Candy Jar Problem

Posters

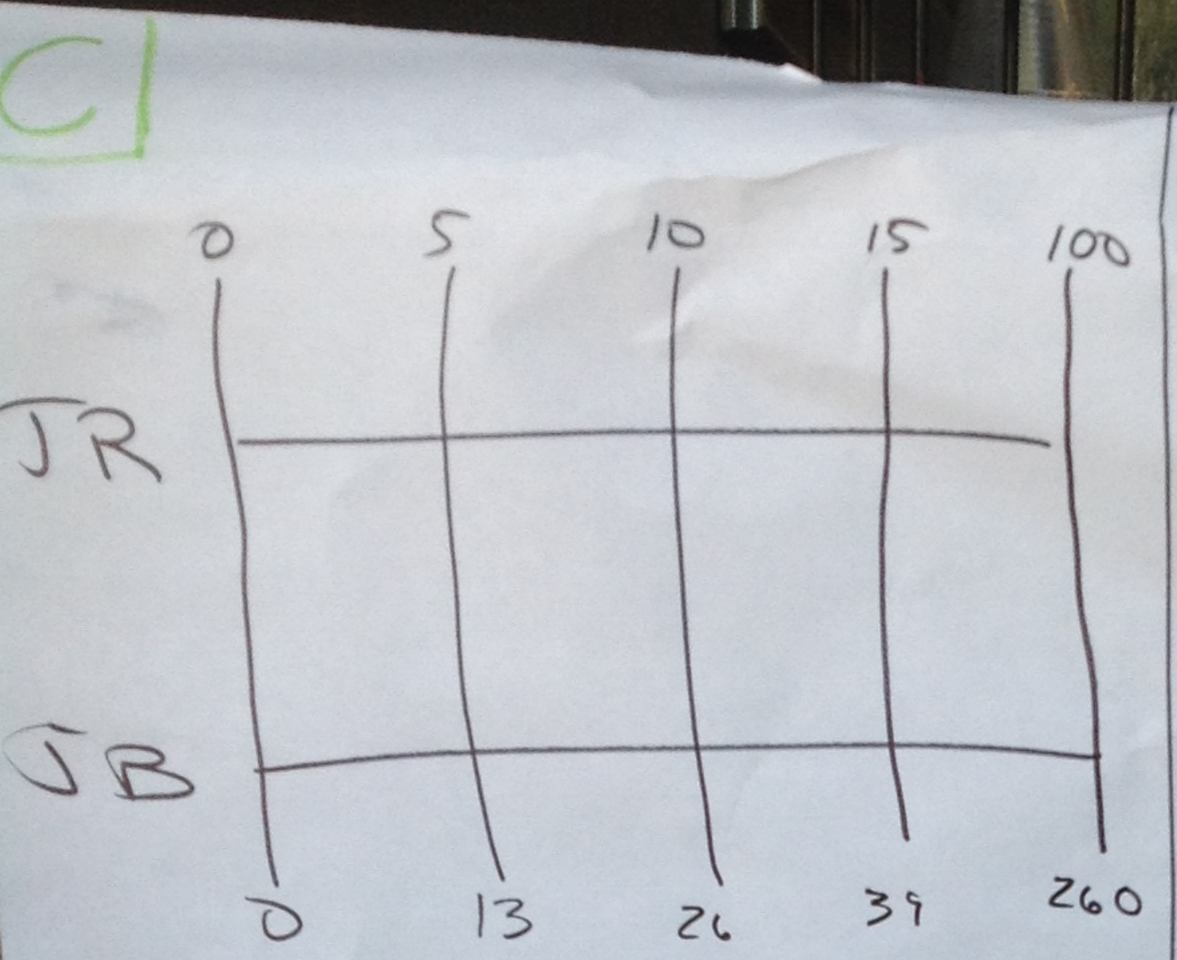
Algebra Readiness Interactive Institute

Professional Development Strand

July, 2014

San Diego, CA

C1



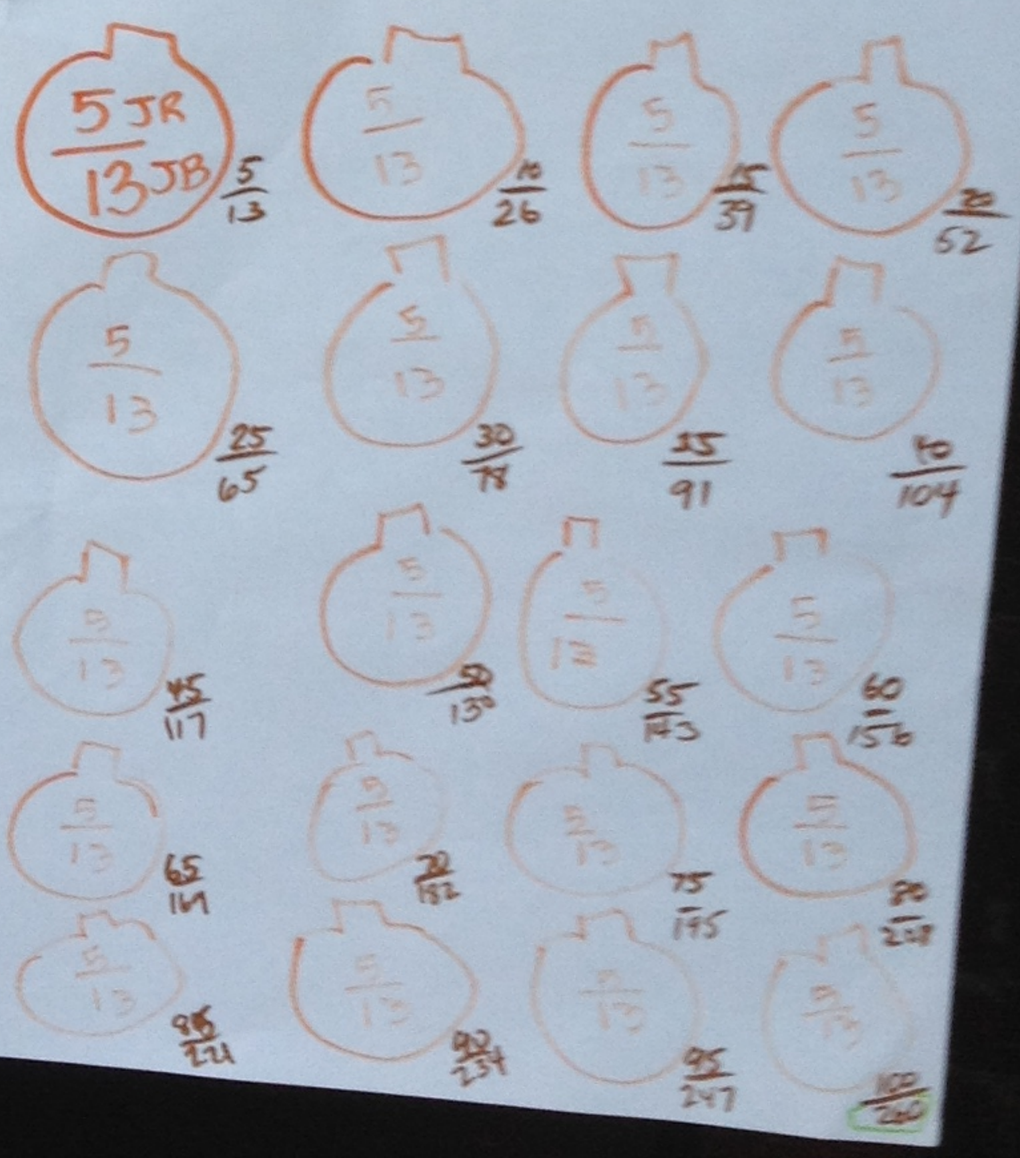
JR $\frac{5}{13}$ $\frac{100}{260}$ ×20

JB 13 = X ×20

13
×20
260 JB

X	Y
5	13
10	26
15	39
20	52
25	65
50	130
100	260

Rule: X · 2.6 = Y



