How Many in One?

Valerie V. Sharon and Mary B. Swarthout

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Fraction division is one of the least understood operations in school mathematics (Van de Walle, Karp, and Bay-Williams 2010). However, just having knowledge of the invert-and-multiply algorithm does not guarantee that students will recognize a problem situation involving fraction division. The National Research Council (2001) recommends teaching fraction division within a real-world context to enable students to build connections between whole number operations and division by fractions.

The Common Core State Standards for Mathematics (CCSSM) calls for students to connect their understanding of division by whole numbers to division by unit fractions (grade 5) and ultimately to use the inverse relationship between multiplication and division to divide by any fraction (grade 6) (CCSSI 2010). The Common Core also emphasizes students' ability to solve word problems involving division by fractions.

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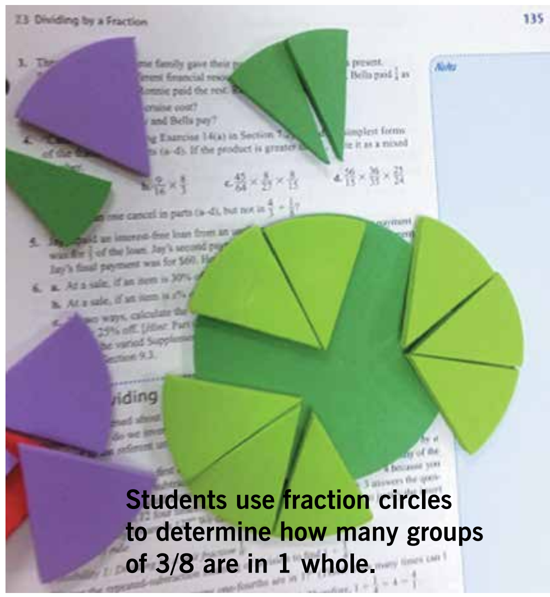
We used the context of serving pizza to a crowd of people to help prospective elementary school teachers understand fraction division. Students worked with pizza models and drew diagrams to show how many thirds are in 1 or more pizzas and then applied proportional reasoning to determine the number of servings in 50 pizzas. To connect the process of determining how many thirds are in a given number to the operation of division, we reminded students that division can be modeled as measuring how many groups of the divisor are in the dividend. For example, to find the quotient of 15 ÷ 3, we count the number of groups of 3 that are in 15. Similarly, division by 1/3 involves counting how many thirds are in the dividend. Students were asked to consider how they could use the unit rate (how many one-thirds are in 1 whole) to find additional quotients. For example, knowing that 1 whole has 3 of the thirds (1 ÷ 1/3) tells us that 2 wholes will have twice as many. Therefore,

$$2 \div \frac{1}{3} = 2 \times \left(1 \div \frac{1}{3}\right) = 2 \times 3 = 6.$$



Students use fraction circles to determine how many groups of 3/8 are in 1 whole.

When the divisor is a unit fraction, the denominator tells us the number of groups in 1 whole, or the unit rate in terms of the divisor, which leads to the general rule that

$$a \div \frac{1}{b} = a \times b \text{ because } 1 \div \frac{1}{b} = b$$

(Sowder, Sowder, and Nickerson 2013). Next, we considered the nonunit divisor 3/8. Using fraction circles, students found 2 2/3 groups of 3/8 in 1 unit pizza (see **fig. 1**). Rewriting

