a set of data (level B, pp. 41–43).

CURRICULUM ENHANCEMENT PRESCRIBED BY CCSSM

CCSSM provides a minimum set of learning expectations and does not claim to address all the appropriate content for a given grade level. Since GAISE contains suggestions about enhancing students' study of statistics, that document can be used as a source of additional appropriate content. Teachers can be alert for opportunities to introduce GAISE-related enrichments as students study statistics.

GAISE recommends that students use dot plots (before box plots and histograms) to compare data sets.

One GAISE-recommended activity alerts students to how changing the wording of a survey question may influence the data obtained (level B, p. 41). For example, when referring to the music preference investigation, respondents may reply differently to closed and open-ended questions. A closed item has predetermined categories:

> Select your favorite type of music from the following list:
> a. Country
> b. Rock
> c. Classical
> d. Pop

An open item may simply ask:

> What is your favorite type of music?

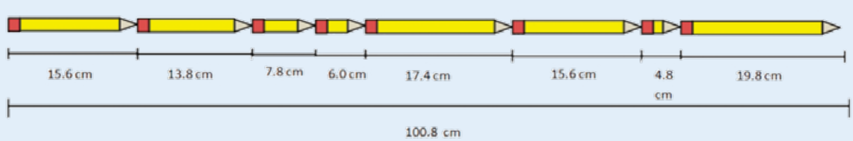
Responses to the open item may include categories not specified in the closed item. Data from the open item may be more difficult to summarize and interpret.

Students can discuss the merits of each type of item and draw on elements of each to form new items, for example, adding the selection "Other (please explain)" to the closed form of the question. As they see how different items elicit different responses, they become more alert to how the wording of opinion polls presented in the media influence the data obtained. Students can also study how leading questions may influence responses (e.g., "Country music is the least popular form of music in school. What is the best type of music to select for our school dance?"). Exceeding the requirements of CCSSM by studying survey item wording gives students a broader view of statistics.

The GAISE guidelines also extend curricula based on CCSSM by

Fig. 3 The "fair share" interpretation of the arithmetic mean, using pencil length, forms a basis for understanding the add-and-divide algorithm.

Lay pencils end to end, and measure:



Determine length per segment if pencils are glued together, and cut into eight equal-length segments ("fair share" per pencil is 12.6 cm):

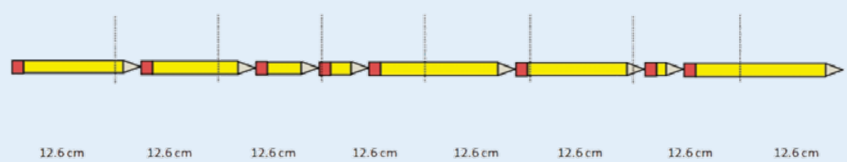
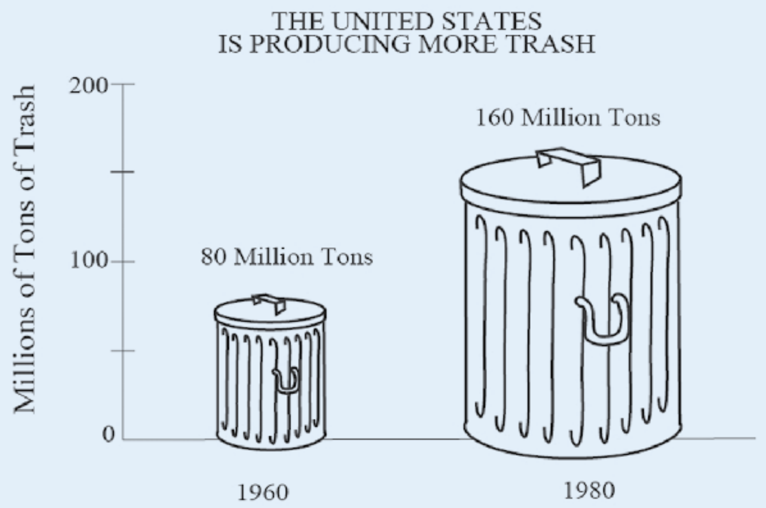


Fig. 4 Pictographs can be misleading if item proportions are not accurate.

Study the pictograph below.



a) Write 1–2 sentences about the information displayed in the graph above.

b) The pictograph shown above is misleading. Explain why.

