SOLUTIONS to calendar

CALENDAR CONTRIBUTORS

The problems and solutions in this month's Calendar were written by Annette Ricks Leitze, graduate advisor in mathematics education at Ball State University in Muncie, Indiana, and several of her graduate students: Barbara Fennell, Angela Greene, Timothy Hildebrand, and Rebecca Wakeman. The drawing of Nick and his brothers (problem 9) was submitted by Joanna Gerr, a student at Thomas Jefferson High School for Science and Technology.

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The Editorial Panel of *Mathematics Teacher* is considering sets of problems submitted by individuals, classes of prospective teachers, and mathematics clubs for publication in the monthly Calendar. Send problems to the Calendar editors. Remember to include a complete solution for each problem submitted.

Department editors

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1. 12 ft. If Kierra is 60 in. tall, then she is 5 ft. tall. The ratio of Kierra's height to the length of her shadow, 5 : 2, equals the ratio of the kite's height to the length of its shadow. The kite is 20 ft. high because 5 : 2 = 20 : 8. If the kite's height increases 10 ft., then it will be flying 30 ft. above the ground. We find the distance d from the kite's new shadow to a point on the ground directly below the kite. Thus, we have $5:2 = 30:d \to d = 12$ ft.

Alternate solution: Find the kite's original height of 20 ft., as before. An increase of 10 ft. represents a 50% increase in height, so the distance to the kite's new shadow must increase 50% also. We have 8 ft. + 0.5(8 ft.) = 12 ft.

2. Approximately 25 cups. Let h be the height of the box in cm. Then the box must measure $(h + 10) \times (30 - h) \times h$, which gives the volume $V(h) = -h(h + 10) \cdot (h - 30)$. This cubic function crosses the x-axis at –10, 0, and 30 and will have $V(h) > 0$ when $0 < h < 30$. Use technology to find the greatest volume: When $h = 18.69$, we find that $V(h) = 6064.6$ cm^3. Convert this measurement to cups: $6064.6/236.588 \approx 25.6$ cups.

3. 10 opossums, 12 owls, and 5 snakes. Let p be the number of opossums, let s be the number of snakes, and let w be the number of owls. Then $p = 2s$; $4p + 2w = 64$; and $p + s + w = 27$. Use the first equation to eliminate s from the third equation; the result is a system of two linear equations in p and w: $4p + 2w = 64$ and $p + p/2 + w = 27$. Simplify both equations: $2p + w = 32$ and $3p/2 + w = 27$. Subtract the second from the first: $p/2 = 5 \to p = 10$. So $w = 12$, and $s = 5$.

4. \$20. If $a^b = 1$, then the exponent $b = 0$ and the base $a \neq 0$, or $a = 1$, or $a = -1$ and b is even. In the first case, we have the exponent $5x - 2 = 0 \to x = 2/5$, with the base $(2x + 9) \neq 0$. In the second case, we have $(2x + 9) = 1 \to x = -4$. In the third case, we have $(2x + 9) = -1 \to x = -5$. Only a positive solution makes sense in context, so Jill will receive 2/5 of \$50—that is, \$20. Note that setting x equal to –5 does not solve the equation because the exponent would not be even.

5. 16,383. We can start with $1 + 2 + 4 + 8 + \cdots = 2^0 + 2^1 + 2^2 + \cdots$. We will have 14 terms in this sum: 13 terms for each "o'clock" from 8 o'clock a.m. to 8 o'clock p.m., inclusive, and 1 term to include Rhonda herself. (That is, three people had heard the rumor by 8:00 a.m.) Use this formula for the sum of a geometric sequence:

$$S_n = \sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r},$$

where r is the common ratio and $n + 1$ is the number of terms. We have

$$S_{13} = \frac{1 - 2^{13+1}}{1 - 2} = \frac{1 - 16384}{-1} = 16383.$$

Alternate solution: Observe that 2^{13} = 8,192 people will hear the rumor at 8:00 pm. At that point, the number of people who have already heard the rumor is 8,192 – 1. The total is 16,383 people.