Let x = the number of jawbreakers in the new jar.

Then $\frac{5}{13} = \frac{100}{x}$

$5x = 1300$

$x = 260$

$5k = 100$

$k = 20$

$13k$ = The number of jawbreakers

$13 \cdot 20$ = # of JB

260 = # of JB

Divide then multiply.

$13\overline{)5}$ then multiply by #

"In and Out Machine"

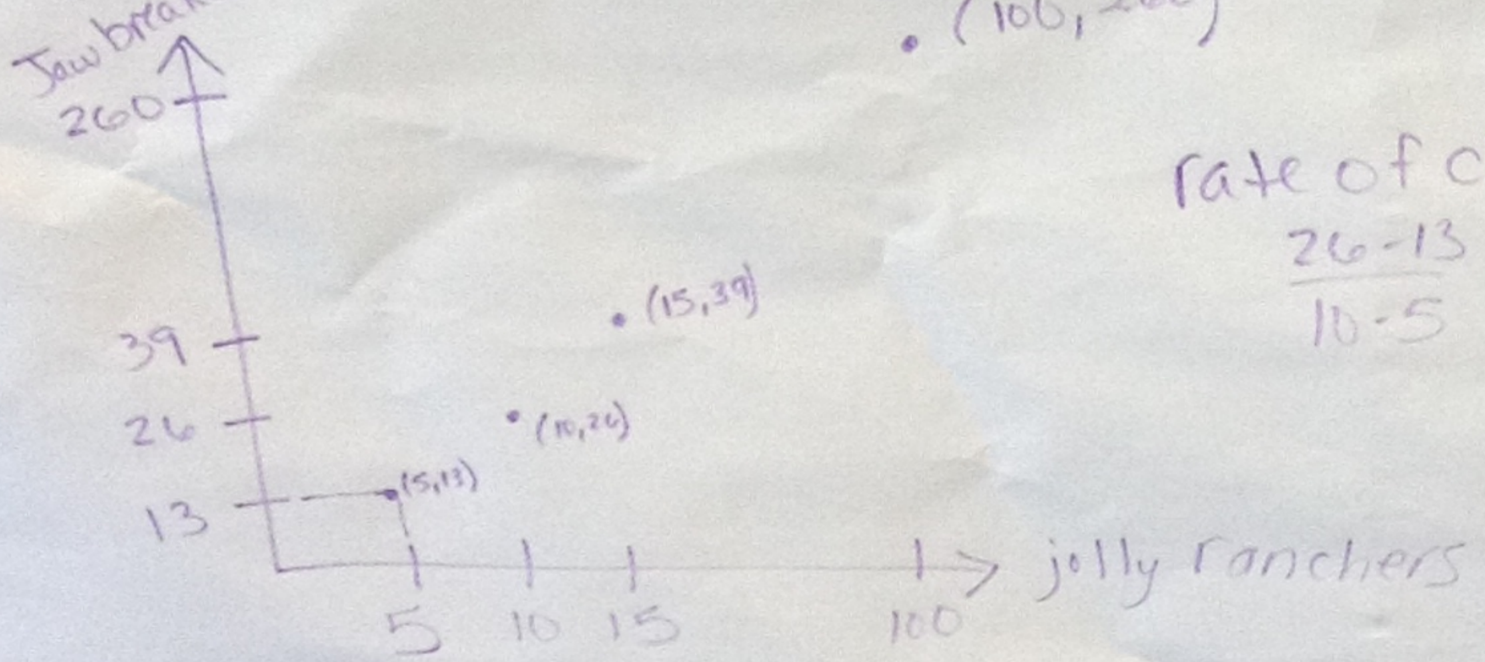
	Old Jar	New Jar
JR	5	100
JB	13	?

