Workshop 1:
Expressions: Grade 6
NCTM Interactive Institute, 2014

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Introductions

Introduce yourself to others at your table:

- Name
- Where do you teach?
- What grades/classes do you teach?
- How long have you been teaching?

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Introduction

There are others that you might like to meet in your group.

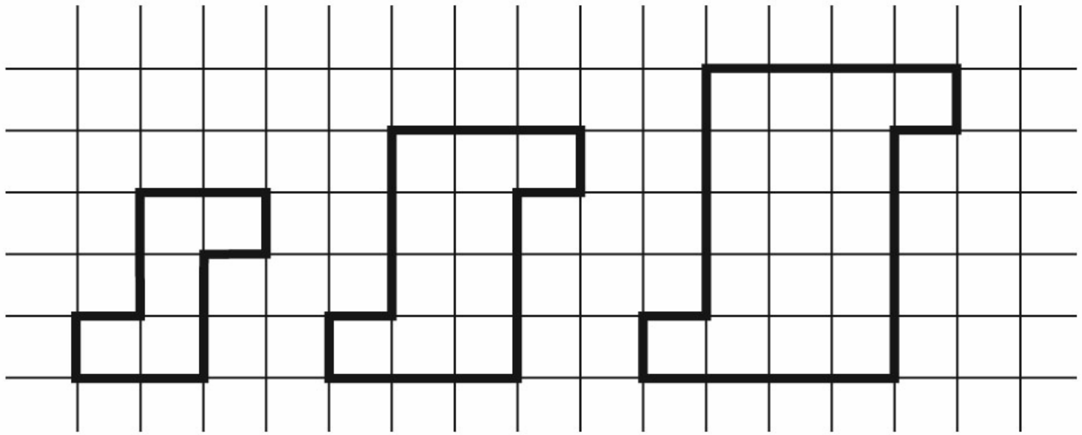
You have 3 minutes to collect as many signatures as you can on the Getting to Know You sheet.

Common Core Standards

This session will address the following:

6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.4	Identify when two expressions are equivalent.
6.EE.9	Write an equation to express one quantity (dependent variable) in terms of the other quantity (independent variable).

Zany Zs



The first three figures in a pattern of tiles are shown above.
The pattern of tiles contains 50 figures.

- Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Explain the reasoning that you used to determine this information.
- Write a description that could be used to define any figure in the pattern.

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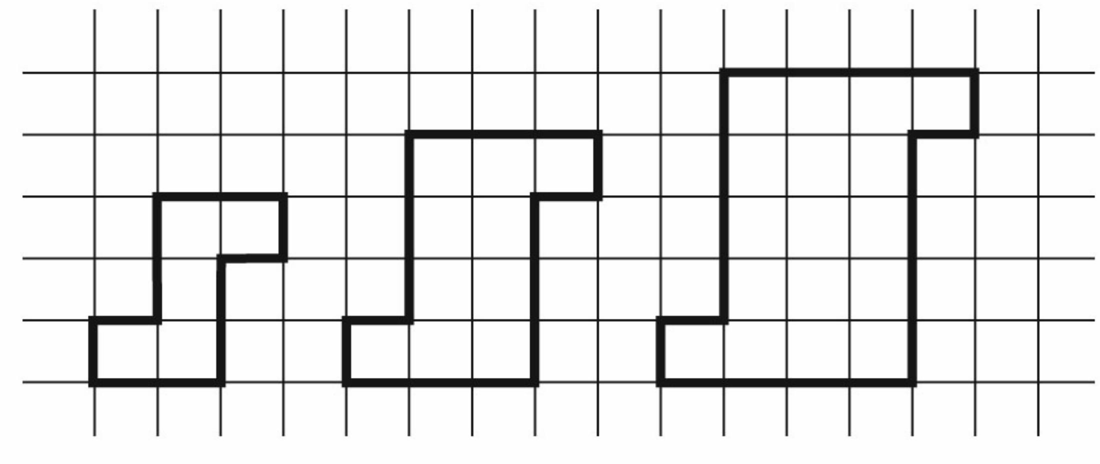
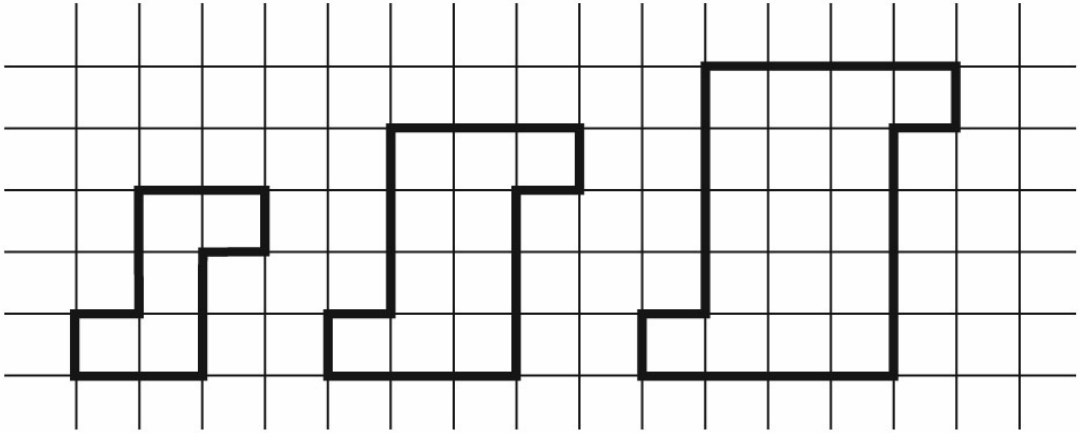
Zany Zs



In your pair, use a variable to write 2 different ways to describe your generalization.

Zany Zs



Share your expressions with your table.

Zany Zs

Student generalizations

n = figure number

$$(n + 1)^2 + 1$$

$$n^2 + 2n + 2$$

$$(n + 2)^2 - 2(n + 1)$$

$$(n + 1)(n + 2) - n$$

Zany Zs

Variant and Invariant Quantities

- Connect the change that occurred from one figure to the next in a recursive manner.
- Use recursive model to write an explicit expression.
- Simplify expression.

Zany Zs

Variant and Invariant Approach

$$n(n + 2) + 2$$

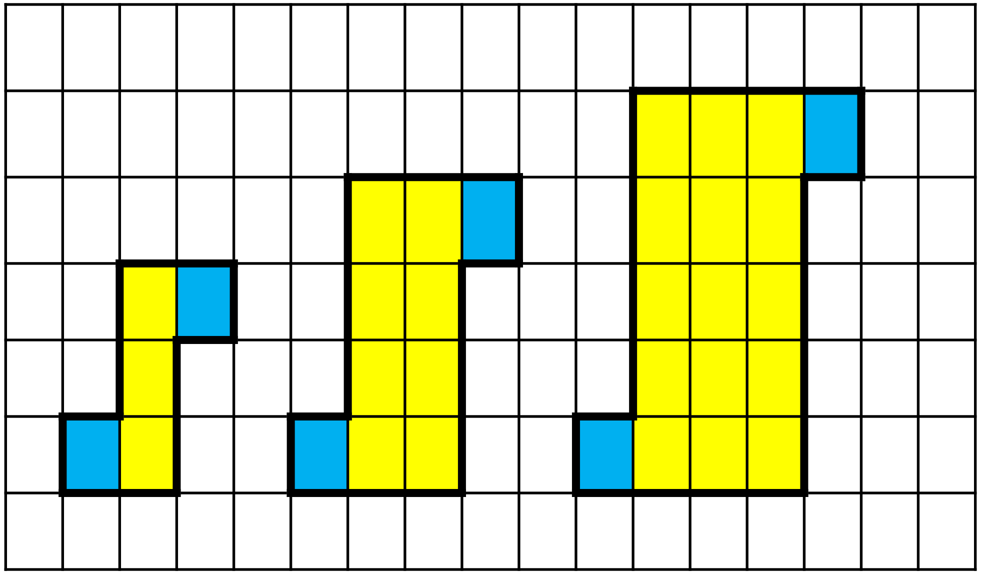


Figure Number	Number of Tiles
1	1 x 3 + 2
2	2 x 4 + 2
3	3 x 5 + 2
4	4 x 6 + 2

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Zany Zs

Variant and Invariant Approach

Figure Number (*n*)	Number of Tiles	
1	1 x 3 + 2	
2	2 x 4 + 2	(1 + 1) x (3 + 1) + 2
3	3 x 5 + 2	
4	4 x 6 + 2	
…	…	
n		

Zany Zs

Variant and Invariant Approach

Figure Number (n)	Number of Tiles	
1	1 x 3 + 2	
2	2 x 4 + 2	(1 + 1) x (3 + 1) + 2
3	3 x 5 + 2	(1 + 2) x (3 + 2) + 2
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…	…	…
n		

Zany Zs

Variant and Invariant Approach

Figure Number (n)	Number of Tiles	
1	1 x 3 + 2	
2	2 x 4 + 2	(1 + 1) x (3 + 1) + 2
3	3 x 5 + 2	(1 + 2) x (3 + 2) + 2
4	4 x 6 + 2	(1 + 3) x (3 + 3) + 2
…	…	…
n		

Zany Zs

Variant and Invariant Approach

Figure Number (*n*)	Number of Tiles	
1	1 x 3 + 2	(1 + 0) x (3 + 0) + 2
2	2 x 4 + 2	(1 + 1) x (3 + 1) + 2
3	3 x 5 + 2	(1 + 2) x (3 + 2) + 2
4	4 x 6 + 2	(1 + 3) x (3 + 3) + 2
…	…	…
n		

Zany Zs

Variant and Invariant Approach

Figure Number (n)	Number of Tiles	
1	1 x 3 + 2	(1 + 0) x (3 + 0) + 2
2	2 x 4 + 2	(1 + 1) x (3 + 1) + 2
3	3 x 5 + 2	(1 + 2) x (3 + 2) + 2
4	4 x 6 + 2	(1 + 3) x (3 + 3) + 2
…	…	…
n		$(1 + (n - 1)) \times (3 + (n - 1)) + 2$

Zany Zs

Variant and Invariant Approach

Figure Number (n)	Number of Tiles	
1	$1 \times 3 + 2$	$(1 + 0) \times (3 + 0) + 2$
2	$2 \times 4 + 2$	$(1 + 1) \times (3 + 1) + 2$
3	$3 \times 5 + 2$	$(1 + 2) \times (3 + 2) + 2$
4	$4 \times 6 + 2$	$(1 + 3) \times (3 + 3) + 2$
…	…	…
n	$n(n + 2) + 2$	$(1 + (n - 1)) \times (3 + (n - 1)) + 2$

Zany Zs

As a table group,
use variant and invariant quantities
to derive the generalized expression for:

$$n^2 + 2(n + 1)$$

$$(n + 1)^2 + 1$$

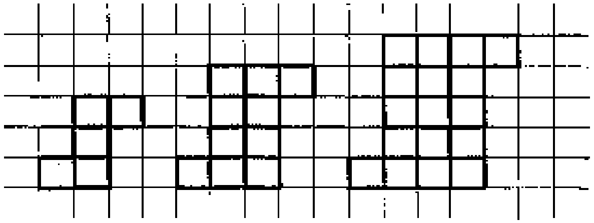
$$n^2 + 2n + 2$$

$$(n + 2)^2 - 2(n + 1)$$

$$(n + 1)(n + 2) - n$$

Student Performance

9. The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

How do you think 4th, 8th, and 12th graders would do on a task like this? Talk about it at your tables.