6. 13,824. If you know none of the three numbers, then there are 25 possibilities for each, so the number of locker combinations is $25^3 = 15{,}625$. However, if you can remember one number, then you reduce the options that you have to try. Suppose, for instance, you remember that one of the numbers is 15. If 15 is the first number, then there are 25 possibilities for each of the following two numbers, resulting in $25^2 = 625$ options to try. There are 625 possibilities if 15 is in the second position and 625 possibilities if 15 in the last position. But using this approach causes overcounting: We have double-counted the 24 options of each of the forms (15, 15, X), (15, X, 15), and (X, 15, 15) with X ≠ 15; and we have counted (15, 15, 15) three times. Eliminating the overcount, we calculate the number of options we must try if we know one of the numbers: 3(625) – 3(24) – 2 = 3(625) – 3(25) + 1 = 1801. This represents a reduction of 15,625 – 1,801 = 13,824 possibilities.

The Venn diagram illustrates the overcounting issue discussed in the

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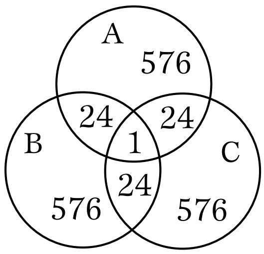
solution above. Let A = {15 is the first number of the combination}; let B = {15 is the second number of the combination}; and let C = {15 is the third number of the combination}. The diagram shows why we count a total of 1801 options when we know exactly one of the numbers but not its position.



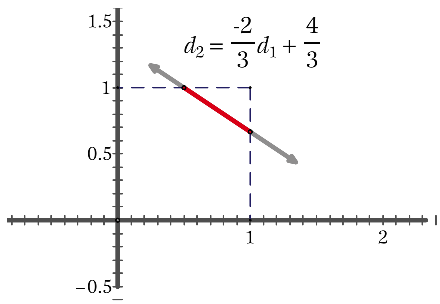
7. 40 firms. Let e be the number of yearbook editors, let s be the number of noneditor staff members without part-time jobs, and let p be the number of noneditor staff members with part-time jobs. We write the following system:

$$\begin{Bmatrix} e+s+p=15 \\ (0.10)(200)e+(0.08)(200)s \\ \qquad +(0.05)(200)p=200 \end{Bmatrix}$$

The second equation simplifies to $2e + (8/5)s + p = 20$, which indicates that s must be a multiple of 5, because $(8/5)s$ must be an integer. If $s = 5$, then $e + p = 10$ and $2e + p = 12$, so $e = 2$. If we let $s = 10$, then $e + p = 5$ and $2e + p = 4$, resulting in a negative value for e. Therefore, there must be 2 editors and each must approach $10\% \cdot 200 = 20$ firms, so they approach 40 firms collectively.

8. 17 gallons. The purchased gas filled $7/8 - 2/8 = 5/8$ tank. If \$42.00 filled 5/8 tank, then d dollars will fill 1 tank. So $d = 42/(5/8) = 42 \cdot 8/5 = \$67.20$ to fill an empty tank. Since the price per gallon was \$3.949, the tank will hold about $67.2/3.949 \approx 17$ gallons.

9. Possible answers include these: (1) The smaller brother sits at the end and the larger brother sits 2/3 m from the fulcrum; or (2) the larger brother sits at the end and the smaller brother sits 1/2 m from the fulcrum. Assign variables as follows: w = Nick's weight; d = Nick's distance from the fulcrum; w_1 = the weight of the smaller brother; d_1 = the smaller brother's distance from the fulcrum; w_2 = the weight of the bigger brother; and d_2 = the bigger brother's distance from the fulcrum. Begin with the fact that Nick must balance his two brothers: $w \cdot d = (w_1 \cdot d_1) + (w_2 \cdot d_2)$. Express the brothers' weights in terms of Nick's weight and replace d with 1 to obtain $1w = (1/2)wd_1 + (3/4)wd_2 \rightarrow 1 = (1/2)d_1 + (3/4)d_2$. Solving for d_2 in terms of d_1 gives us $d_2 = (-2/3)d_1 + 4/3$. Neither d_1 nor d_2 can be greater than 1 since the available length of board is 1 m. Therefore, all solutions lie on the portion of the graphed line highlighted in the figure. In particular, if $d_1 = 1$, then $d_2 = 2/3$; and if $d_2 = 1$, then $d_1 = 1/2$.



10. 361.11 ft. The solution requires using the Pythagorean theorem twice. First, find the distance d between the two towers, which is the hypotenuse of a right triangle with legs 300 and 200: $d^2 = 300^2 + 200^2 = 130{,}000 \rightarrow d \approx 360.56$ ft. This distance is one leg of a second right triangle whose other leg is the difference between the two towers' heights and whose hypotenuse—the length of cable c—we must find: $c^2 = 130{,}000 + (50 - 30)^2 = 130{,}400 \rightarrow c \approx 361.11$ ft.