Group E

$\frac{5}{13} = \frac{100}{jb}$

1	2	3	...	20
$\frac{5}{13}$,	$\frac{10}{26}$,	$\frac{15}{39}$,	...	$\frac{100}{260}$

$y = \frac{13}{5}x$ $y = kx$ direct variation.

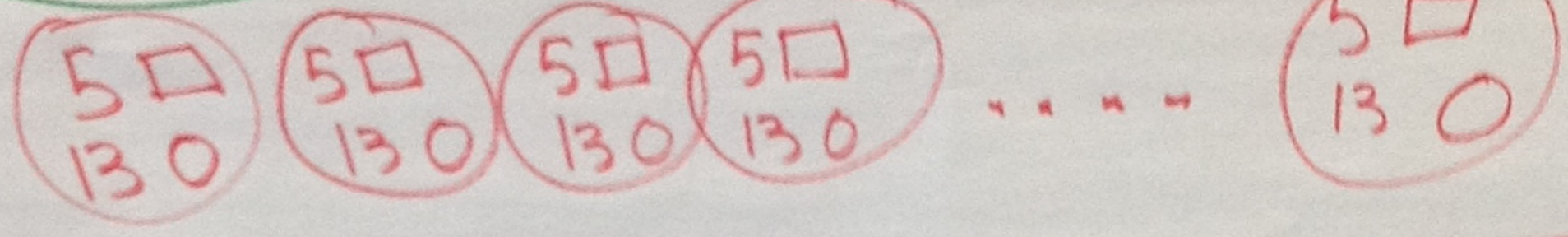


rate of change
$\frac{26-13}{10-5} = \frac{13}{5}$

JR	JB
5	13
10	26
20	52
100	260

Candy Jar Problem

1. **Picture**



2. **Ratio**

$$\frac{5}{13} = \frac{100}{x}$$

$5x = 1300$

$x = 260$

(6) unit scale

JR	5	1
JB	13	2.6

3. **Ratio Table**

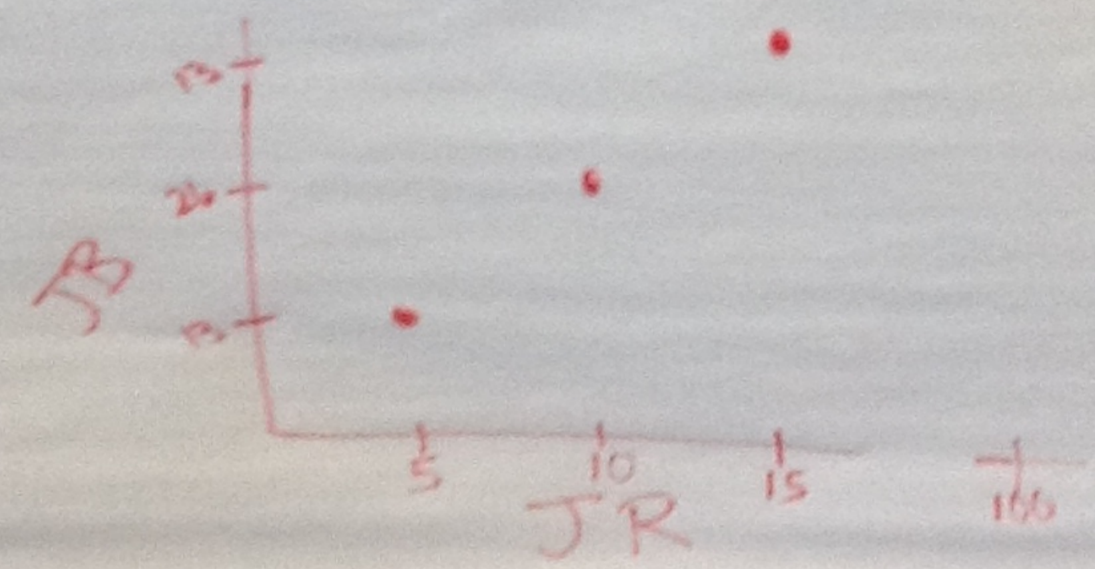
JR	5	100
JB	13	260

(×20 over 5 → 100; ×10 over 13 → 260)

4. **Proportional Table**

JR	JB
5	13
•50	130
100	260

5. **Graph →**



Multiple Strategies

JR: 20 20 20 20 20

JB: 20 20 20 20 20 20 20 20 20 20 20 20 20

260 JB

$$\frac{JR}{JB} \quad \frac{5}{13} \xrightarrow{\times 20} \frac{100}{?}$$

? = 260 JB

$5x = 1300$

$x = 260$

JR: 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 = 100 JR

JB: 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 = 260 JB

Group I

Multiple Strategies

JR: 20 20 20 20 20

JB: 20 20 20 20 20 20 20 20 20 20 20 20 20

100 JR

260 JB

JR / JB

5 / 13 = 100 / ?