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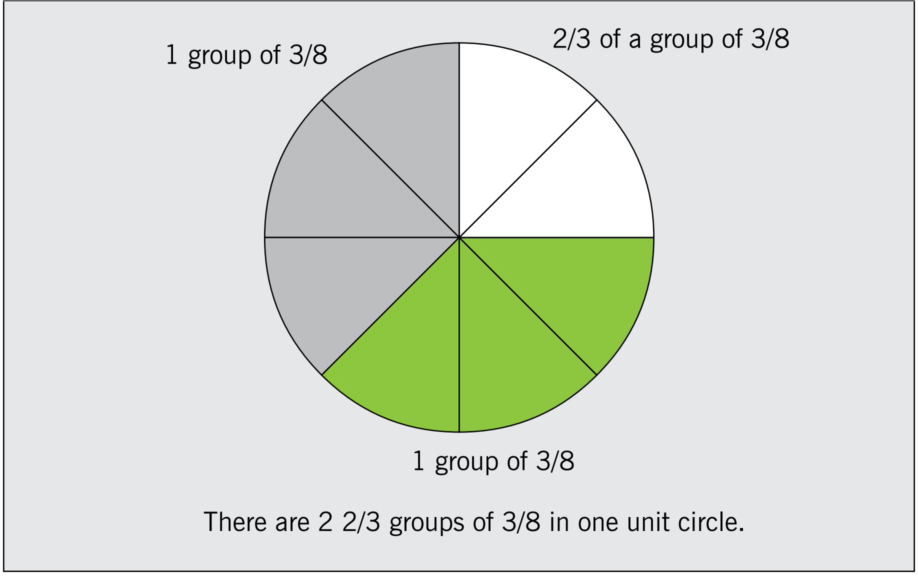
that answer as an improper fraction allowed students to see that the number of servings of size 3/8 in 1 whole pizza, or unit rate of the divisor, 8/3, is actually the reciprocal of the divisor 3/8. Students then used the unit rate to find other quotients since any number of pizzas will have 8/3 times as many servings as 1 whole pizza ($a \div a/b = a \times b/a$ because $1 \div a/b = b/a$). These explorations led our students to a deeper understanding of fraction division, as well as the invert-and-multiply algorithm. See Cavey and Kinzell (2014) for a comprehensive discussion of developing the invert-and-multiply algorithm using the idea of unit rates.

Fig. 1 The shaded regions show two groups of 3/8 in one unit circle. The unshaded regions represent 2/3 of another group of 3/8.



TEACHER INSTRUCTIONS

Plan on using two 45-minute class periods to complete both activities that are provided in this exploration. Our prospective teachers completed these activities in small groups, thus providing opportunities for peer discussion. Each group should have access to several sets (3–4) of fraction circles. You can easily cut fraction circles out of foam or construction paper if you have access to a die-cut press. We used a commercially produced pizza game, which added an element of realism to the activity.

Launch

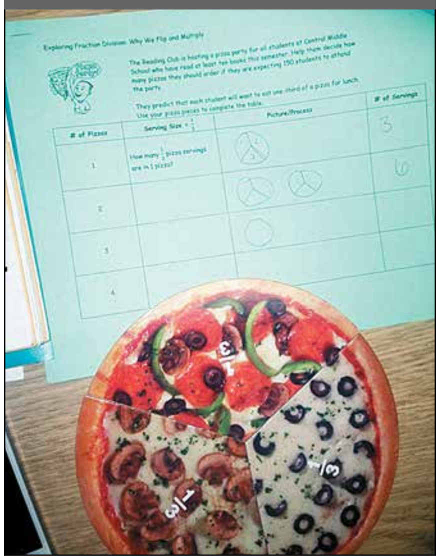
Begin the activity by reminding students what they already know about whole-number division. Introduce a context and ask students to think about division as an equal-sharing situation or as a repeated-subtraction scenario (measurement). For example, consider the context of serving 6 cookies to some students. If there are two students, each student will receive 3 cookies each. In this equal-sharing situation represented by 6 ÷ 2, the divisor, 2, tells us the number of groups to make; the quotient, 3, represents the size of each serving (group). However, suppose we wish to give each student 2 cookies. To how many students can we serve cookies if we only have 6 cookies? The division problem, 6 ÷ 2, now becomes a measurement situation in which the divisor gives us the size of each serving (2 cookies) and the quotient tells us how many servings we can make (3 groups of size 2). Ask students to draw pictures to represent how many servings of size 2 they could make if they started with 9 cookies. They should find that they can make 4 servings of 2 cookies and have 1 cookie leftover. The remaining cookie is enough to make 1/2 serving of size 2, so we have 2 1/2 servings. Gregg and Gregg (2007) provide additional activities on measurement and servings of cookies.

Activity 1

After introducing the context of planning a pizza party, demonstrate how to find the number of servings of size 1/3 in 1 whole pizza using the unit circle and the 1/3 circle pieces. Draw a picture of the solution on the recording sheet, being sure to number each serving (see **fig. 2**). Allow time for students to work with a partner or group members to complete the remaining sections of **activity sheet 1**. Reserve approximately 10 minutes to discuss student responses to the follow-up questions.

