# Appendix 2
## Resources for Teachers

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The following list highlights a few of the many books, articles, videos, and websites that are helpful resources for teaching addition and subtraction in prekindergarten–grade 2. Abstracts from the publishers provide brief descriptions of some of the resources.

## Books

Battista, Michael T. *Cognition-Based Assessment & Teaching of Addition and Subtraction: Building on Students' Reasoning*. Portsmouth, N.H.: Heinemann, 2012.

Using a research-based framework that describes the development of students' thinking and learning in terms of levels of sophistication, a "cognitive terrain" that includes ascents and plateaus, Battista shows how teachers can build on their students' reasoning.

Caldwell, Janet H., Karen Karp, and Jennifer M. Bay-Williams. *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Prekindergarten–Grade 2*. Essential Understanding Series. Reston, Va.: National Council of Teachers of Mathematics, 2011.

"Unpacking" the ideas related to addition and subtraction is a critical step in developing a deeper understanding. To those without specialized training, many of these ideas might appear to be easy to teach. But those who teach in prekindergarten–grade 2 are aware of their subtleties and complexities. This book identifies and examines two big ideas and related essential understandings for teaching early addition and subtraction. The authors examine the ways in which counting leads to addition and subtraction, as well as the role that these operations play in more advanced topics. The discussion highlights challenges in teaching, learning, and assessment and is interspersed with questions for teachers' reflection.

Carpenter, Thomas, Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. *Children's Mathematics/Cognitively Guided Instruction*. Portsmouth, N.H.: Heinemann, 1999.

By the time children begin school, most have already developed a sophisticated, informal understanding of basic mathematical concepts and problem-solving strategies. Too often, however, the mathematics that classroom instruction imposes on them fails to connect with this informal knowledge. The authors developed this book to help teachers understand children's intuitive mathematical thinking and use that knowledge to guide students in learning mathematics with understanding. Based on

more than twenty years of research, the book portrays the development of children's understanding of basic number concepts. The authors offer a detailed explanation and numerous examples of the problem-solving and computational processes that virtually all children use as their numerical thinking develops. They also describe how classrooms can be organized to foster that development. Two accompanying CDs include videos that provide an inside look at students and teachers in real classrooms implementing the teaching and learning strategies described in the text.

Fosnot, Catherine, and Maarten Dolk. *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction*. Portsmouth, N.H.: Heinemann, 2001.

Fosnot and Dolk focus on the way in which children between the ages of four and eight construct their knowledge of the operations of addition and subtraction. The authors—

- emphasize that a deep knowledge of number sense will support students' computational skills;
- provide strategies to help teachers turn their classrooms into math workshops that encourage and reflect mathematizing;
- explore the development of a repertoire of strategies;
- define and give examples of the use of an open number line model; and
- discuss calculation using number sense and the role of algorithms in instruction about computation.

O'Connell, Susan, and John SanGiovanni. *Mastering the Basic Math Facts in Addition and Subtraction: Strategies, Activities, and Interventions to Move Students beyond Memorization*. Portsmouth, N.H.: Heinemann, 2011.

O'Connell and SanGiovanni emphasize the value of helping students develop both automaticity with the facts and understanding of them instead of merely memorizing them. Aligning their ideas with both the Common Core State Standards and the NCTM Principles and Standards, the authors stress the importance of understanding the concepts of addition and subtraction. They share insights into teaching the basic facts, including a variety of instructional strategies and activities.

Van de Walle, John A., Karen S. Karp, and Jennifer M. Bay-Williams. *Elementary and Middle School Mathematics: Teaching Developmentally*. 8th ed. Boston: Pearson, 2013.

The purpose of this book is to help teachers understand mathematics and become confident in their ability to teach the subject to children in kindergarten through eighth grade. The chapters related to teaching and learning basic facts and addition and subtraction provide ideas and insights that will support teachers as they design and implement their lessons.

## Articles

Baroody, Arthur. "Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them." *Teaching Children Mathematics* 13 (August 2006): 22–31.

In this well-known article, Baroody discusses the three phases through which children progress in gaining mastery of addition and subtraction number

combinations: (1) using counting strategies, (2) using reasoning strategies, and (3) producing correct answers efficiently (mastery). He shares several vignettes that contrast "conventional wisdom" and "the number-sense view."

Bofferding, Laura, Melissa Kemmerle, and Aki Murata. "Making 10 My Way." *Teaching Children Mathematics* 19 (October 2012): 164–73.

The authors describe a kindergarten unit of study that focuses on identifying combinations of 10 by using ten frames and counters. Students receive explicit instruction on addition and subtraction problem types, including "join" (result unknown), "compare" (difference unknown), "separate" (result unknown), and "part-part-whole" (part unknown or whole unknown). Samples of students' work are shared to highlight the strategies that the kindergartners used, and a table that includes activity descriptions, needed materials, and goals shows how to differentiate instruction.

Buchholz, Lisa. "Learning Strategies for Addition and Subtraction Facts: The Road to Fluency and the License to Think." *Teaching Children Mathematics* 10 (March 2004): 362–69.