×20 ×20

? = 260 JB

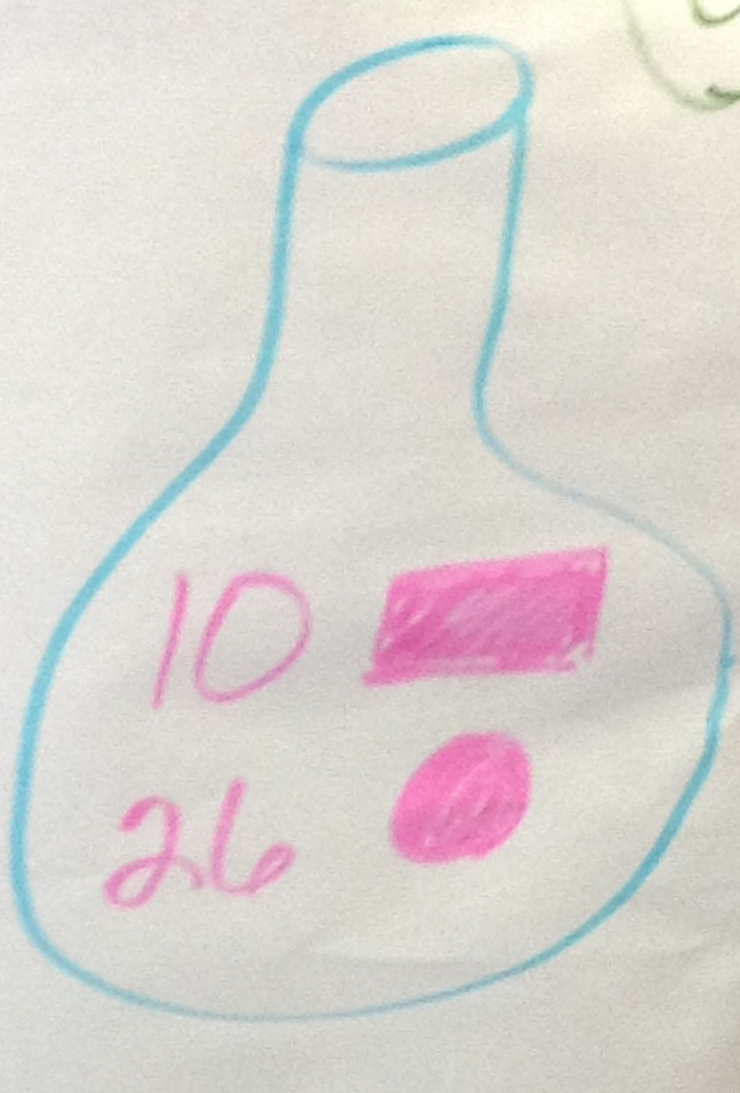
$5x = 1300$

$x = 260$

JR: 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 = 100 JR

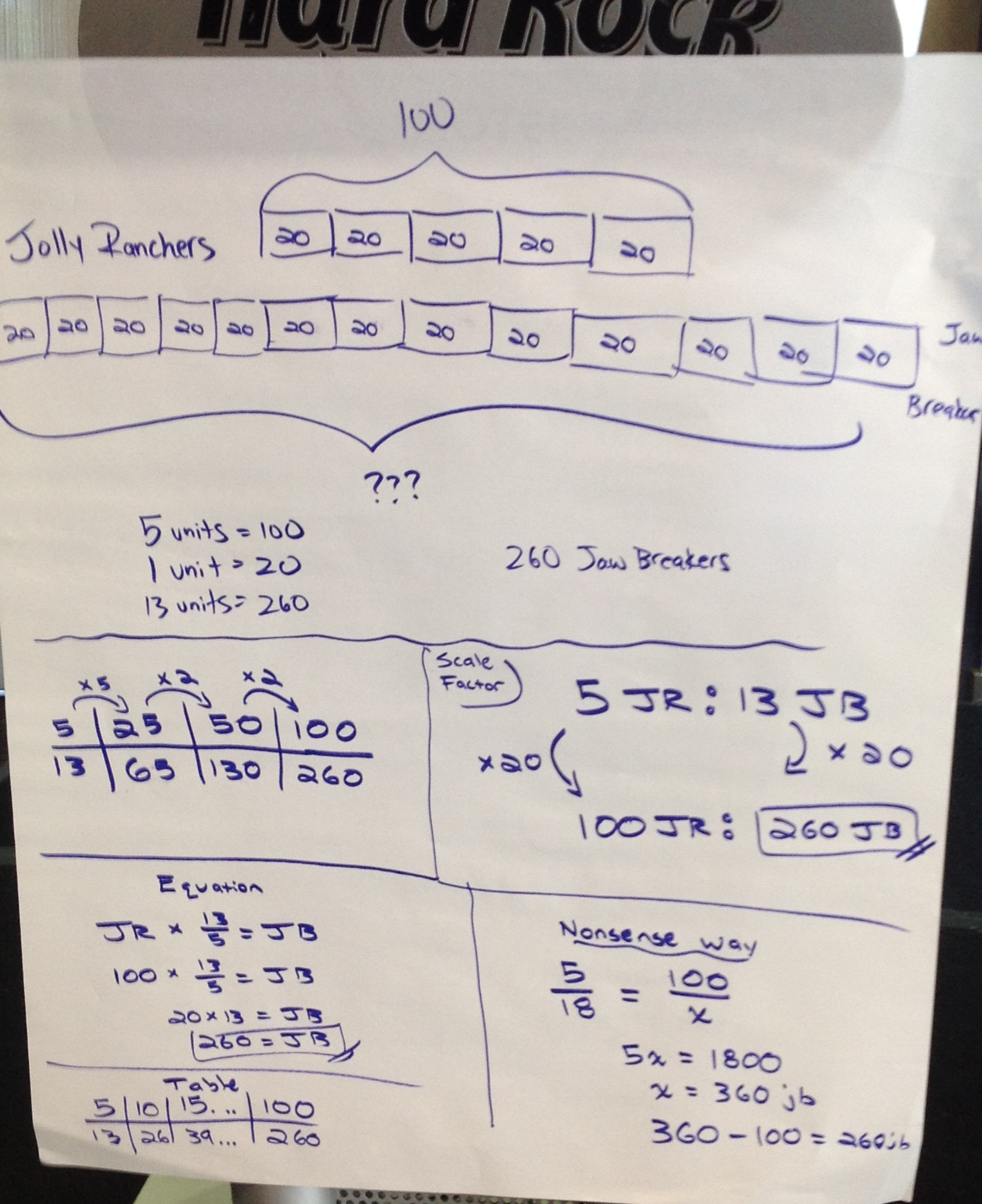
JB: 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 13 = 260 JB