Student Performance

8th grade, on a similar item: 7.4% correct

12th grade, 19.54% correct

Variables

- In the Zany Zs task, you used a variable.
- At your table, describe different ways that we can think about a variable. In general, what can it represent?

Use of Variable

- Variables have many different meanings, depending on context and purpose.
- Using variables permits writing expressions whose values are not known or vary under different circumstances.
- Using variables permits representing varying quantities. This use of variables is particularly important in studying relationships between varying quantities.

(*Developing Essential Understandings of Expressions, Equations, and Functions, 6–8,* [NCTM 2011], p. 9)

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Use of a variable

Suppose that Cassie and Winston are student members of the service club and are planning to participate in the 5K run. Cassie runs at a constant rate of 11.2 km per hour (186.7 m per minute) and Winston runs at a constant rate of 13.7 km per hour (228.3 m per minute).

- What are some of the variables?
- What would it mean to think about the variables as unknowns?
- What would it mean to think about the variables as changing quantities?

Lloyd, Gwendolyn, Herbel-Eisenmann, B., & Star, Jon. (2011). *Developing essential understanding of expressions, equations, and functions for teaching mathematics in grades 6–8. Reston, VA: NCTM>*

Use of a variable

- When you think about your curriculum materials, what is the most often used role of a variable?
- How does that impact student understanding of variable?

How would students respond?

Bart said, "$t + 3$ is less than $5 + t$."

Always true -- Sometimes true -- Never true

Explain your answer.

Cupcakes cost c cents each and doughnuts cost d cents each. I buy 4 cupcakes and 3 doughnuts. What does $4c + 3d$ stand for or represent?

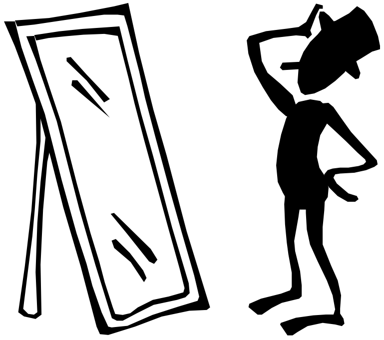
Dougherty, B. J., Foegen, A., Deleeuw, W., Olson, J. (2013). *Progess monitoring tools: conceptual.* Ames, IA: Iowa State University, University of Missouri.

Reflection

Individual writing – Pair discussion

In the Zany Zs task,

- What was the role of the teacher (instructor)?
- What was the role of the student?



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Reflection

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

(*Principles to Actions: Ensuring Mathematical Success for All* [NCTM 2014], p. 29)

Grade 6:
Expressions & Equations
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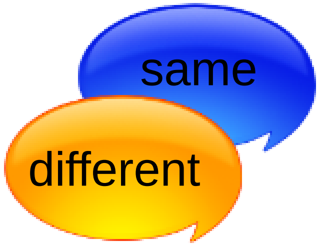
Introductions…

With your table, decide the similarities and differences about the four phrases below:



- **Numerical expression**
- **Numerical equation**
- **Algebraic expression**
- **Algebraic equation**

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Common Core Standards

This session will address the following:

6.EE.2	Write, read, and evaluate expression in which letters stand for numbers.
6.EE.4	Identify when two expressions are equivalent.
6.EE.9	Write an equation to express one quantity (dependent variable) in terms of the other quantity (independent variable).
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$.

Algebra Magic

- Think of a number.
- Multiply the number by 3.
- Add 8 more than the original number.
- Divide by 4.
- Subtract the original number.

Compare your answer to others at your table.

Why did this happen? Show in 2 different ways.

Algebra Magic

What could be done to the steps in order to get the number you started with?

- Think of a number.
- Multiply the number by 3.
- Add 8 more than the original number.
- Divide by 4.
- Subtract the original number.

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Writing Expressions

- Enter the first three digits of your phone number.
- Multiply by 80.
- Add 1.
- Multiply by 250.
- Add the last four digits of your phone number.
- Repeat the above step.
- Subtract 250.
- Divide by 2.

Describe the number you have.

How did the problem work?

Algebra Magic

Which of the following steps can you reverse without changing the result? Why?

1) Think of a number.
2) Subtract 7.
3) Add 3 more than the original number.
4) Add 4.
5) Multiply by 3.
6) Divide by 6.

Algebra Magic

The following trick is missing the last step.

- Think of a number.
- Take its opposite.
- Multiply by 2.
- Subtract 2.
- Divide by 2.
- ??????????

Decide what the last step should be for the given condition so final result is:

a) One more than original number.
b) Opposite of original number.
c) Always 0.
d) Always -1.

Algebra Magic

Make up a separate algebra magic trick with at least five steps that will meet one of the bullets listed below:

- Final result is one more than the original number.
- Final result is 0.
- Uses all four operations.
- Result is same, whether steps are done backwards or forward.

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Interpreting Algebraic Expressions

What errors might occur as students translate the following sentences into algebraic expressions?

- Multiply ***n*** by 5 then add 4.
- Add 4 to ***n*** then multiply your answer by 5.
- Add 4 to ***n*** then divide your answer by 5.
- Multiply ***n*** by ***n*** then multiply your answer by 3.
- Multiply ***n*** by 3 then square your answer.

Matching Expressions, Words, Tables, & Areas

Work collaboratively with your tablemates.

- Match cards to make a complete set with an equivalent expression, description, table, and area cards.

- If there is not a complete set, make a card for the missing type(s) with one of the blank cards.

Matching Expressions, Words, Tables, & Areas

Large group discussion:

- Which, if any, of the groups of expressions are equivalent to each other? How do you know?
- What will students learn as a result of this activity?
- What challenges might students encounter with this activity?

Expressions to Equations

$$8 + 4 = \square + 7$$

What responses do students give for box?

Operational vs Relational

“answer” vs “equivalence”

Equality

The notion of equality

is surprisingly complex,

is often difficult for students to comprehend, and

should be developed

throughout the curriculum.

Equality

- Many students at all grade levels have not developed adequate understanding of the meaning of the equal sign.

"Limited conception of what the equal sign means is one of the major stumbling blocks in learning algebra. Virtually all manipulations on equations require understanding that the equal sign represents a relation."

Carpenter, Thomas, Megan Franke, and Linda Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School*. 2003

Equality

Is the number that goes in the box the same number in the following two equations?

$2 \text{ X } \square + 15 = 31$ $\qquad$ $2 \text{ X } \square + 15 - 9 = 31 - 9$

In the equation $\square + 18 = 35$, the number that goes in the box is 17. Can you use this fact to figure out what number goes in this box:

$$\square + 18 + 27 = 35 + 27$$

Transitioning to Relational Thinking

- No calculators – No computations
- Use relational thinking to justify answer.

True or False:

$471 - 382 = 474 - 385$

$674 - 389 = 664 - 379$

$583 - 529 = 83 - 29$

$37 \times 54 = 38 \times 53$

$5 \times 84 = 10 \times 42$

$64 \div 14 = 32 \div 28$

$42 \div 16 = 84 \div 32$

Transitioning to Relational Thinking

- No calculators – No computations
- Use relational thinking to justify answer.

What is the value of variable?

$73 + 56 = 71 + d$

$67 – 49 = c – 46$

$234 + 578 = 234 + 576 + d$

$94 + 87 – 38 = 94 +85 – 39 + f$

$92 – 57 = 94 – 56 + g$

$68 + 58 = 57 + 69 – b$

$56 – 23 = 59 – 25 – s$

Relational Thinking

What properties are important
to developing relational thinking with students?

$a + 0 = a$ $\qquad$ $a - 0 = a$

$a \times 1 = a$ $\qquad$ $a \div 1 = a$

$a + b = b + a$ $\qquad$ $a \times b = b \times a$

$a + b = (a + n) + (b - n)$ $\qquad$ $a + b = (a - n) + (b + n)$

$a - b = (a + n) - (b + n)$ $\qquad$ $a - b = (a - n) - (b - n)$

$ab = (na)\,(\frac{1}{n}b)$ $\qquad$ $\frac{a}{b} = \frac{na}{nb}$

Equality Sign Caution

3 + 5 = 8 + 2 = 10 + 5 = 15

Equality strings
written by students (and teachers!)
provide opportunity to discuss
meaning of equal sign and its proper use.

3 + 5 = 8

8 + 2 = 10

10 + 5 = 15

Interpreting Equations

Which is greater, x or y? Explain your reasoning.

y* = 4*x

x is greater because it's multiplied by 4.

It depends what x and y are.

***y* is greater because it is four times the size of *x*.**

Interpreting Equations

Let ***e*** represent the <u>number</u> of eggs.

Let ***b*** represent the <u>number</u> of egg boxes.

There are 6 eggs in each box.

Find an equation linking ***e*** and ***b***.

$b = 6e$

$e = 6b$

Interpreting Equations

Let ***e*** represent the cost of an egg.

Let ***b*** represent the cost of a box of eggs.

The price per egg is the same whether you buy them separately or in a box.

Find an equation linking ***e*** and ***b***.

$b = 6e$

$e = 6b$

Interpreting Equations

Working together at your tables:

- Match an equation card with a statement card.
- Explain/challenge reasoning.
- Use blank cards to write equation or statement cards so that each card is grouped with at least one other card.

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Solving Equations

An equation states that two expressions are equivalent for certain values of a variable.

Equations become useful in investigating relationships between two expressions.

Tables and graphs can begin the investigation before applying series of symbolic transformations.

Solving Equations

- Kids Help Club has one-time enrollment fee of \$20 and a \$10 monthly membership dues.
- United Service Club has one-time enrollment fee of \$50 and a \$5 monthly membership dues.

Create a table for 12 months and a general expression for each club.

Solving Equations

Kids Help Club

Month	Cost	Ordered Pairs
0	20	(0, 20)
1	30	(1, 30)
2	40	(2, 40)
3	50	(3, 50)
4	60	(4, 60)
5	70	(5, 70)
6	80	(6, 80)
7	90	(7, 90)
8	100	(8, 100)
x	$10x + 20$	$(x, 10x + 20)$

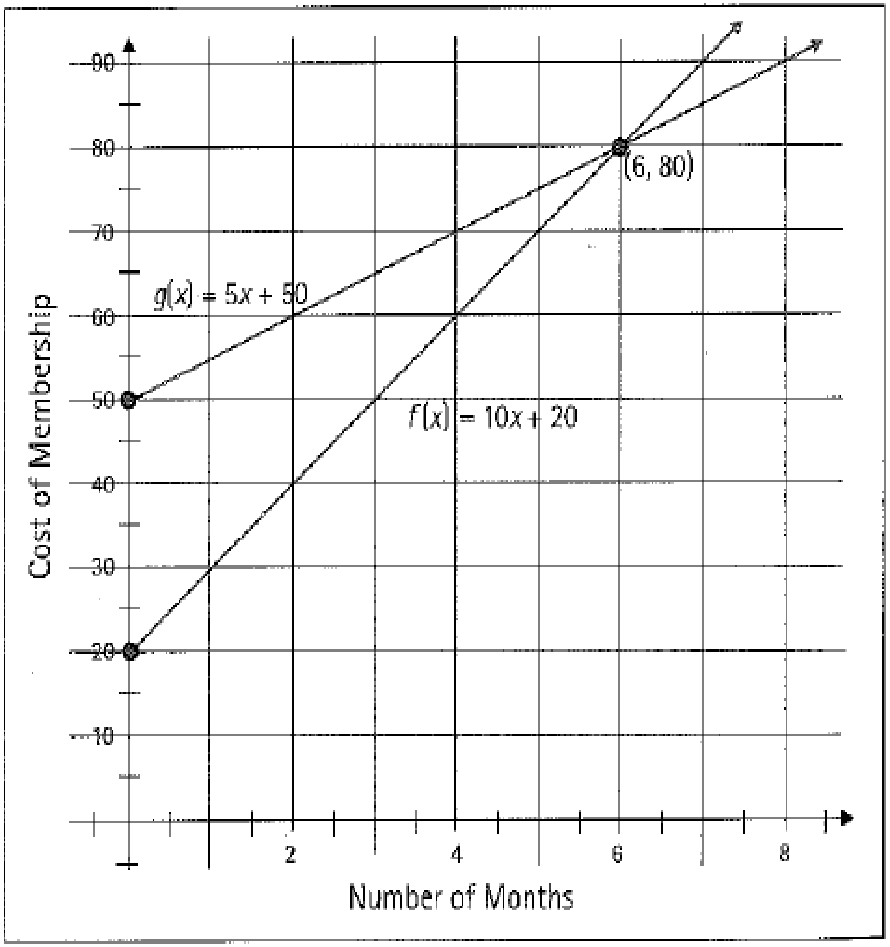
United Service Club

Month	Cost	Ordered Pairs
0	50	(0, 50)
1	55	(1, 55)
2	60	(2, 60)
3	65	(3, 65)
4	70	(4, 70)
5	75	(5, 75)
6	80	(6, 80)
7	85	(7, 85)
8	90	(8, 90)
x	5x + 50	(x, 5x + 50)

Solving Equations

- Graph ordered pairs for two expressions:
  $10x + 20$ and $5x + 50$
- Graph shows solution to equation
  $10x + 20 = 5x + 50$
- Solution is point of interception (6, 80)

  After 6 months, both clubs will have paid $80.