11. 18.9 ft. Since the 50 ft. tower and the 30 ft. tower are separated by a distance of approximately 360.555 ft. (see the solution to the problem for November 10), the slope of the first zip line will be $(30 - 50)/360.555$, approximately -0.0555. The new tower will be 200 ft. from the 30 ft. tower. Designate the height of the new tower as h and the position of the 30 ft. tower as zero. Solve the equation to find h: $(h - 30)/(200 - 0) = -0.0555 \rightarrow h \approx 18.9$ ft.

12. $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

We start by examining cases for 1×1, 2×2, 3×3, and 4×4 checkerboards before considering the general case (see solution 12 table).

From the first four rows of the table, we see that the entries are all perfect squares. Therefore, we write the total number of squares on each checkerboard as the sum of the squares from 1 to n:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution 12

Dimension	Number of 1×1 Squares	Number of 2×2 Squares	Number of 3×3 Squares	Number of 4×4 Squares	Number of $n \times n$ Squares	Total Number of Squares
1×1	1					1
2×2	4	1				5
3×3	9	4	1			14
4×4	16	9	4	1		30
⋮	⋮	⋮	⋮	⋮		
$n \times n$	n^2	$(n-1)^2$	$(n-2)^2$	$(n-3)^2$	1	

13. Approximately 4.9 in. Begin by considering a two-dimensional cross section of the paper towel roll, sliced perpendicular to the interior cardboard tube. Calculate the area of the outer circle: $A = \pi r^2 = 16\pi$. The radius of the hollow tube is 0.5 in.; subtract the area occupied by the tube in the cross section: $16\pi - 0.25\pi = 15.75\pi$ in.^{2} of paper towels. The area of the cross section will be proportional to the number of towels. The promotion states that the special roll will have 50% more towels, so we can multiply by 1.5 to find the area occupied solely by paper towels on the special roll: $15.75\pi \cdot 1.5 = 23.625\pi$ in.^{2}. We still must find the radius of the special roll. The area of paper plus the area of hollow tube equals the area of the cross section: $23.625\pi + 0.25\pi = 23.875\pi$ in.^{2}. Solving for the radius, we have 23.875π in.^{2} = $\pi r^2 \to r \approx 4.89$ in.

14. 798. If r is the common ratio, we can rewrite the sequence as 6, $6r$, $6r^2$, $6r^3$, and $6r^4$. Since $6r^4 = 14{,}406$, we find $r = 7$. Then $a = 6(7) = 42$; $b = 6(7^2) = 294$; and $c = 6(7^3) = 2058$. The arithmetic mean is $(42 + 294 + 2058)/3 = 798$.

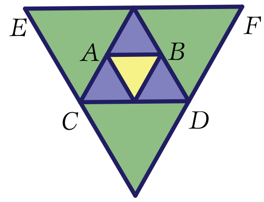
Alternate solution: Solve for r, represent the mean algebraically, and then substitute for r: $(6r + 6r^2 + 6r^3)/3 = 2r(1 + r + r^2) = 14(1 + 7 + 49) = 14 \cdot 57 = 798$.

15. 35.26°. Segment XZ in the figure is the space diagonal. Segment YZ is a face diagonal that lies in the base of the cube. We need the measure of angle XZY; let $\theta = m\angle XZY$. Let $XY = a$. By the Pythagorean theorem, $YZ = a\sqrt{2}$. Then

$$\tan\theta = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1}\frac{1}{\sqrt{2}} \approx 35.26°.$$

16. $T_n = 1.5 \cdot 2^n$; or $T_n = 2T_{n-1}$ with $T_1 = 3$. A theorem states that the segment joining the midpoints of two sides of a triangle is half the length of the third side. Then AB is half CD, which is, in turn, half EF. Since the innermost triangle has perimeter 3, the next larger triangle has perimeter 6, and third triangle has perimeter 12. Each triangle's perimeter is twice the perimeter of the previous triangle in the sequence. If T_n represents the perimeter of the nth triangle, then a recursive definition is given by $T_n = 2T_{n-1}$ with $T_1 = 3$. For n equal to 1, 2, 3, and 4, we have T_n equal to 3, 6, 12, and 24 or, equivalently, $3 \cdot 2^0$, $3 \cdot 2^1$, $3 \cdot 2^2$, and $3 \cdot 2^3$. So an explicit expression for T_n is $3 \cdot 2^{n-1}$. We can have the exponent match the term number as follows: $T_n = 3 \cdot 2^{n-1} = 3(0.5) \cdot 2^n = 1.5 \cdot 2^n$.