This article suggests strategies to offer students to help them establish and retain addition and subtraction facts, including the use of doubles, combinations that sum to 10, and "fact families," as well as "doubles plus 1" and "doubles minus 1" (or plus or minus 2), "count up," "add 1 to 9," "add 10," "count back," "think addition," and "subtract from 10." A worthwhile discussion deals with a parent's initial wish to have her son learn more about "borrowing" and "carrying."

Griffin, Sharon. "Laying the Foundation for Computational Fluency in Early Childhood." *Teaching Children Mathematics* 9 (February 2003): 306–9.

Drawing on research on number sense and the use of strategies, the author provides a diagnostic tool for determining where students are in their thinking about a basic addition situation. The tool distinguishes five levels of possible performance by students, ranging from those who do not know how to respond to a basic situation to those who use retrieval strategies efficiently. The author shares a series of instructional "next steps" to help students who are performing at a particular level advance to the next one.

Mann, Rebecca L. "Balancing Act: The Truth behind the Equals Sign." *Teaching Children Mathematics* 11 (September 2004): 65–69.

This article moves students from considering the equal sign as a signal that the answer is coming next to a symbol that indicates a "balanced," or equivalent, relationship between the quantities on the two sides of an equation. Building on a discussion of seesaws, students model the actions of seesaws that have them sitting on one end as different imaginary items are placed in their hands. Teacher and students create a set of generalizations that help students see the connection with the equal sign. The author includes challenging activities that might be successful with students who finish early or are gifted.

Postlewait, Kristian B., Michelle R. Adams, and Jeffrey C. Shih. "Promoting Meaningful Mastery of Addition and Subtraction." *Teaching Children Mathematics* 9 (February 2003): 354–57.

The authors posit that students' ability to compose and decompose number is a basis for computational fluency, and they argue that this skill allows students to move away

from a reliance on rules and procedures to a conceptual understanding of number and operations.

Wenrick, Melanie, Jean L. Behrand, and Laura C. Mohs. "A Pathway for Mathematical Practices." *Teaching Children Mathematics* 19 (February 2013): 354–62.

The authors show students exploring the commutative property of addition, the decomposition of numbers, relationships among numbers, and the meaning of the equal sign. First, the students decide whether equations are true or false, and then they rewrite equations that are false to make them true. Students share strategies as they discuss their thinking about how they solved the problems. The authors highlight the importance of selecting appropriate problems for use in instruction, and they share several examples of student work that offer opportunities for students to discuss the work of others.

Wicket, Maryann. "Tuheen's Thinking about Place Value." *Teaching Children Mathematics* 16 (November 2009): 256.

Students who were given the expression "59 + 67 =" (written horizontally in this way) were asked to solve the problem by using any strategy that made sense to them and that they could explain. Although the students were familiar with the standard algorithm, several found the regrouping process challenging. A sample of work by one student, Tuheen, together with a detailed account of his thinking and his written record, is perfect to share with a class as the basis for discussion.

## Videos

Integrating Mathematics and Pedagogy (IMAP)

http://www.sci.sdsu.edu/CRMSE/IMAP/video.html

*IMAP: Select Videos of Children's Reasoning* is a CD containing twenty-five video clips of elementary school children engaged in mathematical thinking. The CD runs on both PC and Mac platforms and comes with an interface that includes the transcript (full or synchronized) and background information for each clip. Also included on the CD is a video guide containing questions to consider before and after viewing each video clip, interviews that teachers or prospective teachers can use when working with children, and other resources.

See also Carpenter and colleagues (1999), under "Books."

*Children's Mathematics/Cognitively Guided Instruction* has two accompanying CDs that provide an inside look at students and teachers in real classrooms implementing the teaching and learning strategies described in the text.

## Online Resources

NCTM Illuminations Lessons

http://illuminations.nctm.org/Lessons.aspx

The NCTM Illuminations website has hundreds of lessons. Select the type of lessons that you want as well as the appropriate grade band, and click Search.

Illustrative Mathematics Project

http://www.illustrativemathematics.org

Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that support implementation of the Common Core State Standards. One tool on this website is a growing collection of mathematical tasks that are organized by standard for each grade level and illustrate each standard's important features. The tasks on the website are not meant to be considered in isolation but are presented in sets that illustrate a particular standard. Eventually, the site will showcase sets of tasks for each standard that—

- illuminate the central meaning of the standard and also show connections with other standards;
- clarify what is familiar about the standard and what is new with the advent of the Common Core;
- include both teaching and assessment tasks; and
- reflect the full range of difficulty expected.

Thinking Blocks: Addition and Subtraction

www.thinkingblocks.com/ThinkingBlocks_AS/TB_AS_ Main.html

Thinking Blocks are teacher-developed tools that link to the various problem structures. They use two-digit numbers and problems with multiple steps, including "compare," "part-part-whole," and "change" examples. Because the ideas are presented as games, viewing the introduction is a necessary preliminary for playing.