Developing Essential Understanding of Expressions, Equations & Functions Grades 6-8 (p. 34)

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Solving Equations Strip Diagram Method

- Helps students conceptualize the characteristics of the problem to solve

  Make sense of variable to represent unknown quantity

- Helps students formulate an algebraic equation to solve the problem

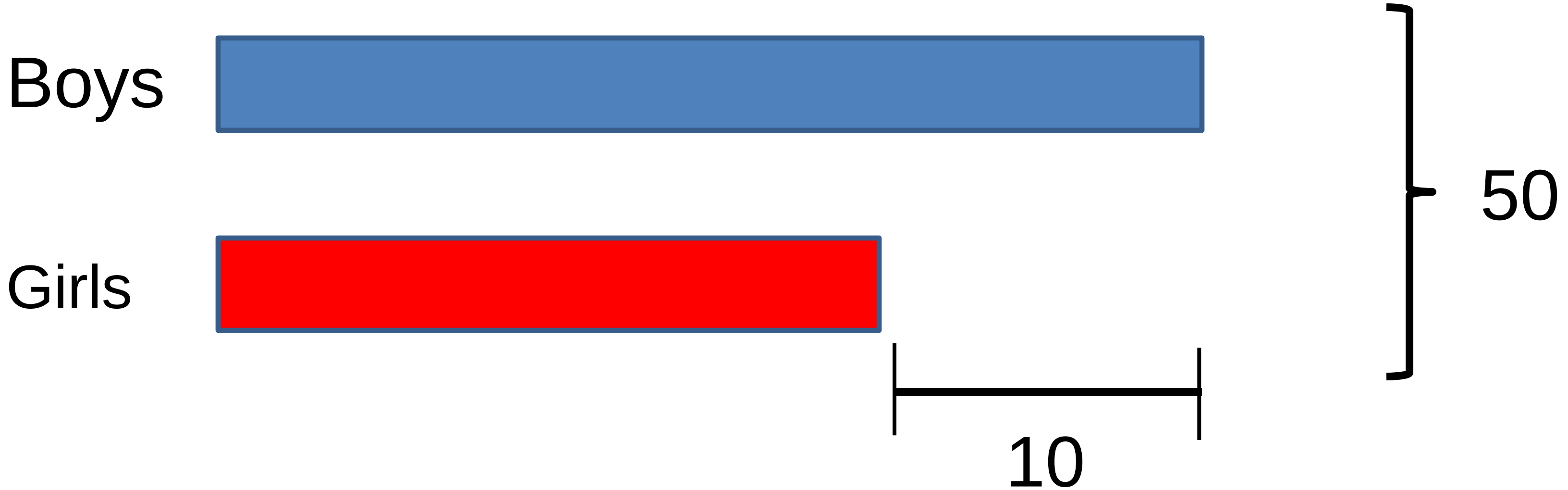
  Analyze relationship(s) between components of problem

- Helps empower students

  Develop competence and confidence in using the algebraic method.

Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

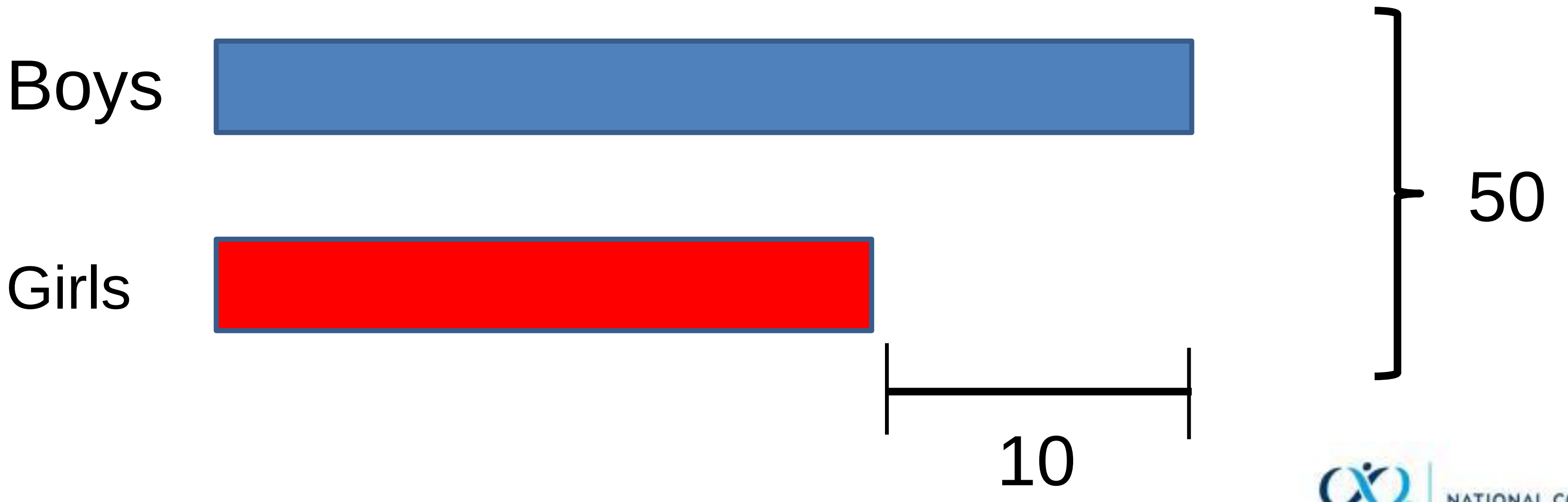


Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let *x* be the number of <u>girls</u>.

What could be possible algebraic equation(s)?



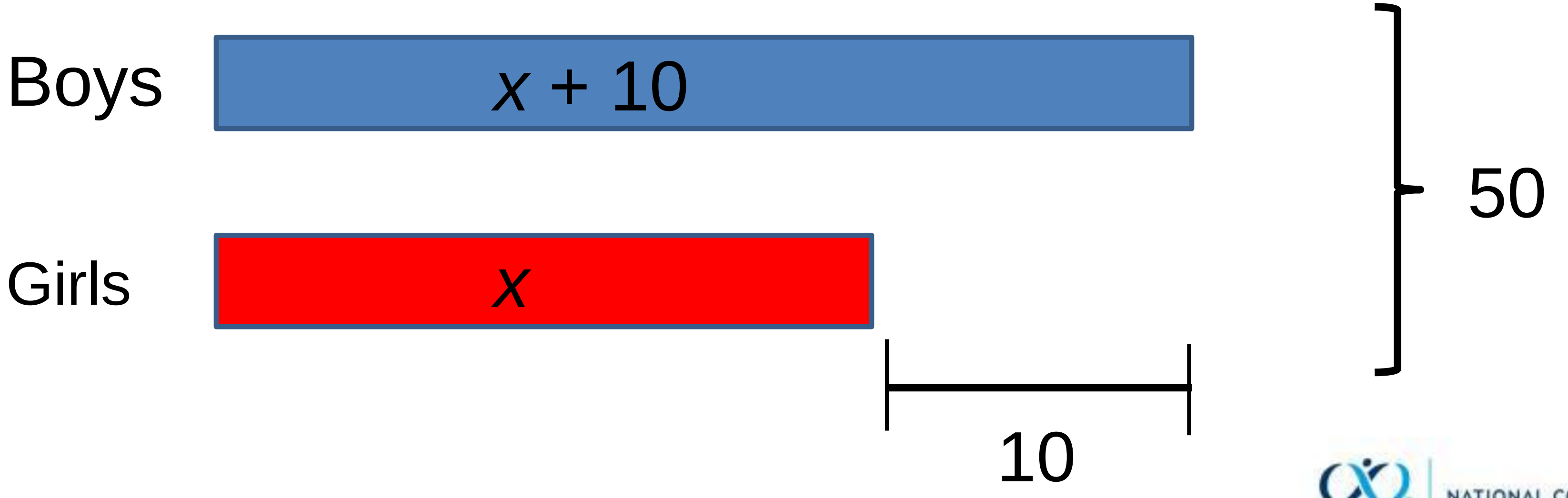
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let *x* be the number of <u>girls</u>.

What could be possible algebraic equation(s)?

$$x + (x + 10) = 50$$



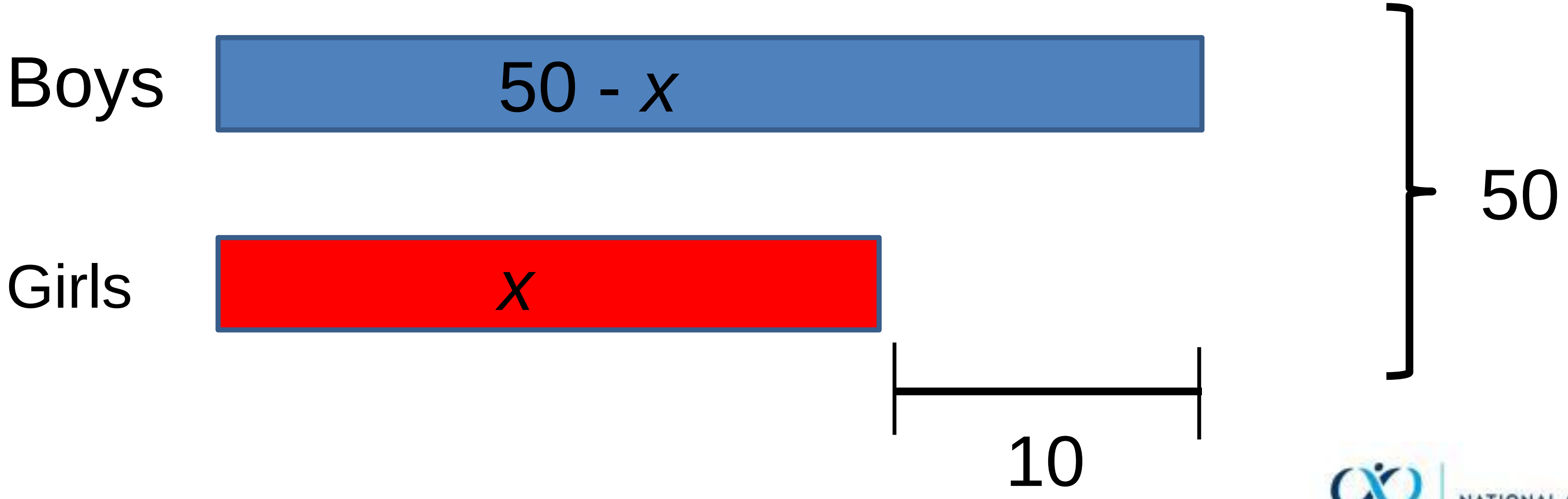
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let *x* be the number of <u>girls</u>.