During a class discussion, ask students how they used the results from the table to answer question 1 (How many servings of size 1/3 pizza could they get from 10 pizzas?). Some students may have drawn 10 circles,

Fig. 2 Problem 1 from **activity sheet 1** asked students to find how many servings of 1/3 are in 1 pizza.



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each divided into thirds, and counted the number of thirds; others may have multiplied 10 by 3. Revoice their explanations by stating that once we know that there are 3 servings of size 1/3 in 1 pizza, we can find the number of thirds in any quantity of pizzas by multiplying the number of pizzas by 3. This idea of unit rates is also used to answer questions 2 and 3.

Some students may struggle to understand why 1/2 divided by 1/3 is 3/2, or 1 1/2 (see question 4b). Have students use their fraction circles to see how many servings of size 1/3 they can make with 1/2 circle. Cover the 1/2 circle with sixths and then layer one of the thirds on top to show that it takes two of these slices (sixths) to make 1 whole serving of size 1/3 with one slice remaining, so each of the slices (sixths) represents 1/2 serving of size 1/3. Since there are 3 of these 1/2 servings of size 1/3 in 1/2 pizza, we know that 1/2 divided into groups of 1/3 is 3/2 or 1 1/2.

Activity 2

The second activity serves as an extension of the first activity and is designed to reveal the unit rate for a nonunit fraction, a/b. The exploration demonstrates why we multiply by the reciprocal of the divisor to find the quotient. Begin the activity by reading the treasurer's decision to change the serving size to 3/8 pizza. Demonstrate how to use the fraction pieces to divide 1 whole pizza into servings of size 3/8. Students will find that 1 pizza has 2 whole servings of size 3/8 with 2/8 leftover. Ask what part of a serving the remaining pizza will make. Some students will want to name the remaining part based on the whole pizza. Emphasize that the 2/8 represents 2 of the 3 parts needed for 1 whole serving. Therefore, 1 whole pizza will make 2 2/3 servings of size 3/8.

The remaining questions provide the opportunity for students to apply the unit rate of 2 2/3 to find quotients involving other dividends. Be sure that students recognize the reciprocal relationship between the divisor (3/8) and the unit rate (see in question 2). Although students may want to use their fraction pieces to verify answers to questions 3 and 5, encourage them to also think about how they could apply the unit rate to solve each problem. You may wish to have students share their word problems for 2 ÷ 3/4 (question 6). Verify that each question accurately represents the situation because some students may have difficulty identifying the appropriate unit of the divisor or may write a problem that involves multiplying by 3/4 instead of dividing.

UNDERSTANDING FOR SUCCESSFUL SOLVING

Van de Walle, Karp, and Bay-Williams (2010) suggest that carefully designed tasks, such as the ones described here, can be used to help students understand fraction division. They note that working with a set of similar problems involving whole-number dividends and fraction divisors will lead students to recognize that they need only multiply the dividend by the reciprocal of the divisor. Understanding fraction division as more than "easy as pie, just invert and multiply" will enable students to successfully solve fraction division problems.

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activity sheet 1

Name ______________________________

DIVIDING BY A UNIT FRACTION

The Reading Club at Central Middle School is hosting a pizza party for all students who have read at least 10 books this semester. Help the club members decide how many pizzas they should order if they are expecting 150 students to attend the party.

They predict that each student will want to eat 1/3 pizza for lunch. Use your pizza pieces to complete the table.

No. of Pizzas	Serving Size = 1/3 pizza	Picture/Process	No. of Servings
1	How many 1/3 pizza servings are in 1 pizza?		
2	How many 1/3 pizza servings are in 2 pizzas?		
3	How many 1/3 pizza servings are in 3 pizzas?		
4	How many 1/3 pizza servings are in 4 pizzas?		

Use the results from your table to answer the following questions:

1. How many servings of size 1/3 pizza could they get from 10 pizzas?

2. How many pizzas do they need to order for 150 students if each student gets 1/3 pizza?

3. The club president does not want to order that many pizzas. She suggests that they limit each person to 1/6 pizza. How many pizzas will they need to order for 150 students if each student gets 1/6 pizza?