x 2 =



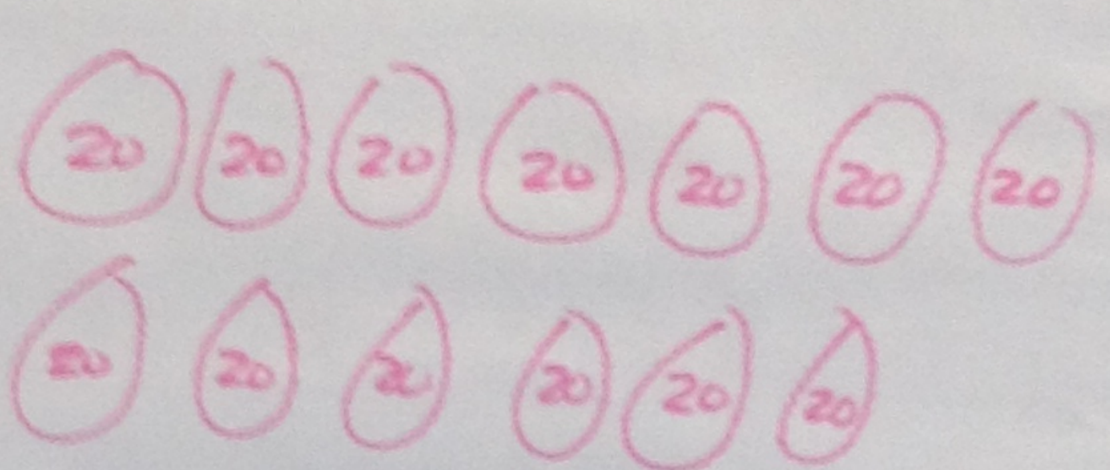
5 □ x 20 = 100 □

130 ○ x 20 = 260 ○


100
Jolly Ranchers
20 20 20 20 20
20 20 20 20 20 20 20 20 20 20 20 20 20
Jaw Breaker
???
5 units = 100
1 unit = 20
13 units = 260
260 Jaw Breakers
×5 ×2 ×2
5 | 25 | 50 | 100
13 | 65 | 130 | 260
Scale Factor
5 JR : 13 JB
×20 ×20
100 JR : 260 JB
Equation
JR × 13/5 = JB
100 × 13/5 = JB
20 × 13 = JB
260 = JB
Table
5 | 10 | 15... | 100
13 | 26 | 39... | 260
Nonsense way
5/18 = 100/x
5x = 1800
x = 360 jb
360 − 100 = 260 jb

20	20	20	20	20

(100)



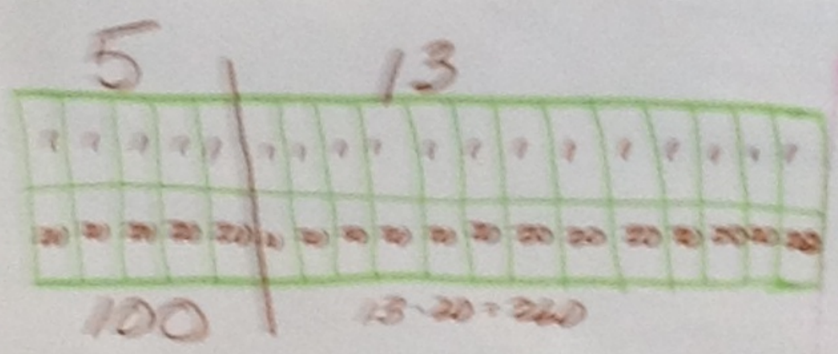
(260)

20	20	20	20	20														
R	R	R	R	R	B	B	B	B	B	B	B	B	B	B	B	B	B	B

100

$13 \cdot 20 = 260$

JR	JB
5	13
10	26
20	52
40	104
80	208
100	260



JR	JB
5	13
10	26
20	52
40	104
80	208

$20 + 80 = 100$

$52 + 208 = 260$

$JR = 5 : 18$

$JB = 13 : 18$

$\frac{5}{18} x = 100$

$x = 100 \left(\frac{18}{5}\right)$

$x = 360$ total

$JR = 100$

$JB = 260$

Strategies

① Proportion (cross multiply)