What could be possible algebraic equation(s)?

$$(50 - x) - x = 10$$



Solving Equations
Strip Diagram Method

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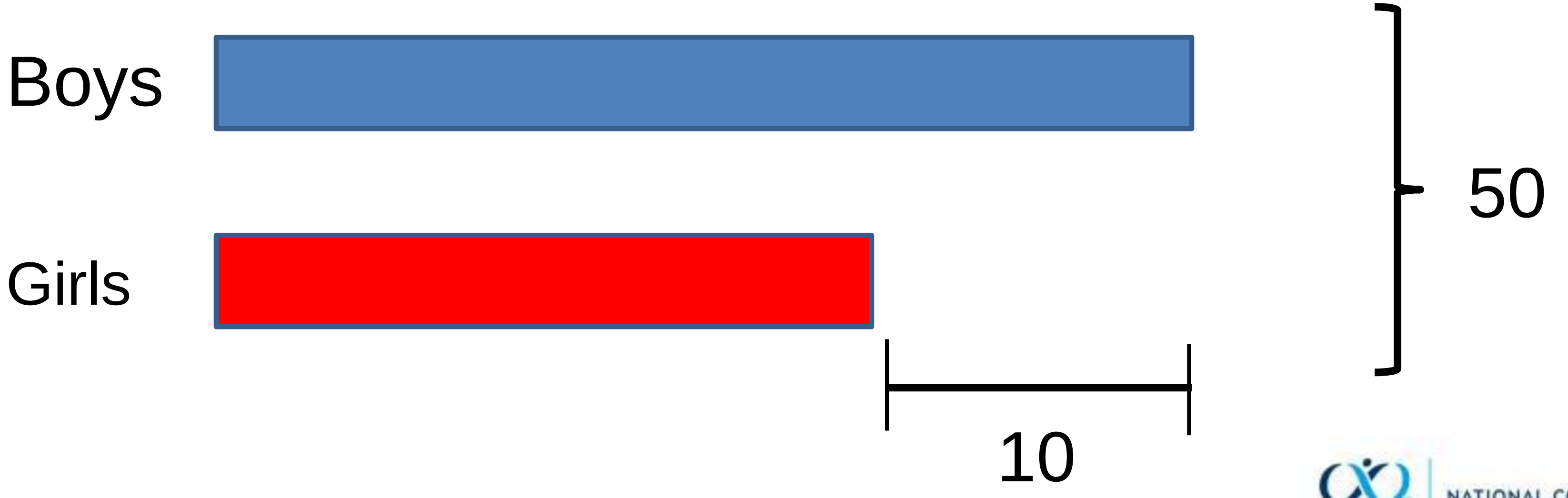


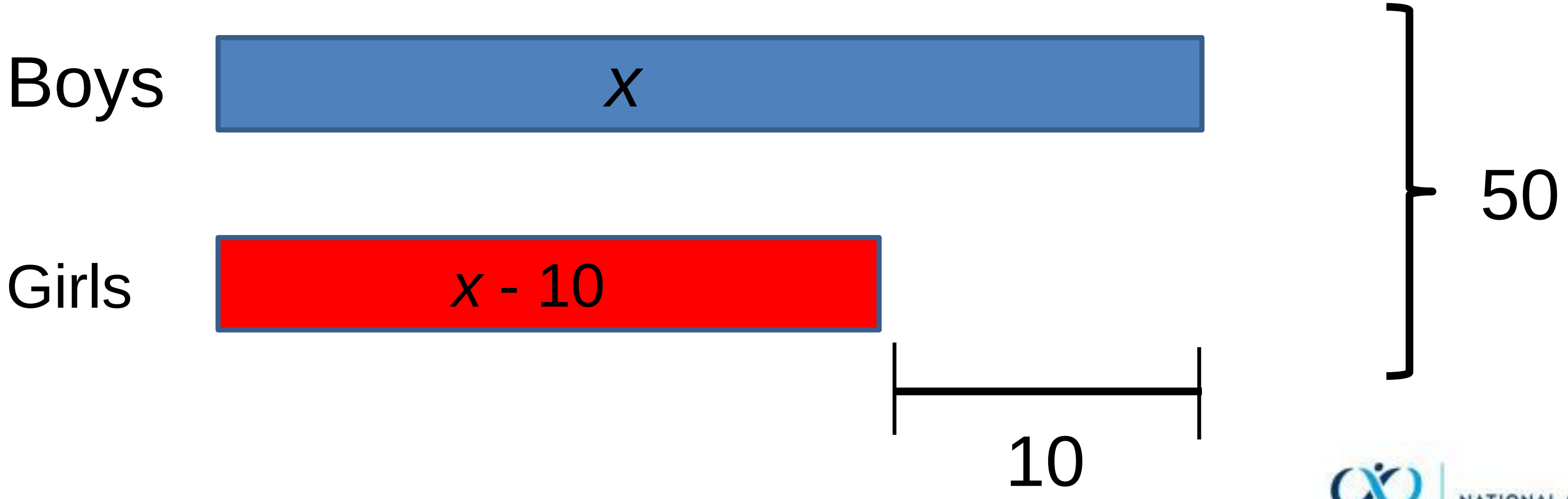
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let *x* be the number of <u>boys</u>.

What could be possible algebraic equation(s)?

$$x + (x - 10) = 50$$



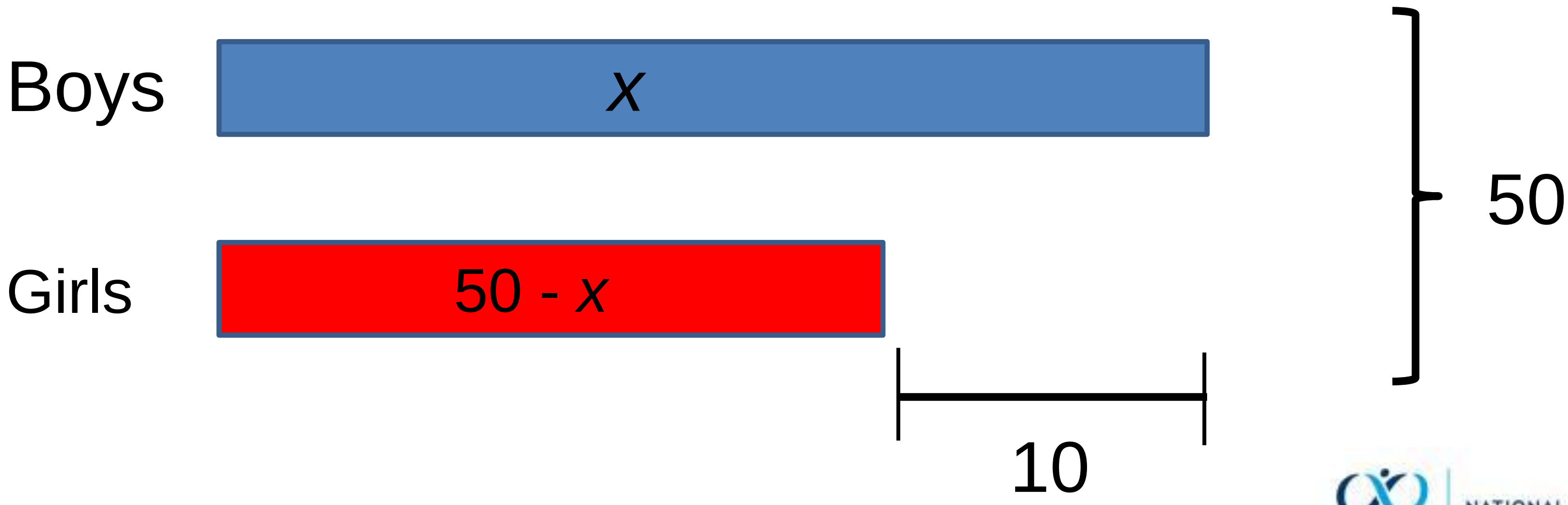
Solving Equations
Strip Diagram Method

There are 50 children in a dance group. If there are 10 more boys than girls, how many girls are there?

Let *x* be the number of <u>boys</u>.

What could be possible algebraic equation(s)?

$$x - (50 - x) = 10$$



Solving Equations
Strip Diagram Method

Use the Strip Diagram Method to solve the problems on the handout.

Set up the diagram/algebraic equations in as many ways as possible.

Solving Equations

Cover up method: $5+\frac{3x-1}{4}=7$

Solving Equations
Cover Up Method

Cover up method: $5+\dfrac{3x-1}{4}=7$

$5 + \square = 7$	$\square = 2$	$\dfrac{3x-1}{4} = 2$

Solving Equations
Cover Up Method

Cover up method: $5 + \frac{3x - 1}{4} = 7$

$5 + \boxed{} = 7$	$\boxed{} = 2$	$\frac{3x - 1}{4} = 2$
$\frac{\boxed{}}{4} = 2$	$\boxed{} = 8$	$3x - 1 = 8$

Solving Equations
Cover Up Method

Cover up method: $5 + \dfrac{3x - 1}{4} = 7$

$5 + \square = 7$	$\square = 2$	$\dfrac{3x - 1}{4} = 2$
$\dfrac{\square}{4} = 2$	$\square = 8$	$3x - 1 = 8$
$\square - 1 = 8$	$\square = 9$	$3x = 9$

Solving Equations
Cover Up Method

Cover up method: $5 + \frac{3x - 1}{4} = 7$

$5 + \square = 7$	$\square = 2$	$\frac{3x - 1}{4} = 2$
$\frac{\square}{4} = 2$	$\square = 8$	$3x - 1 = 8$
$\square - 1 = 8$	$\square = 9$	$3x = 9$
$3\,\square = 9$	$\square = 3$	$\boldsymbol{x = 3}$

Solving Equations
Cover Up Method

- Practice solving equations using the Cover Up Method with your tablemates.

- What will students learn as a result of this activity?
- What challenges might students encounter with this activity?

Reflection

- What new idea(s) do you want to implement into your classroom?

- What challenges did you encounter during this session?

Reflection

Build procedural fluency from conceptual understanding Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.

(*Principles to Actions: Ensuring Mathematical Success for All* [NCTM 2014], p. 47)

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Reflection

Build procedural fluency from conceptual understanding Teacher and student actions, *continued*	
What are *teachers* doing?	**What are *students* doing?**
Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Striving to use procedures appropriately and efficiently.

(*Principles to Actions: Ensuring Mathematical Success for All* [NCTM 2014], p. 48)

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Grade 6:
Representing Ratios in Various Formats
NCTM Interactive Institute, 2014

Arlene P. Mitchell
Shawn Towle
J. Matt Switzer

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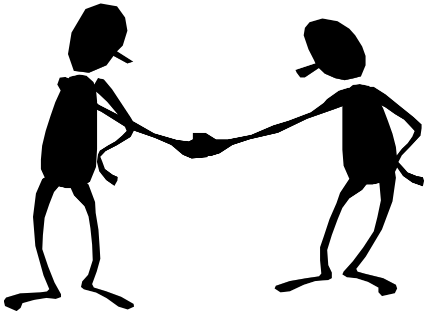
Introductions….