4. The process of dividing a pizza into servings of 1/3 pizza can be written symbolically as $1 \div 1/3$. Because there are 3 servings of 1/3 in 1 whole pizza, we write $1 \div 1/3 = 3$.

a. Write one expression using division and one expression using multiplication to find how many servings of 1/3 pizza will be in 12 pizzas.

b. Write an expression to find how many servings of 1/3 pizza will be in 1/2 pizza. Calculate the answer and explain why the answer makes sense.

5. Explain how to divide a number by a unit fraction like 1/3.

6. Draw a picture to show how many servings of size 1/3 pizza would be in 1 1/2 pizzas. Explain your answer using your knowledge of the number of servings of this size in 1 pizza.

7. Write an expression for finding out how many servings of 1/3 pizza would be in 1 1/2 pizzas. Use arithmetic to calculate the answer. Compare your answer with your solution to question 6.

activity sheet 2

Name ______________________________

DIVIDING BY A NONUNIT FRACTION

The treasurer complains that 1/6 pizza is not enough, and besides, the pizzeria cuts each pizza into 8 slices, not 6 slices. He suggests that they allow each student to have 3/8 pizza. Use this information to complete the table. Include partial servings in your answer.

No. of Pizzas	Serving Size = 3/8 pizza	Picture/Process	No. of Servings
1	How many 3/8 pizza servings are in 1 pizza?		
2	How many 3/8 pizza servings are in 2 pizzas?		
3	How many 3/8 pizza servings are in 3 pizzas?		
5	How many 3/8 pizza servings are in 5 pizzas?		

1. About how many pizzas should they order for all 150 students if each student is served 3/8 pizza? Explain your answer.

2. With a serving size of 3/8 pizza, there are 2 2/3 servings in one whole pizza, 1 ÷ 3/8 = 2 2/3. We can rewrite the mixed number as an improper fraction, 2 2/3 = 8/3. What is the relationship between the unit rate (8/3) and the divisor (3/8)?

3. Use the idea that 1 pizza has 8/3 of a group (serving) of 3/8 to answer the following questions. Write any mixed numbers as improper fractions.

a. Write an expression to find how many servings of 3/8 pizza will be in 4 pizzas.

b. Write an expression to find how many servings of 3/8 pizza will be in 3/8 pizza. Explain why the answer makes sense.

4. Explain how to divide a number by a nonunit fraction like 3/8.

5. Draw a picture to show how many servings of size 3/8 would be in 1 1/2 pizzas. Explain your answer using your knowledge of the number of servings of this size in 1 pizza.

6. Write a word problem that could be solved by 2 ÷ 3/4.

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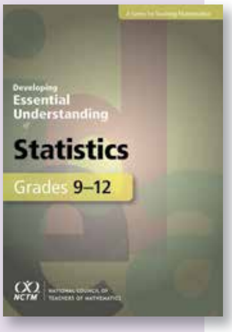

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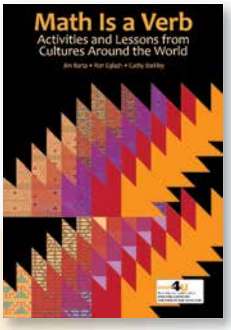

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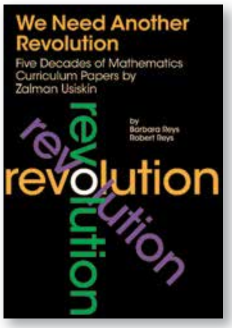

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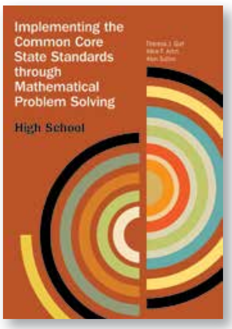

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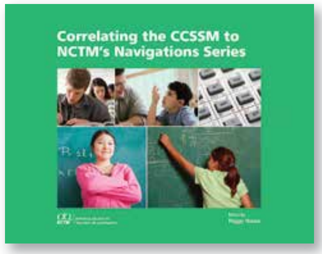

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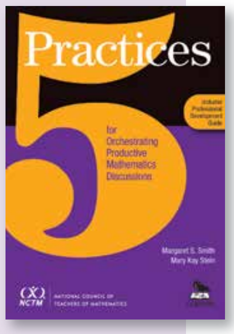

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