② Pictoral/Visual representation

③ Ratio Table

④ Scale Factor/Proportion

⑤ Equation

⑥ Repeated addition

Mathematical Teaching Practices

Posters

Algebra Readiness Interactive Institute

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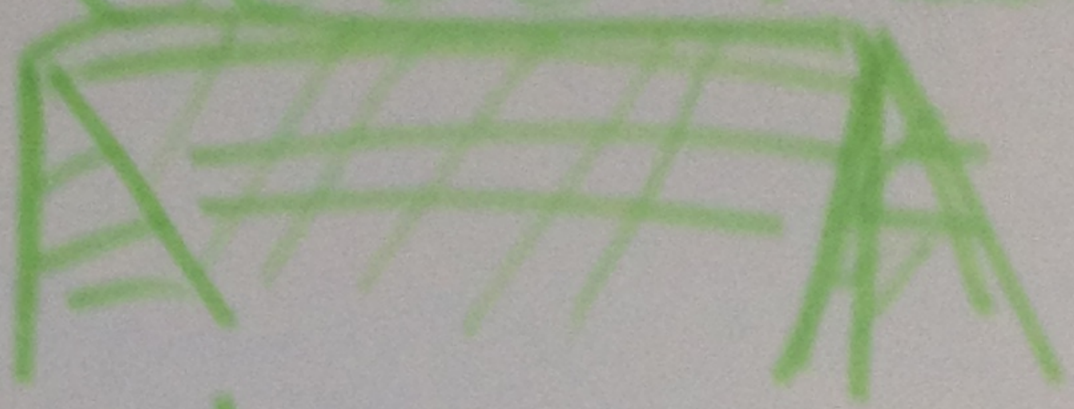
Group E
Practice 1 MTP1

1) Key Elements:
   a) clear goals
   b) learning progressions
   c) goals guide instruction

2) Analysis of Mathematical tasks
   a) use proportional reasoning to determine the number of jaw breakers as compared to the given number of jaw breakers
   b) understanding rate relationships between two quantities. Modeling with mathematics and using known facts to describe the relationship
   c) Gives students opportunities to interpret multiple representations of proportional relationships

#1

GOO-ALS!



Key Elements

Learning goals
Goals w/ learning progressions
Goals guide Instruction

Analysis

Learning Goals (A) Use ratio reasoning → rwp

What students are learning (B)
- Scale factor
- multiple entry points to solve rwp
- x
- patterns/entry points/meta cog

Support Student Learning (C)
Multiple ways to solve a problem
Real world connections

2. Implement Tasks that promote Reasoning and Problem Solving

Analysis of Candy Jar task:

A. level 3 (High level - procedures w/ connections)

- this problem had multiple solution paths

B. This problem could be dropped to level 2 if multiple exposures to same type of problem occurred. Students would find procedure to solve.

C. Challenges:

- matching task to student experience + ability
- tasks w/ multiple entry points

(2) Implement Tasks that Promote Reasoning and Problem-Solving

A

KEY ELEMENTS:

a. Meaningful tasks promote conversation and debate and lead to multiple ways of solving.
b. Need to pick tasks that balance breadth and depth and enable greatest leaps in thinking.
How do we achieve fluency and mastery w/o compromising task complexity?

ANALYSIS of TASK:

a. Could be taught at ~~lowest level~~ lower-level demand
(rotely ~~not~~ set up ratio, cross-multiply, etc)
Can stretch to higher-level demand by explaining answer, or stretching to write a general rule for any circumstance
b. Lowest-level: Present ratio, ask student to solve for x.
Higher levels: -Solve in a different way.
-Present numbers, and ask student to craft story problem.

4 Facilitate Meaningful Math Discourse

(A) Focus on tasks that promote reasoning and problem Solving

(B) Determine how to build on & honor student thinking when ensuring that the goal is being met

(C) Students must have opportunities to talk with, respond to and question one another as a class

Key Elements

(3.) MTP

1. Understanding that students have different learning methods
2. Allow students to explaining their reasoning
3. Making Connections

Analysis of the Task

A. Multiple reps shows the teacher the depth of understanding of the student

B. Diverse way of solving the problem

C. Teachers need to be comfortable w/ multiple approaches and understand the approaches

3. Use + Connect Mathematical Represent

A. Visual representations are tools to support discourse
see structure in mathematics

B. Different representations are not different methods;
the representations are related.

C. Moving between different representations is Ke
in problem solving + teacher moves facilitate

Task Analysis

A. tables, tape diagrams, equations, proportions, visual
drawings
- Misconceptions become more evident

B.

C.

4. Facilitate Meaningful Mathematical Discourse

A. Discussions should not be "show-and-tell" – they should be PURPOSEFUL.

B. Teachers' role is as the facilitator, which requires purpose and PLANNING.

C. Must have a "safe environment" and sense of community so that students are comfortable & confident in sharing.

- Need to anticipate student responses.
- Guage how long to let them struggle.
- Choose which methods to share first.
- Decide which questions and the order in which they should be asked
  ↳ Sequence questions
- Have extension (hip-pocket) questions ready.

Choose which strategies to share & in what order

Solution Order:

- Visual to show multiplicative
- table and/or double number line
- make the chart into (x,y) graph

→ This all depends on what students' responses are.

- This requires teacher to monitor student work, plan questions, decide on a sequence of questions/student work

Extension: Could you ever have exactly 139 Jaw Breakers?

Oooh!

#5. Pose Purposeful Questions

Key Elements

- Purpose is to assess understanding
- Guides students to different thinking
- Teachers model questions < type / pattern

Examples - Types of Questions

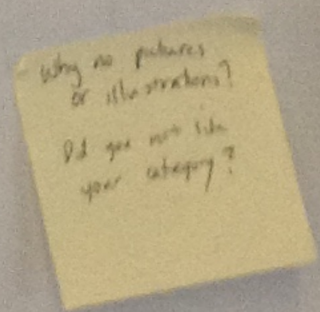
Gathering Info: "What is the relationship between . . . "