- Introduce yourself to others at your table.



- Discuss success and challenges you encounter when teaching the topic of **ratios** to students in your classroom.

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Common Core Progressions

GRADE 6	GRADE 7	GRADE 8
Understand ratio concepts and use ratio reasoning to solve problems.	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	**Understand the connections between proportional relationships, lines, and linear equations.**
• Concept of ratio • Use ratio language • Concept of unit rate • Use ratio and rate reasoning to solve real-world and mathematical problems (tables, diagrams, double number lines, equations)	• Compute unit rates • Represent proportional relationships between quantities • Use proportional relationships to solve multistep ratio and percent problems	• Graph proportional relationships, interpreting the unit rate as the slope of the graph • Use similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line

Big Ideas for Ratios

A ratio is an ordered pair of numbers or measurements that expresses a comparison between the numbers or measures.

– Reasoning with ratios involves attending to and coordinating two quantities.

Forming a ratio involves isolating one attribute from other attributes.

– A ratio is a multiplicative comparison of two quantities or it is a joining of two quantities in a composed unit.

Ratios

- **Ratios are expressed in fraction notation.**
  - Ratios and fractions do not have identical meaning.
  - Ratios and fractions can be conceived as overlapping sets.
- **Fractions are ratios, but not all ratios are fractions.**
  - Some ratios make "part-part" comparisons or relate more than two parts.
- **Rates are ratios, but not all ratios are rates.**

Ratios are building blocks for proportions and proportional reasoning.

Types of Ratios

- **Part-to-Whole Ratios**

  Compare two measures of same type

  Nonfiction books to all books in library, percentages, probabilities

- **Part-to-Part Ratios**

  Compare two measures of same type

  Fiction books to nonfiction books in library, odds of an event

- **Rates as Ratios**

  Comparison of measures of two different things / quantities

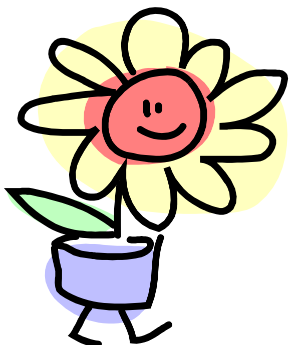
  Prices, time and distance, miles per gallon, inches per foot

- **Special Ratios**

  Golden ratio, π, slope of line, geometric similarity, trigonometric functions from right triangles

Ratios:
Student Thinking

Two weeks ago, two flowering plants were measured at 8 inches and 12 inches. Today they are 11 inches and 15 inches tall, respectively.



Which flowering plant grew more –
the 8-inch or 12-inch flower?

Defend two different “answers” to this problem.

Additive Versus Multiplicative Reasoning

Multiple Representations

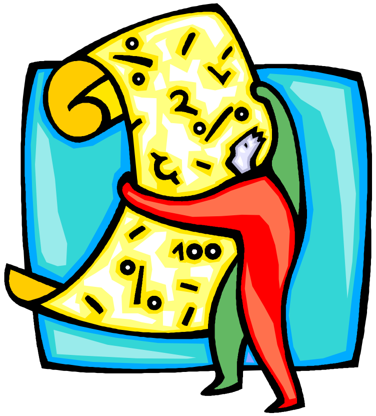
During this session, we are going to use various representations to help students develop conceptual understanding of ratios.

- Unit ratio
- Ratio table
- Double number line
- Tape diagram
- Coordinate graph

Comparing Ratios

Unit rate

Distinguish equivalency between 2 or more ratios



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Comparing Ratios

Two camps of Scouts are having pizza parties.

The Bear Camp ordered enough so that every 3 campers will have 2 pizzas.

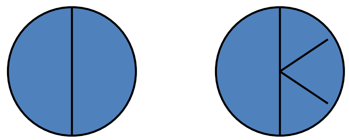
The leader of the Raccoons ordered enough so that there would be 3 pizzas for every 5 campers.

Which campers had more pizza to eat: the Bear campers or the Raccoon campers?

Unit Rate

"Pizzas per Camper" Approach

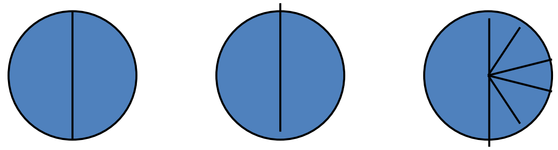
Bear Campers



Each of the 3 campers will get $\frac{1}{2}$ pizza and $\frac{1}{6}$ pizza.

$\frac{2}{3}$ pizza per camper

Raccoon Campers



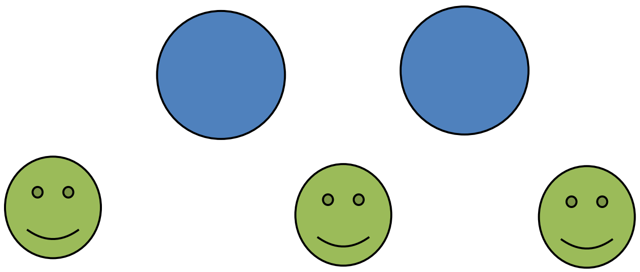
Each of the 5 campers will get $\frac{1}{2}$ pizza and $\frac{1}{10}$ pizza.

$\frac{3}{5}$ pizza per camper

Unit Rate

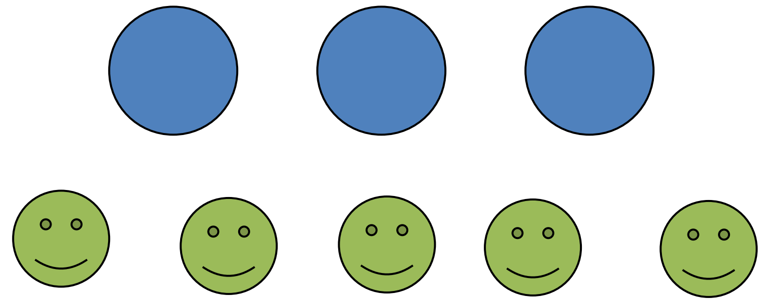
“Campers per Pizza” Approach

Bear Campers



$1\frac{1}{2}$ **campers per pizza**

Raccoon Campers

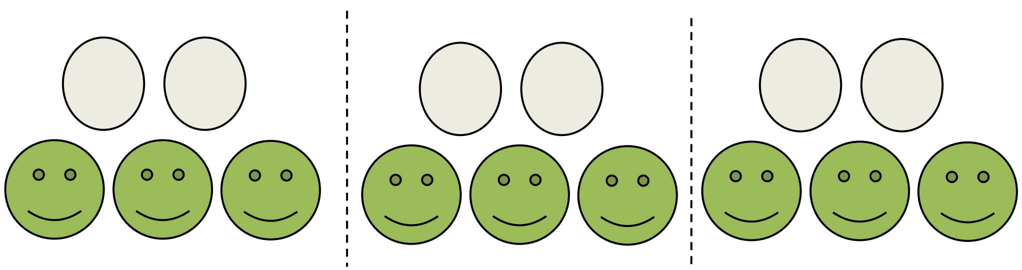


$1\frac{2}{3}$ **campers per pizza**

Unit Rate

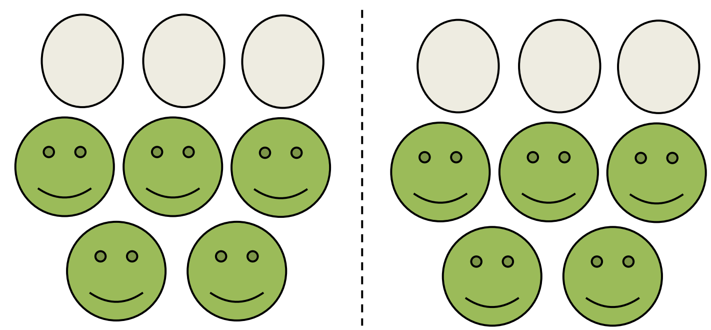
Compare equivalent number of pizzas to number of campers

Bears



6 pizzas for 9 campers

Raccoons

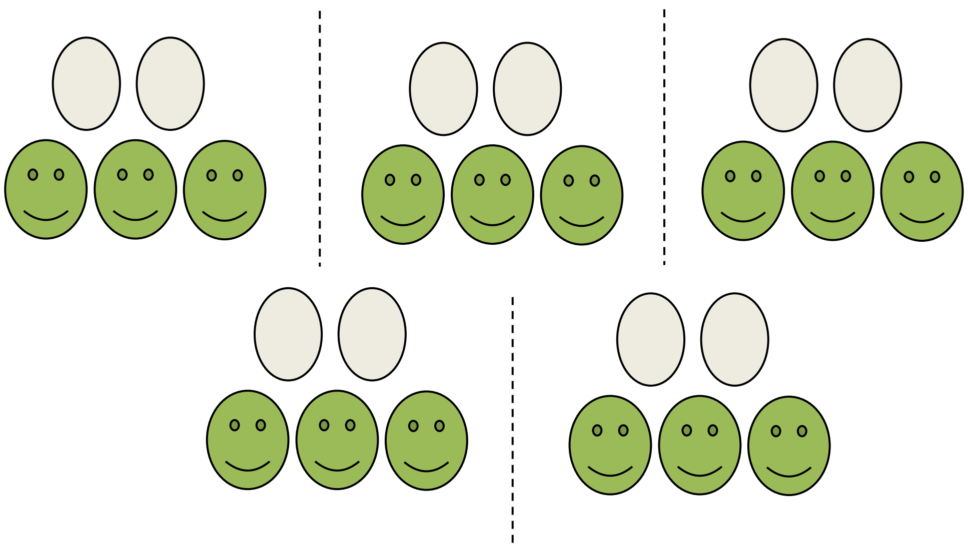


6 pizzas for 10 campers

Unit Rate

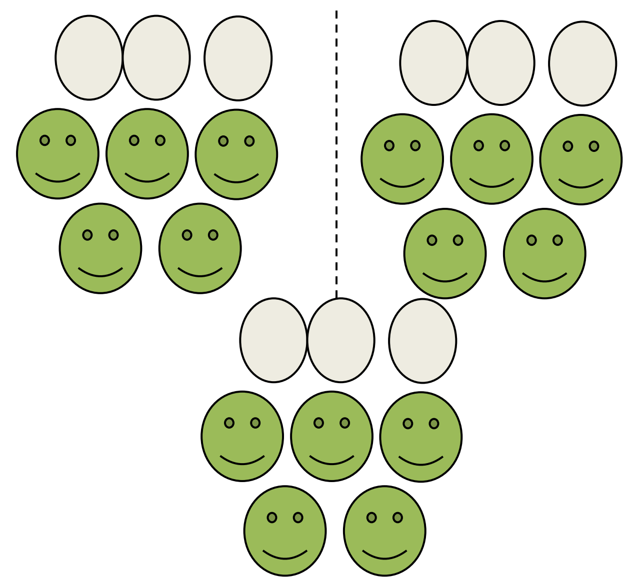
Compare equivalent number of campers to number of pizzas

Bears



15 campers for 10 pizzas

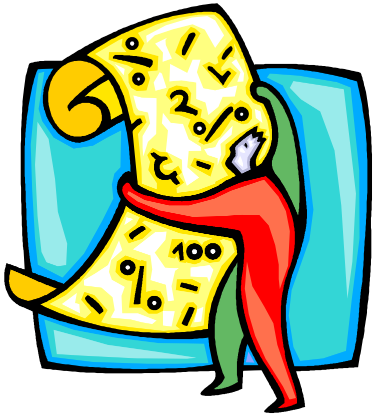
Raccoons



15 campers for 9 pizzas

Comparing Ratios

Using multiplicative comparisons is a powerful proportional reasoning strategy which is an important element in algebra.