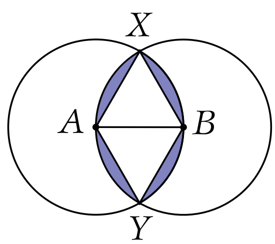
17. $\dfrac{4\pi - 3\sqrt{3}}{6} \approx 1.23$ units^2.

Label the points of intersection of the two circles X and Y. Construct $\overline{XA}$, $\overline{XB}$, $\overline{YA}$, $\overline{YB}$, and $\overline{AB}$. Each of these has length 1 because each is a radius of circle A or circle B. The region common to both circles has been divided into two congruent equilateral triangles and four congruent circular segments. To find the area of the segment bounded by $\overset{\frown}{XB}$ and $\overline{XB}$, subtract the area of triangle AXB from the area of sector XAB. Sector XAB has a central angle of 60°, so its area is $\pi/6$. Triangle AXB has area $s^2\sqrt{3}/4 = 1^2\sqrt{3}/4 = \sqrt{3}/4$, so the area of the segment is $\pi/6 - \sqrt{3}/4 = (2\pi - 3\sqrt{3})/12$. Combine the separate areas in one expression:

$$2 \cdot \frac{\sqrt{3}}{4} + 4 \cdot \frac{\left(2\pi - 3\sqrt{3}\right)}{12} = \frac{4\pi - 3\sqrt{3}}{6}$$

$$\approx 1.23 \text{ units}^2.$$

This value is between 1/3 and 1/2 the area of one of the circles, which seems to be a reasonable answer.



18. 16. Since Andrew earned 0 points for his 6 unanswered questions, lost 1/3 point for each of his 6 incorrect answers, and earned 1 point for each of the 24 correct answers, his score is $0(6) - (1/3) \cdot (6) + 1(24) = 22$. Blake's score is then $(2/3)22 = 44/3 = 14\ 2/3$.

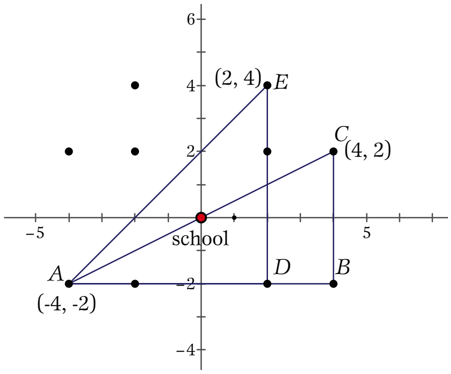
Let x represent the number of questions that Blake missed, and let y represent the number of questions that he answered correctly. Then $-(1/3)x + 1y = 44/3$. Since Blake did not leave any questions blank, we know that $x + y = 36$. Solve the system by substitution: $x + 44/3 + (1/3)x = 36 \to 4x + 44 = 108 \to x = 16$.

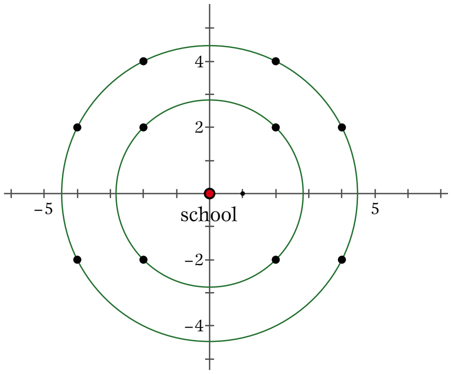
19. 4. Since the ratio is 3 : 4, 3/7 of the 28 students are boys, meaning that the class has 12 boys. This number remains the same when the ratio changes to 3 : 5. So 12 boys are 3/8 of the larger class. If c is the new class size, then $(3/8)c = 12 \to c = 32$. This is an increase of 4 students, who must be girls.

20. 42. If $\sqrt{1050x}$ is a whole number, then $1050x$ must be a perfect square. Begin by finding the prime factorization of 1050, which is $2 \cdot 3 \cdot 5^2 \cdot 7$. To make the product $1050x$ a perfect square, every prime factor must be squared, so x must be $2 \cdot 3 \cdot 7 = 42$.