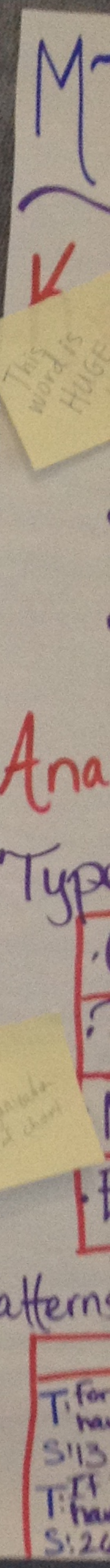
Probing Thinking

Making Thinking Visible

Reflection

*Funneling vs. Focusing → question patterns





MTP #5: Pose Purposeful Questions

Key Elements

- Preplanned open-ended questions
- Anticipate student responses
- Knowledge of students

Analysis of Mathematical Task

Types of Questions

- Gathering Information - How many JR and JB? How do you Know? (Right There)
- Probing/Thinking - How did you get your answer? Are there and other ways to get the same answer?
- Math The Math Visible - How does your table relate to the JR and JBs.
- Encourage Reflection/Justification - How do you Know there are 260 JBs?

Patterns of Questions

FUNNELING	FOCUSING
T: For every 5 JRs how many JBs wou[ld] have?	What relationship did you notice between the JRs and JBs?
S: 13	S There are always more JBs than JRs
T: If I had 10 JRs, how many JBs would you have?	T How many more? Is it the same each time?
S: 26 T: How do you Know	S 8 more. Yes

GROUP E

#6 BUILD FLUENCY FROM CONCEPTUAL UNDERSTANDING

- A. PROVIDE STUDENTS WITH CONCEPTUAL UNDERSTANDING + CONCRETE EXAMPLES BEFORE ALGORITHMS

- B. FLUENCY = OWNERSHIP + ABILITY TO SOLVE A PROBLEM USING A VARIETY OF METHODS

- C. ABILITY TO EXPLAIN + JUSTIFY + REASON HOW THEY OBTAINED THEIR ANSWERS!

AS RELATED TO CANDY JAR PROBLEM

A.
- SHOW THE DIAGRAM
- SHOW TABLE
- SHOW EQUIVALENT RATIOS
- SET UP PROPORTION
- CREATE A GRAPH
- EQUATION FROM GRAPH
- CHECK FOR REASONABLENESS
- JUSTIFY ANSWER/RESULTS

- TIES TOGETHER ALGEBRAIC REPRESENTATIONS

B. RATIO/PROPORTION
UNIT RATE
DIRECT VARIATION
SLOPE

#6

Key Elements

1. Exploration & discussion of mathematical concepts
2. PRACTICE & Rehearse strategies and procedures that are carefully selected and moderate in number
3. Explain & justify reasoning

Use problems that make students want to learn a procedure for efficiency

Analysis of M.T.

A) 1. Multiplication
2. Develop different representations

B) 1. PARTS to Whole (Porportional Reasoning)
2. Analyzing reasonableness of Answer

sup 7

6: Build Procedural Fluency from Conceptual Understanding

A. Fluency is more than just quickly getting the right answer It involves the flexibility to utilize

2. Analyzing reasonableness of results

6. Build Procedural Fluency from Conceptual Understanding

A. Fluency is more than just quickly getting the right answer. It includes the flexibility to utilize different strategies

B. Students need opportunities to practice strategies & procedures to solidify their knowledge.

C. Learning and applying [illegible] strategies to different problems rather than memorizing formulas

Mathematical Tasks

Practices: Using equations to solve equations

Concepts: Pictures / Models, Sort [illegible] as

MTP #7

Key Elements:

- Plan to include tasks in lessons that require kids to struggle productively
- Plan for support based on misconceptions and struggles
- Growth mindset is key!

Analysis of TASK:

A. "Rescue Attempts" → Ask leading questions / ~~correct misconceptions~~ teaching in isolation

B. "Avoid Rescuing" → Ask guiding questions / connect to prior knowledge / collaborative thinking / support and validate multiple approaches / students give hints to others

- "challenges" → valuing all strategies / management

T. Supporting Productive Struggle in Learning Mathematics

KEY ELEMENTS:

Ⓐ "If you are not struggling, you are not learning" – new idea of what it means to do math.

Ⓑ THE ROLE OF THE TEACHER – planning for misconceptions & how to support the student moving foward w/out removing demand.

Ⓒ PROVIDING ~~FEEDBACK~~
- how well did you engage in the stuggle?
- what did you do when you got stuck?

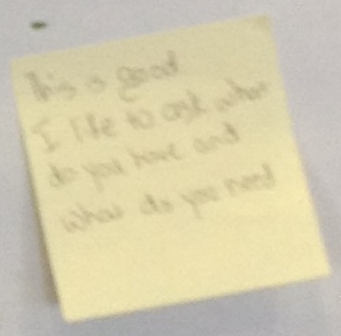
what MP did you engage in? How?

No Pencil Grabbing!

#7 Support Productive Struggle

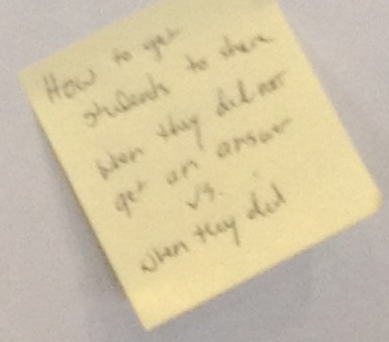
- No rescuing
- growth mindset
- drawing out multiple methods

A. We rescue & scaffold.
"Let me show you..."



B. Ask the student, "What do you know?"
Have students discuss problem w/o paper.
"Coach" with positive reinforcement

C. Reluctant Learners
Changing Teacher Habits
Classroom management



(8) Elicite : Use Evidence of Student Thinking

Key Elements

1) Identify what counts as evidence of student progress toward math learning goals.