Multiplicative Comparisons Using Equations

Bear Camp	
# of Campers	# of Pizzas
3	**2**
1	
	1
Raccoon Camp	
# of Campers	# of Pizzas
5	**3**
1	
	1

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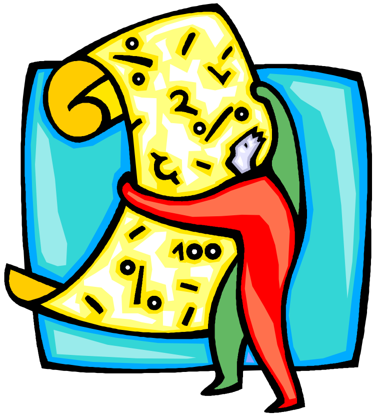
Comparing Ratios

Ratio table

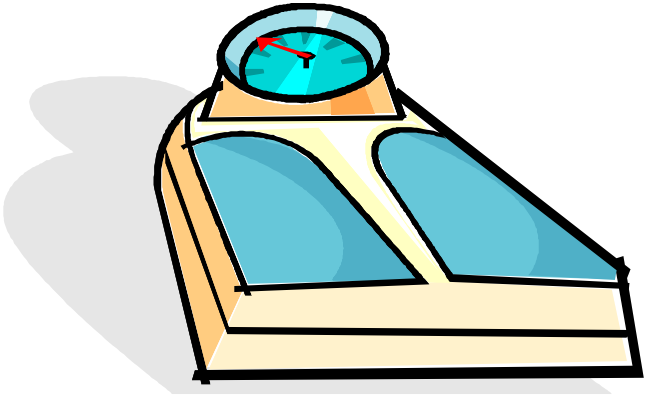
Relationship of two variable quantities



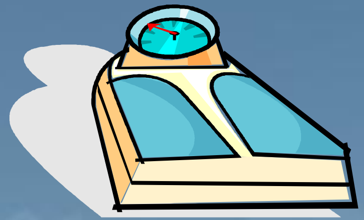
Ratio Table

A person who weighs 160 pounds on Earth will weigh 416 pounds on the planet Jupiter.

How much will a person weigh on Jupiter who weighs 120 pounds on earth?



Ratio Table



÷ 2 ÷ 2 X 3

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312

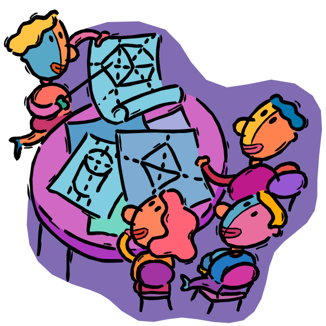
Add

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312

Ratio Table

- At your table, use a ratio table to solve your assigned problem.
  - Find a variety of ways to use the ratio table with the problem.
  - What success/challenges might students encounter using the ratio table?

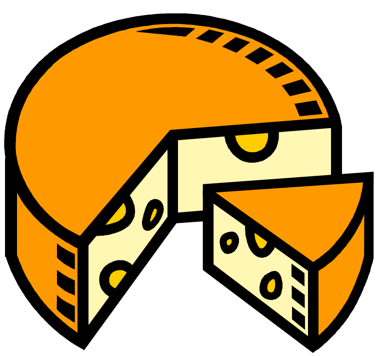
- Share your problem with large group.



Ratio Table

Cheese is $4.25 per pound.

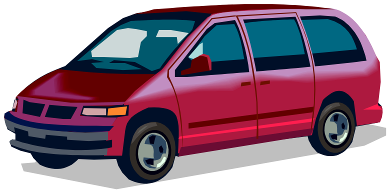
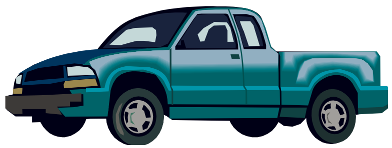
How much will 12.13 pounds cost?



	Lbs	Cost	Notes
A	1	4.25	Given
B	10	42.50	A x 10
C	2	8.50	A x 2
D	0.1	0.425	A ÷ 10
E	12.1	51.125	B + C +D
F	0.01	0.0425	D ÷ 10
G	0.03	0.1275	F x 3
H	12.13	51.5525	E + G

Compare Ratios

Use a tape diagram to solve the problem:

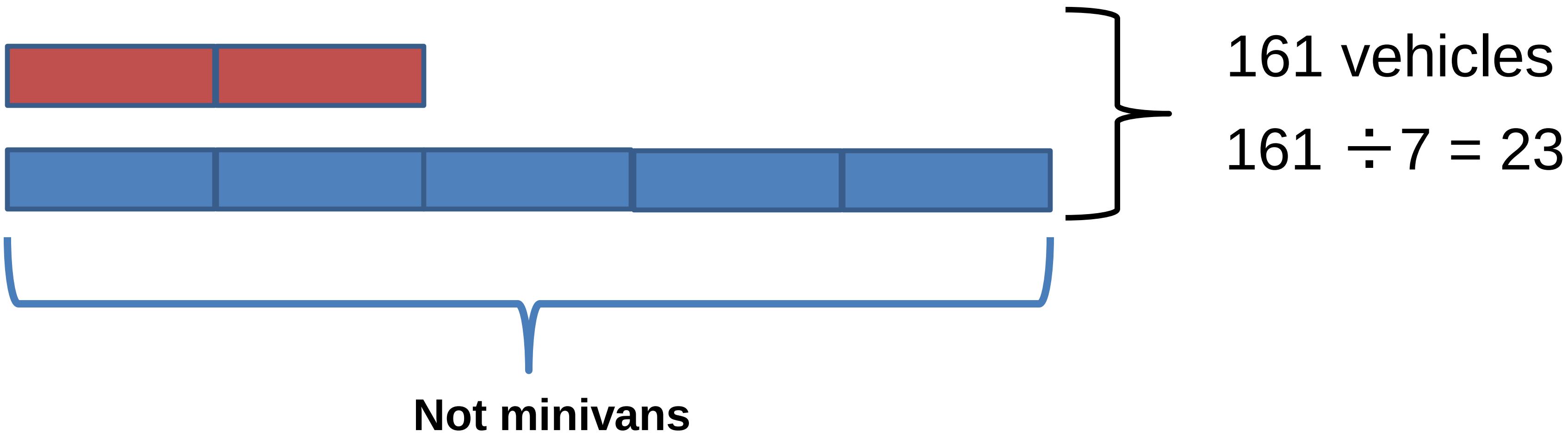
The school parking lot holds 161 vehicles.

When Carla looked at the filled parking lot at school, she noticed there were 2 minivans for 5 other types of vehicles.

How many of the vehicles are not minivans?

Tape Diagram

161 vehicles in ratio of 2:5

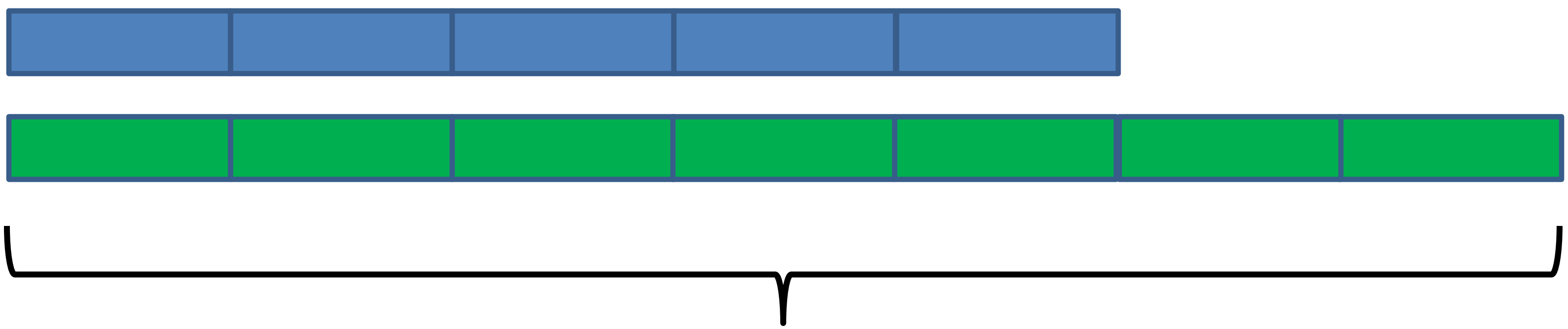


Vehicles that are not minivans

23 X 5 = 115

Tape Diagram

161 vehicles in ratio of 5:7



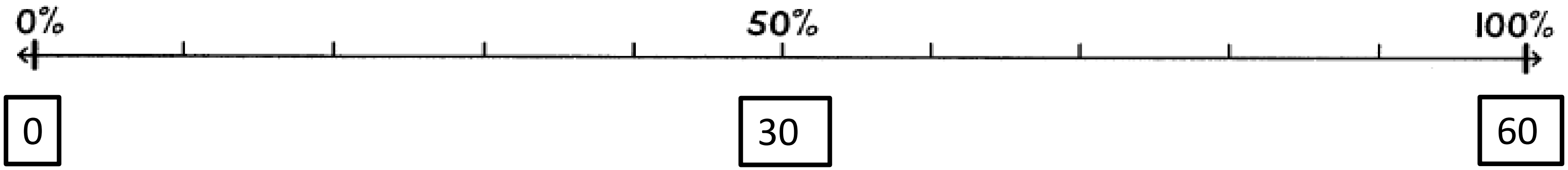
161 total vehicles

$161 \div 7 = 23$

Vehicles that are not minivans

$23 \times 5 = 115$

Solve Percent Problems Using Double Number Line



Explore percent problems

(1) 45% of 70 = ________

(2) 30% of _______ = 75

(3) ______% of 75 = 30

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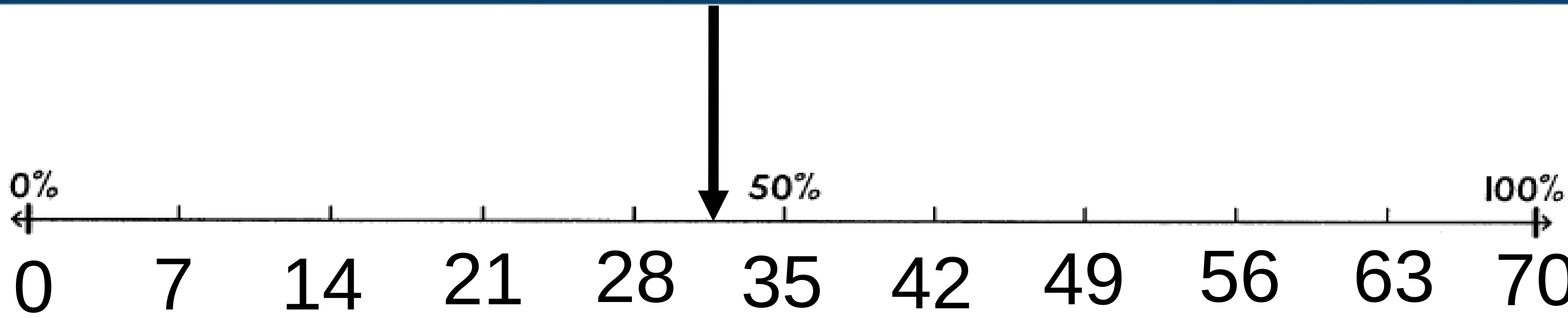
Solve Percent Problems Using Double Number Line