Source: Eighth-grade NAEP test item, block: 1992-8M15, number 17 (NAEP 2002); National Assessment of Educational Progress (as cited in Harper 2004, p. 341)

explicitly recommending that students study common misuses of statistics. Students who are aware of strategies that use statistics to deceive can develop a skeptical, questioning, and analytic stance toward reports they read in the media and elsewhere. For instance, pictographs are often deceiving because proportions within them are not accurate.

The National Assessment of Educational Progress (NEAP) item shown in **figure 4** is an example of a misleading pictograph. The volume of the larger can is more than twice that of the smaller can, even though the value corresponding to the larger can is exactly twice that of the smaller. In other words, the picture shows the height of the larger can to be exactly twice the height of the smaller can (160 and 80, respectively), however, the volume of the larger can will be larger than twice the volume of the smaller can. As represented in the picture, the larger can appears to double both the height and width of the smaller can, thus increasing the volume by more than twice. If all dimensions are doubled, the area of the trash can *representation* is quadrupled, and the volume of the *actual* trash can is eight times as large.

More examples of potentially misleading graphs can be found in the GAISE document (e.g., level B, pp. 56–58). As students correct flaws in such graphs, they work toward not only standards for graph construction (e.g., CCSSM 6.SP.4, p. 45) but also standards for geometry when constructing graphs involving three-dimensional objects (e.g., CCSSM 6.G.2, p. 45). Therefore, the enrichment activity of analyzing misuses of statistics is not at all a diversion from the goals of CCSSM. Rather, it provides a rich setting for attaining learning goals from multiple standards.

CONCLUSION

The richness of the GAISE document certainly has not been captured in its entirety. However, we do hope that the article presents a frame of reference as well as helpful insights for those charged with implementing CCSSM. Readers are encouraged to examine GAISE for themselves for additional thoughts on implementing CCSSM.

As educators use ideas from GAISE, students can engage in the process of statistical investigation, connect statistics to other disciplines, experience a coherent learning sequence, and adopt a critical stance toward misuses of statistics. Because of GAISE's potential to enhance students' learning in these ways, it is most profitable to view it as a companion document to CCSSM rather than as one supplanted by it. The middle grades bear a great deal of the load for developing students' statistical thinking in CCSSM, and GAISE provides a catalyst for educators to think creatively about attaining its goals.

REFERENCES

Balka, Don S., Ted H. Hull, and Ruth Harbin Miles. 2010. *A Guide to Mathematics Leadership: Sequencing Instructional Change.* Thousand Oaks, CA: Corwin.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.
Common Core State Standards Initiative (CCSSI). 2011. "Myths vs. Facts." http://www.corestandards.org/about-the-standards/myths-vs-facts.
Franklin, Christine, Gary Kader, Denise Mewborn, Jerry Moreno, Roxy Peck, Mike Perry, and Richard Scheaffer. 2007. *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework.* Alexandria, VA: American Statistical Association.
Harper, Suzanne R. 2004. "Students' Interpretation of Misleading Graphs." *Mathematics Teaching in the Middle School* 9 (February): 340–43.
National Assessment of Educational Progress (NAEP). 2002. *The Nation's Report Card Sample Questions Tool Version 2.0.* Washington, D.C.: National Center for Educational Statistics. nces.ed.gov/nationsreportcard/itmrls/.
National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.
National Research Council (NRC). 1996. *National Science Education Standards.* Washington, DC: National Academies Press.
Rubin, Andee, Bertram Bruce, and Yvette Tenney. 1991. "Learning about Sampling: Trouble at the Core of Statistics." In *Proceedings of the Third International Conference on Teaching Statistics,* Vol. 1, edited by David Vere-Jones, pp. 314–19. Voorburg, the Netherlands: International Statistical Institute.


Randall E. Groth, regroth@salisbury.edu, teaches mathematics and mathematics education courses at Salisbury University in Salisbury, Maryland. He is interested in strategies for developing the statistical thinking of students and their teachers. **Anna E. Bargagliotti**, abargagl@lmu.edu., teaches mathematics and statistics at Loyola Marymount University in Los Angeles, California. She is interested in how a teacher's understanding of mathematics and statistics influences teaching practices and in turn influences student achievement.