- Elicit : gather evidence of student understanding
  : ask questions to clarify ~~mis~~understandings & misconceptions
- Assessment → interpret ~~pupil~~ student thinking, to assess student learning. based on indicators

---

A) Evidence : • That students under ratio in some form.

- Does answer make sense?
- If thinking is mathematically sound?
- Can explain thinking?

B Indicators

- academic vocabulary
- if stuck → use similar but simpler problem
- multiple approaches
- good discussion re: problem
- asking appropriate Qs, critically analyzing others work, comments etc.

MTP 8
Elicit & Use Evidence of Student Thinking

Key Elements

What counts as evidence

Immediate & Long Term Corrective Feedback

Provide / Training using response frames to explain

Potential Ideas

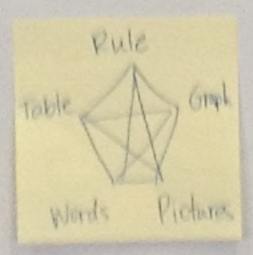
Pictures

Tables

Graphs

Ratios

Equations



Indicators

Process Errors

Computational Errors

Positive indicators also

C

MTP 8
Elicit & Use Evidence of Student Thinking

Key Elements

What counts as evidence

Immediate & Long Term Corrective Feedback

Provide / Training using response frames to explain

Potential Ideas

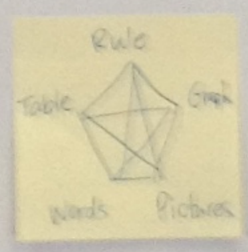
Pictures

Tables

Graphs

Ratios

Equations



Indicators

Process Errors

Computational Errors

Positive indicators a/so

Professional Development Models

Posters

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Professional Development Strand

(I)

Types of PD Models

- Trainer-of-trainer
- Webinars
- Assigned coaches to school sites
- MSPs
- PLCs
- Early release days → district
  site
  grade level
- Facilitated planning using UbD
- Face-to-face (ongoing)
- common planning time
- PD for admin.

GROUP E

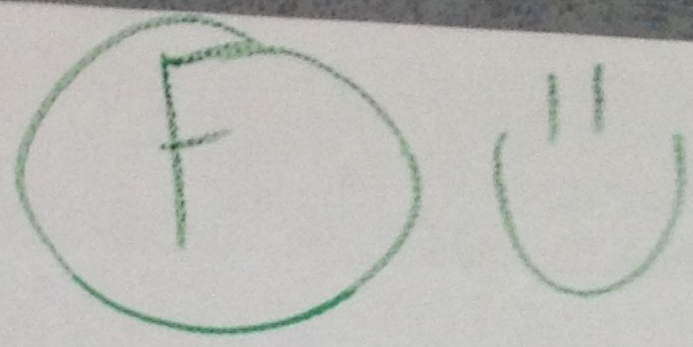
PROFESSIONAL DEVELOPMENT INVOLVING

- CALCULATOR USAGE/IMPLEMENTATION/ TRAINING
- IN-SERVICE DAYS
- AFTER SCHOOL TRAINING
- INSTRUCTIONAL STRATEGIES
- DATA TEAM MEETINGS

PD Models

Group "G"

- Conferences → share
- Technology Training
- Social Media sharing
- Training Trainers
- Regional / State breakdown
- Online ex: Webinars
- Monthly Face-to-Face
- Site-Based
- Workshops / Classroom Coaching



teacher leaders

P.L.C.

coaching sessions

curriculum mapping

instructional design

conferences

workshops

data design teams

Blended Learning (cohort)

informal discussions of best practices

teacher - teacher

vertical planning

Labs

turn keys / train the trainers

PD Models

PD Models

D

- Lesson Studies - development/observations/ reflection
- Algebra Institutes - district level
- Multidistrict
- Online - webinars, modules
- Workshops with consultants
- Blended models (hybrids)
- Coaching - O3's
- PLC's < book studies, problem solving
- University Partners
- Practice Labs
- Coteaching
- Ongoing seminars
- Fellowships

Group B

✓1) Training of Trainers

2) Video Study
   * rubric to evaluate student-teacher
   * Protocol-Tuning

3) Role Playing
   — "How to lead a discussion — Talk Moves" Suzanne Chapin PD
   Classroom Discussion - using Math Talk to help students learn 360 (online Professional development)

✓4) Simple K-12

5) Textbook Selection Process

✓6) PLC (Professional)

7) Japanese Lesson Study

PD Work

Group A

1) Job Embedded
2) Coop lesson planning
3) Consultants
4) 1x week Planning
5) 360° curriculum review

learning sessions

Group C

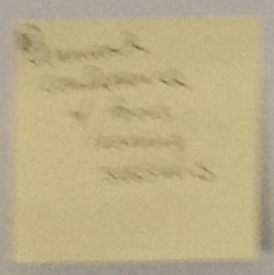
PD Models

+ Collaborative
+ PLC / GLC
+ T.O.T. (Trainer of Trainers)
+ 1 to 1 (Each teacher attends their strand of P.D)
+ Centralized
- Staff Development @ Site
+ On-line P.D.
- SMART Pages Sessions (after school)(voluntary)
+ Coaching Cycles
+ Lesson Demos from Coaches
+ Book Studies
+ Conferences

Observing Evidence of Learning

H group

- Observing for evidence of learning OEL
- content modules
- one-on-one coaching
- PLC's
- book study
- examining student work
- examining/analyzing data
- Two plus one
- Series of PD
- workshop
- task writing sessions
- stand alone PD



GROUP C

PD Models

- Collaborative
- PLC / GLC
- T.O.T. (Trainer of Trainers)
- 1 to 1 (each teacher attends their strand