(1) 45% of 70 = 31.5

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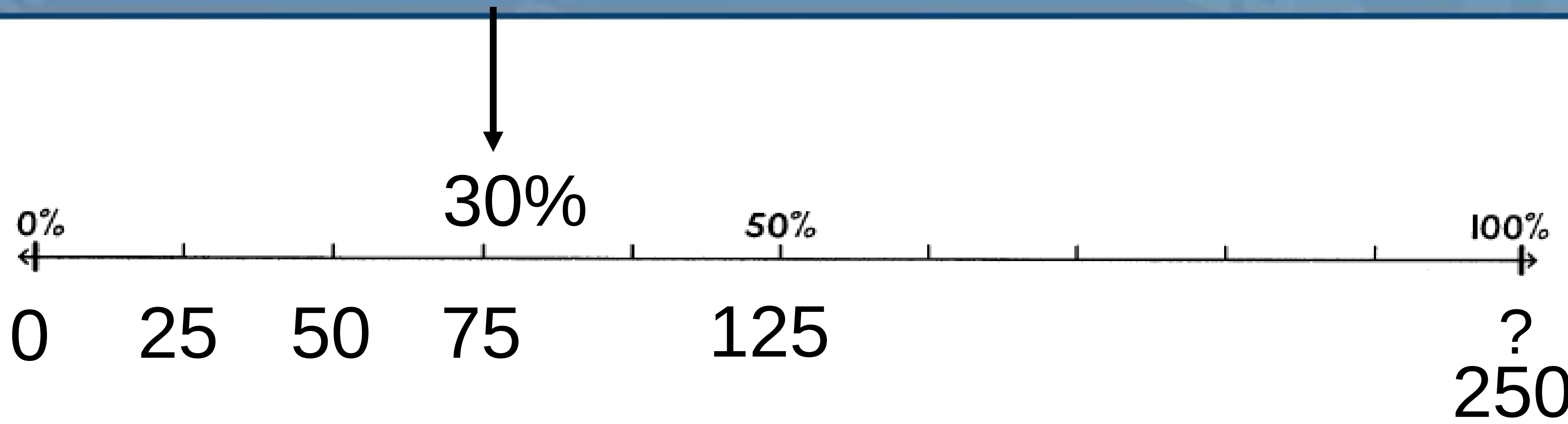
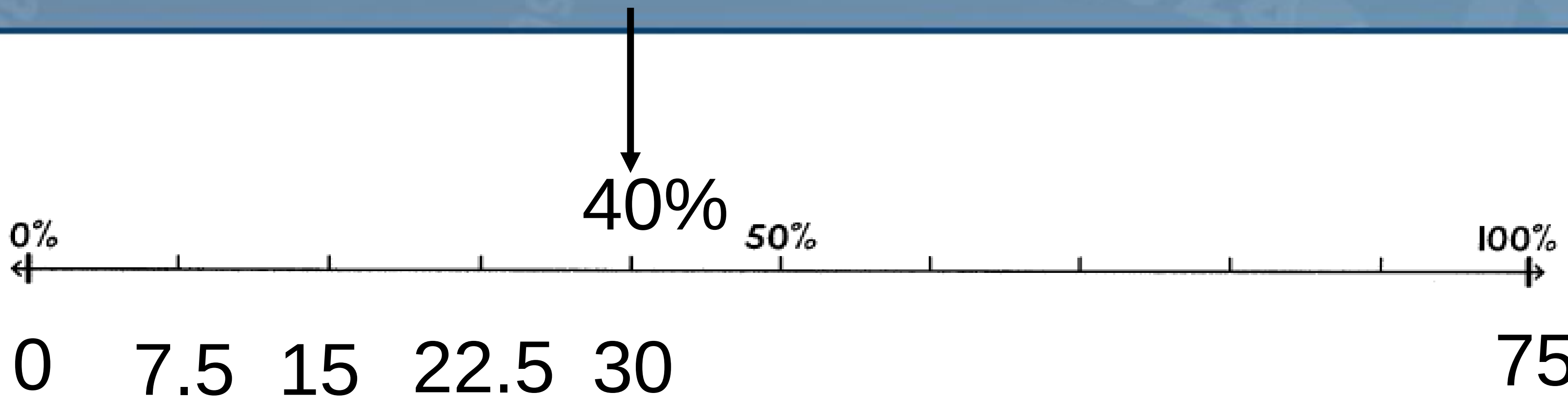
Solve Percent Problems Using Double Number Line



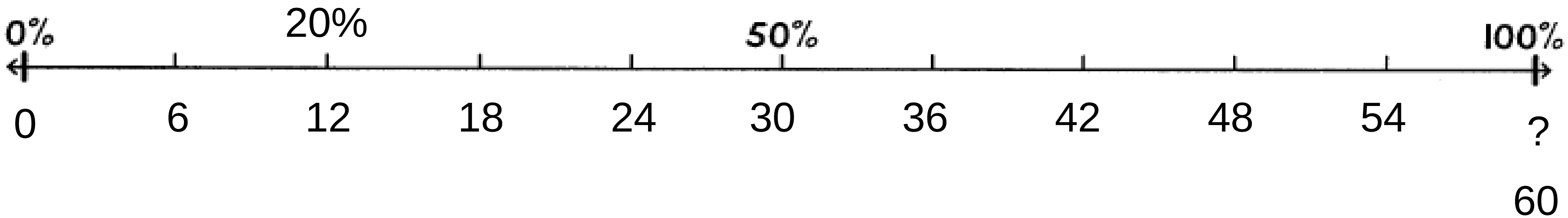
(2) 30% of **250** ________ = 75

Solve Percent Problems Using Double Number Line



(3) ___40___% of 75 = 30

Double Number Line for Percents



20% of Ms. Thompson's show dogs are Labradors. She has 12 Labradors.

How many show dogs does she have?

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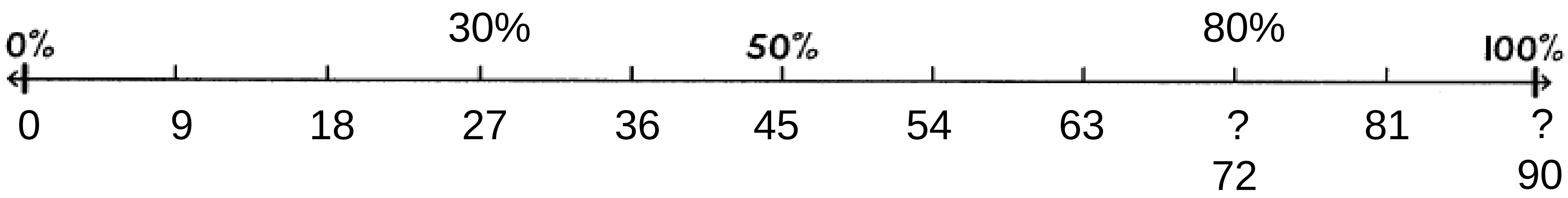
Double Number Line for Percents



Jan has completed 27 items which is 30% of the test. She needs to complete 80% to move on.

- How many items does she need to complete to move on?
- How many items are there on the test?

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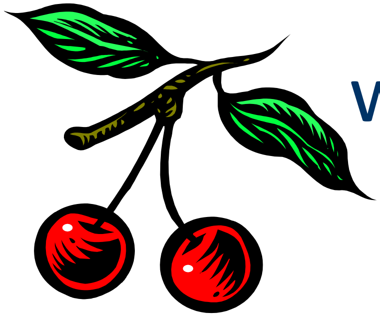
Ratios in Context

Luis mixed 6 ounces of cherry syrup with 53 ounces of water to make a cherry-flavored drink.

Martin mixed 5 ounces of the same cherry syrup with 42 ounces of water.



Who made the drink
with the stronger cherry flavor?

Give mathematical evidence
to justify your answer.

Ratios in Context

Student Work for Cherry Syrup Problem

Divide syrup by water

$\frac{6}{53} = .1132$

$.1190 > .1132$

$\frac{5}{42} = .1190$ Martin's is stronger

Divide water by syrup

$53 \div 6 = 8.8\overline{3}$

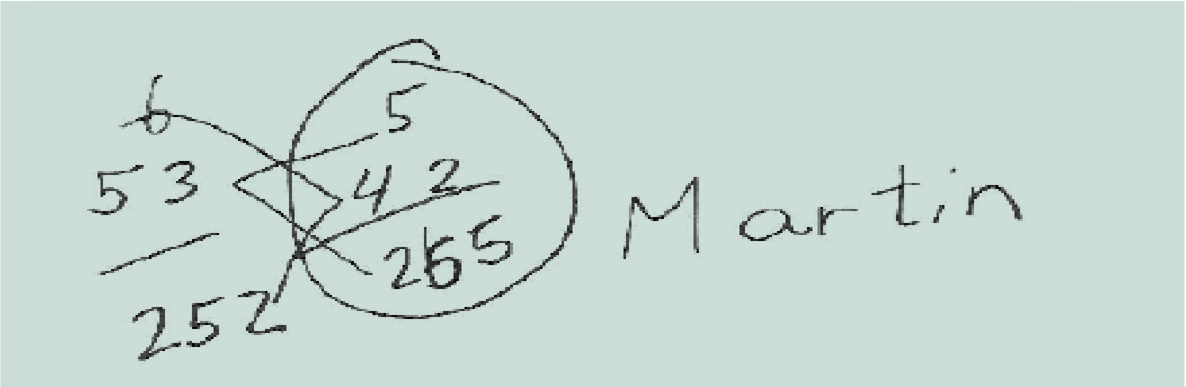
$42 \div 5 = 8.4$

Luis made it stronger

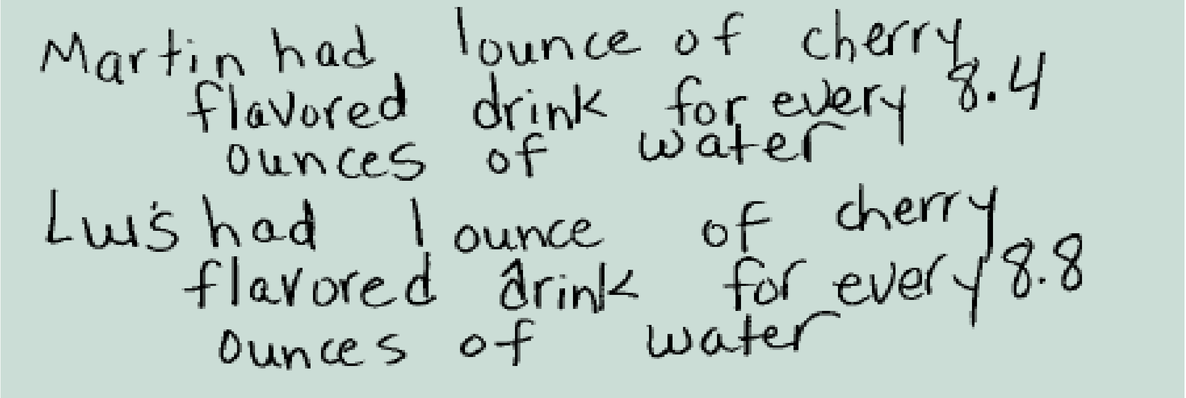
Ratios in Context

Student Work for Cherry Syrup Problem

Multiply syrup and water.