21. 1.12 mi. If we use a coordinate plane and put the school at (0, 0), we end up with the following possible locations for Tom's ending location: (±2, 4); (±2, ±2); or (±4, ±2). We are looking for the farthest Tom could end up from Mark's house, so we need to find the points that have the greatest distance between them. After plotting the points on the coordinate plane, we make two observations. First, because of symmetry, more than one pair of points will have the same "greatest distance" between them. Second, we do not need to check more than two distances—namely, the distance between (–4, –2) and (4, 2) and the distance between (–4, –2) and (2, 4). These two distances are shown as AC and AE in the figure. Segment AC is the hypotenuse of $\triangle ABC$; the legs have lengths 8 and 4. Segment AE is the hypotenuse of $\triangle ADE$; the legs have lengths 6 and 6.

Since $8^2 + 4^2 = 80 > 6^2 + 6^2 = 72$, $AC = \sqrt{80} \approx 8.94$ is the longer distance. The units in the problem are blocks of length 1/8 mi., so Tom can end up at most 8.94(1/8) ≈ 1.12 mi. from Mark's house.





22. Mine B, mine C, and mine A. The abbreviation BTU stands for British Thermal Unit. A BTU is the amount of heat required to raise the temperature of one pound of water one degree Fahrenheit. The abbreviation MBTU means 1 million BTUs.

The coal from mine A costs \$42 per ton, which is equivalent to \$42 per 2000 lbs. If the coal contains 11,000 BTU per pound, then 2000 lbs. contain 22,000,000 BTU, or 22 MBTU. So the cost is 42/22 = \$1.91 per MBTU. The coal from mine B costs \$43 per ton and contains 11,500 BTU per pound. So the cost is 43/(11.5 • 2) = \$1.87 per MBTU. The coal from mine C costs \$41 per ton and contains 10,800 BTU per pound. So the cost is 41/(10.8 • 2) = \$1.90 per MBTU. The ranking from lowest cost to highest cost follows: coal from mine B, coal from mine C, and coal from mine A.

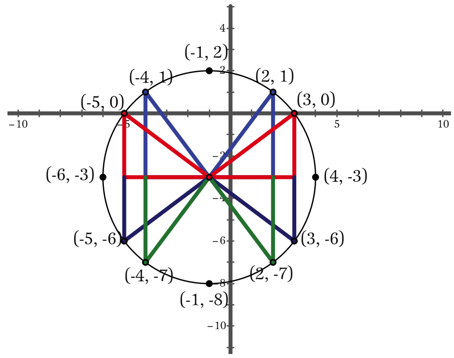
23. The bid from mine A. Add the transportation cost per ton to the coal cost per ton before dividing by the coal quality in MBTU per ton to find the true cost per MBTU for the electric utility. The cost of purchasing and transporting coal from mine A is (42 + 4)/22 = \$2.09 per MBTU; the cost from mine B is (43 + 7)/23 = \$2.17 per MBTU; and the cost from mine C is (41 + 5)/21.6 = \$2.13 per MBTU. The utility should accept the bid from mine A because the cost per MBTU is the lowest when the transportation cost is included in the total cost.

24. 19. The time required for a 100-car train to be loaded, to travel to the utility, and to be unloaded is 6 hr. + 14 hr. + 3(100) min. = 25 hr. The return trip requires 80% of 14 hr. = 11.2 hr.; therefore, one round trip takes 25 + 11.2 = 36.2 hr. The month of November has 30 days, or 30 • 24 = 720 hr. A maximum of

$$\left\lfloor \frac{720}{36.2} \right\rfloor = 19 \text{ round trips}$$

can be completed in November.

25. Twelve points: $(-1, -3) + (0, \pm 5)$, $(-1, -3) + (\pm 5, 0)$, $(-1, -3) + (\pm 3, \pm 4)$ and $(-1, -3) + (\pm 4, \pm 3)$. Two ordered pairs will have the same x-coordinate: $(-1, 2)$ and $(-1, -8)$. Two will have the same y-coordinate: $(-6, -3)$ and $(4, -3)$. Any other ordered pair that is 5 units from $(-1, -3)$ will be a vertex of a right triangle with hypotenuse 5. There is only one Pythagorean triple with hypotenuse 5: the 3-4-5 triangle. The figure at the top of column 3 shows all possible arrangements of this triangle, such that $(-1, -3)$ is one endpoint of the hypotenuse.