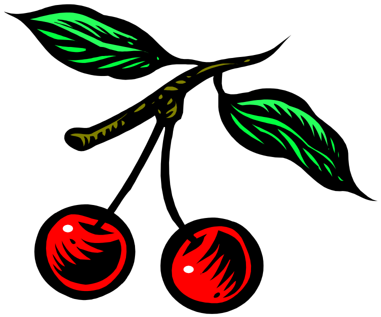
Apply rates



Graphing Ratios

With your table group:

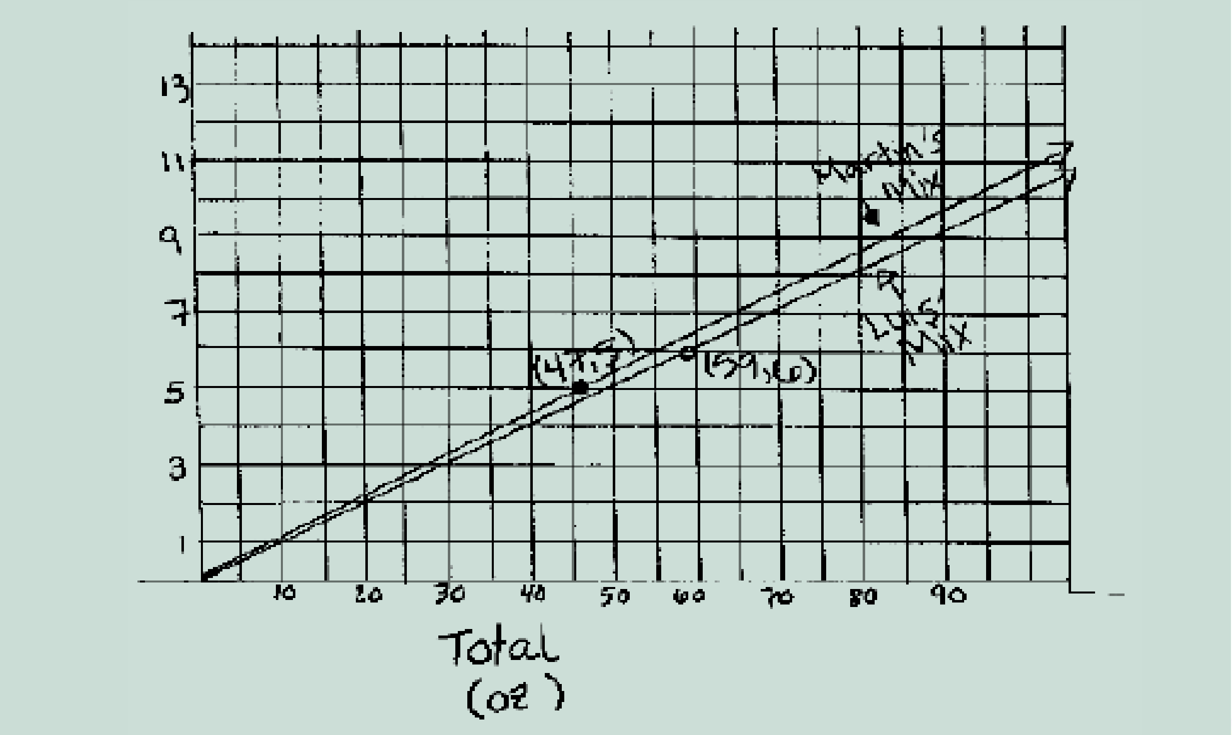
- Use a variety of coordinate graphs to compare the mixtures to justify your decision about which mixture tastes more "*cherrier*."



Graphing Ratios

Student work

- Total ounces (x-axis) vs Ounces of syrup (y-axis)



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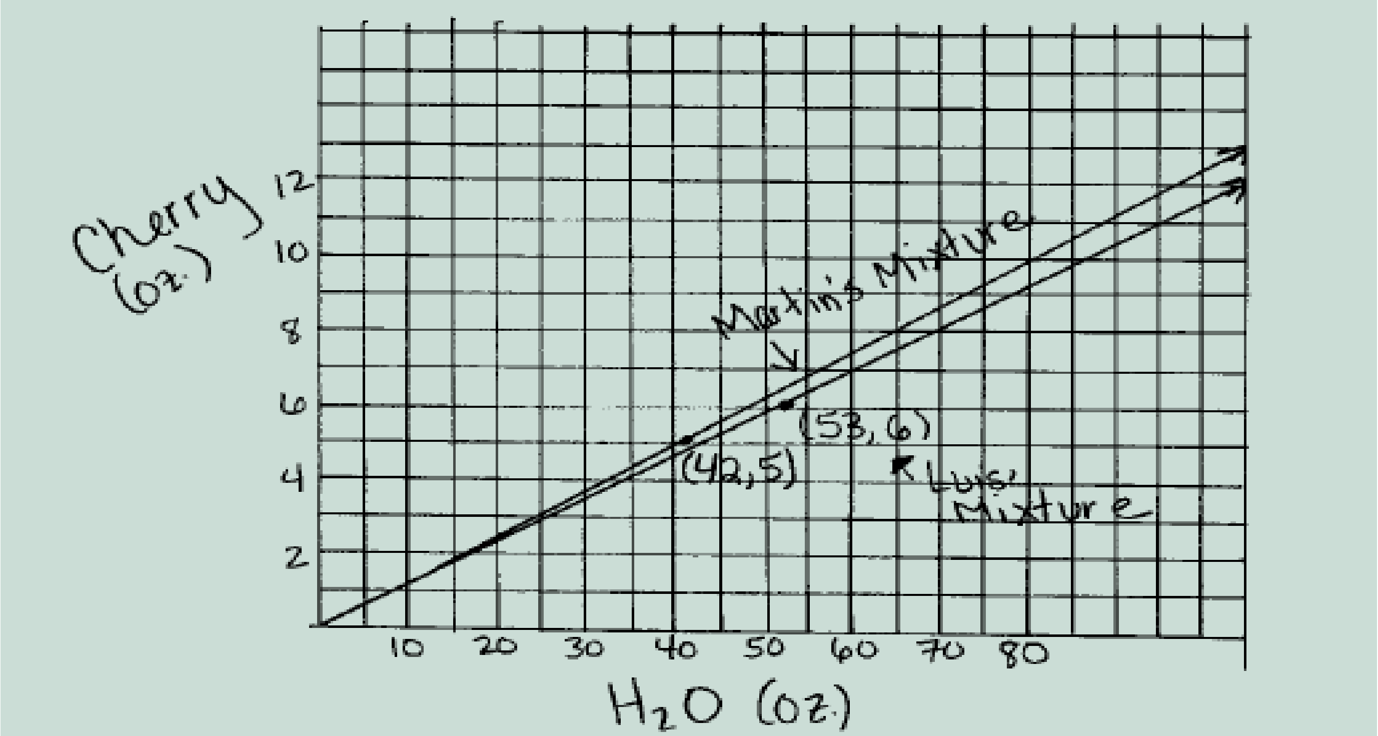
Graphing Ratios

Student Work

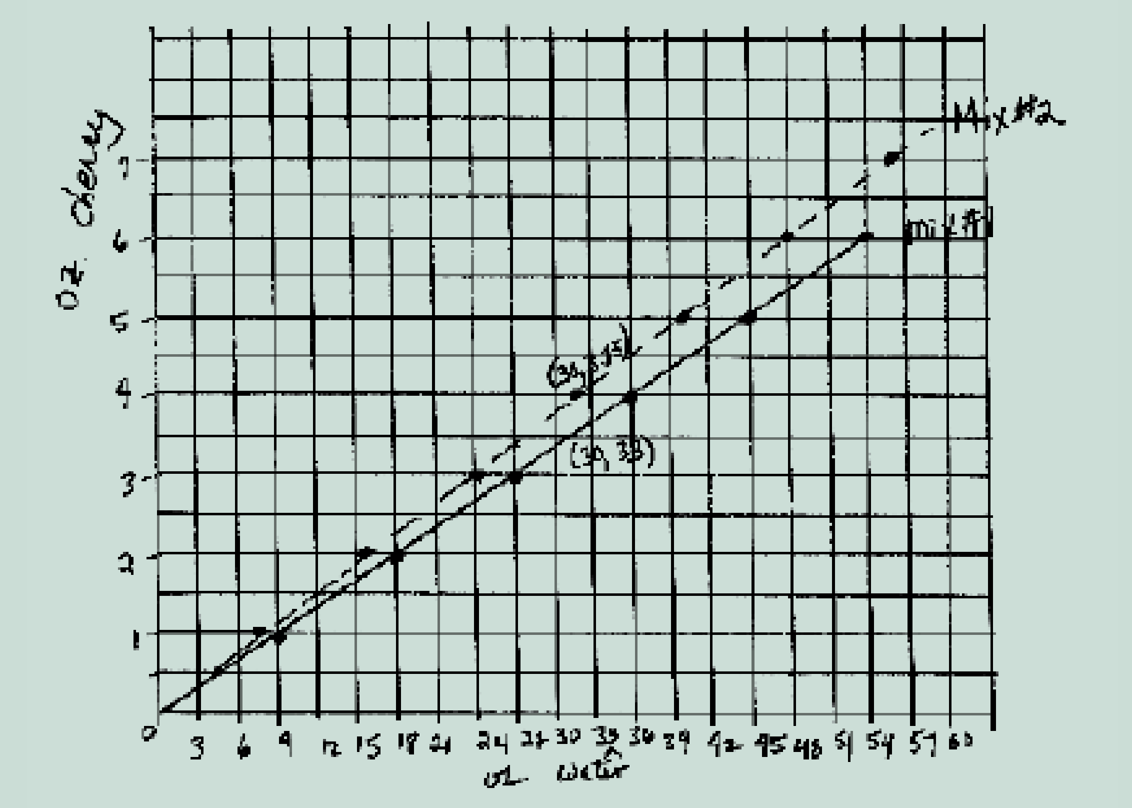
Total ounces water (x-axis) vs Ounces of syrup (y-axis)



Graphing Ratios

Student Work

Total ounces water (x-axis) vs Ounces of syrup (y-axis)

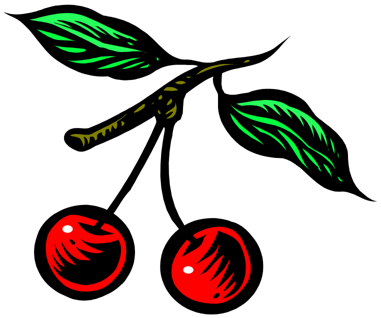


Graphing Ratios

With your table group, discuss:

- How will graphing ratios help students to understand ratios beyond computation?

- What strategies will help students to compare different ratios through the use of graphing?



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Multiple Representations

A car magazine is writing a story about four different cars, reporting the number of miles driven for different amounts of gas.

- With your **Expert group**, describe the gas mileage for your assigned car using multiple representations (words, table, equation, and graph).




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Multiple Representations

- Each **Home group** will have one member from the **Expert group** to discuss their assigned car.
- Use the various representations to decide:
  - The ordering of the cars from greatest number of miles per gallon to least number of miles per gallon.
  - The car Krystal likely bought if she drove 924 miles and used 28 gallons of gas.

Summary

There are several ways the same collection of equivalent ratios can be represented. These include:

- ratio tables,
- tape diagrams,
- double number lines,
- equations, and
- graphs on coordinate planes.

Reflection

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

(*Principles to Actions: Ensuring Mathematical Success for All* [NCTM 2014], p. 29)

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Exit Ticket

Describe how the various representations might contribute to the learning of ratios by students.

- Ratio table
- Double number line diagram
- Tape diagram
- Equation
- Coordinate graph

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