26. 32.8 in. The distance traveled in 1 revolution equals the tire's circumference. We are given that traveling 1 mile requires 615 revolutions. Use dimensional analysis to change the units to inches per revolution:

$$\frac{1 \cancel{\text{mi.}}}{615 \text{ rev.}} \bullet \frac{5280 \cancel{\text{ft.}}}{1 \cancel{\text{mi.}}} \bullet \frac{12 \text{ in.}}{1 \cancel{\text{ft.}}}$$

$$= \frac{63360 \text{ in.}}{615 \text{ rev.}} = 103.02 \text{ in./rev.}$$

This gives us the circumference, so $C = \pi d = 103.02$ in. $\rightarrow d \approx 32.8$ in.

27. 0.017 mph. The tip of the minute hand completes 1 revolution in 1 hr. We need to change this rate to mph:

$$\frac{1 \cancel{\text{rev}}.}{1 \text{ hr.}} \bullet \frac{2\pi \bullet 14 \cancel{\text{ft}}.}{1 \cancel{\text{rev}}.} \bullet \frac{1 \text{ mi.}}{5280 \cancel{\text{ft}}.}$$

$$= \frac{28\pi \text{ mi.}}{5280 \text{ hr.}} \approx 0.017 \text{ mi./hr.} = 0.017 \text{ mph}$$

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28. 840 ft. The tip of the minute hand makes 1 complete revolution each hour. If its average speed were 1 mph, the circular path it traveled would be 1 mile in circumference. If $C = 2\pi r = 1$ mi., then $r = 1/(2\pi) \approx 0.159$ mi. is the approximate length of the minute hand in miles. Multiply by 5280 ft./mi. to convert to feet: $0.159 \cdot 5280 \approx 840$ ft.

29. 9 gallons and 2 Super Slurps or 4 gallons and 12 Super Slurps. Set up and solve a system of linear equations to find the price of a gallon of gas and a Super Slurp. Let g represent the price of a gallon of gas; let s represent the price of a Super Slurp. Then $10g + 2s = 33.1$, and $7g + 3s = 24.85$. Multiply the first equation by 3, multiply the second by 2, and subtract the second from the first: $30g + 6s = 99.3$, and $14g + 6s = 49.7 \rightarrow 16g = 49.6 \rightarrow g = \$3.10$ and $s = \$1.05$.

Frank used two bills no larger than $20 bills and received no change. We will assume that he did not pay with a $2 bill and used only one or two of the following: $1, $5, $10, or $20. We can make lists for the cost of g gallons of gas and s Super Slurps.

Solution 29

Number of Gallons of Gas	Cost	Number of Super Slurps	Cost
1	3.10	2	2.10
2	6.20	4	4.20
3	9.30	6	6.30
4	12.40	8	8.40
5	15.50	10	10.50
6	18.60	12	12.60
7	21.70	14	14.70
8	24.80	16	16.80
9	27.90	18	18.90
10	31.00	20	21.00
11	34.10	22	23.10
12	37.20	24	25.20
		26	27.30
		28	29.40
		30	31.50
		32	33.60
		34	35.70
		36	37.80
		38	39.90

We need not concern ourselves with odd numbers of Super Slurps because their cost would have a cent value ending in 5. All the possibilities for the cost of gas end in 0. Therefore, it would be impossible to pair up any number of gallons of gas with an odd number of Super Slurps and get a total that could be attained with only two bills (see solution 29 table).

Looking at the table, we see that the only combinations that would add up to an amount that could be paid with exactly two bills are these:

- 9 gallons of gas and 2 Super Slurps, for a cost of $30. Frank would have paid with a $20 bill and a $10 bill.
- 4 gallons of gas and 12 Super Slurps, for a total of $25. Frank would have paid with a $20 bill and a $5 bill.
- 0 gallons of gas and 20 Super Slurps, for a total of $21. Frank would have paid with a $20 bill and a $1 bill. This possibility stretches the rules because Frank did not buy any gas.

Alternate solution: Notice that to have an integral number of dollars, each increase of 1 gallon of gas decreases the number of Super Slurps by 2. That is, $1g + 18s = \$22$; $2g + 16s = \$23$; and so on. The only sums greater than or equal to 22 that satisfy the two-bill restriction are $25, $30, and $40. To get $25, we need $4g$ and $12s$; to get $30, we need $9g$ and $2s$; there is no way to get $40.

30. 12 gallons. Assign variables as follows: let t be the time for Donald to fill the dunk tank; let r be the fill rate for the dunk tank; let g be the number of gallons in the duck-pull pond. Donald's work can be modeled by $rt = 120$. Daisy's work can be modeled by $r/5 \cdot t/2 = g \rightarrow rt = 10g$. Substituting 120 for rt, we have $120 = 10g \rightarrow g = 12$.

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