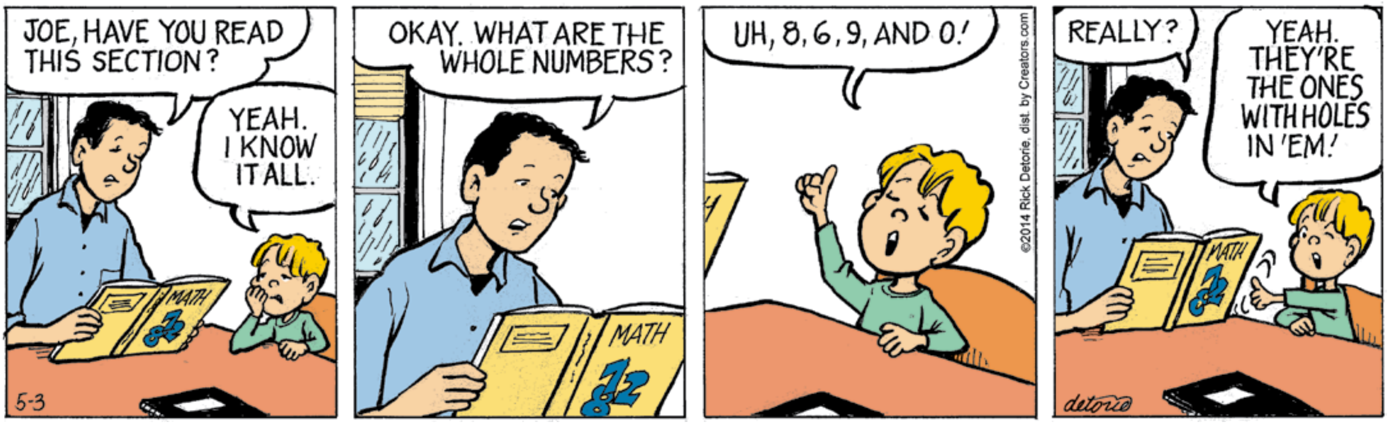
cartoon corner

Name ______________________________

ONE BIG HAPPY® by Rick Detorie



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LET'S BE RATIONAL ABOUT NUMBERS

1. a. Describe the set of *whole numbers*, and point out the "hole" in Joe's thinking.

b. How many whole numbers use exactly 2 digits?

c. Is the difference between 2 whole numbers always a whole number? Explain.

2. a. An *integer* is any number from the set {. . . , –3, –2, –1, 0, 1, 2, 3, . . .}. If all members of a set A are members of a set B, then set A is a subset of set B. Are the integers a subset of the whole numbers? Explain.

b. Which is greater, –5 or –4? Describe a real-world example that supports your answer.

c. Is the quotient of 2 nonzero integers always an integer? Explain.

3. a. A *rational number* is a number that can be written as a fraction (or mixed number) with integers in the numerator and denominator (where the denominator is not equal to 0). Are the integers a subset of the rational numbers? Explain.

b. Explain why $0.\overline{3}$ is a rational number.

c. Circle each number below that is *not* a rational number.

$0 \quad -5 \quad \frac{11}{7} \quad -1\frac{3}{4} \quad -0.75$

$0.1\overline{6} \quad 3.14 \quad \pi \quad \sqrt{2} \quad 5^3$

d. How many rational numbers are between 1/3 and 2/3? Explain.

4. a. A decimal that does not terminate and does not repeat is an *irrational number*. For example, $\sqrt{3}$ is irrational because its decimal equivalence, 1.73205 . . . , is an infinite nonrepeating decimal. Circle each number below that is irrational.

$0.222\ldots \quad 0.08\overline{3} \quad \sqrt{4} \quad -0.\overline{142857}$

$\pi \quad \sqrt{5} \quad -(0.05)^3 \quad \sqrt{10}\cdot\sqrt{100}$

b. What is "irrational" about the date 3/14/15? How often does that date occur? Give the 4-digit year when that date will next occur.

CHALLENGE

5. Write a mathematical argument (that does not involve rounding) to show that 1 is equal to the repeating decimal $0.\overline{9}$.

from the March 2015 issue of mathematics teaching in the MIDDLE SCHOOL

Edited by **David B. Spangler**, McGraw-Hill Education, and **Katie A. Hendrickson**, Athens Middle School, Athens, Ohio. Classroom teachers interested in field-testing or submitting a cartoon should contact Cartoon Corner at **mtms@nctm.org**. The cartoons must include the date and the newspaper syndicate that holds the copyright.

SOLUTIONS

1. a. The set of whole numbers is {0, 1, 2, 3, . . .}. The numbers 8, 6, 9, and 0 are just a subset of the set of whole numbers.

b. There are 90 whole numbers that use exactly 2 digits. The numbers are 10 through 99.

c. No. When you subtract a larger whole number from a smaller whole number, the difference is not a whole number because it is negative.

2. a. No. There are members of the set of integers that are *not* members of the set of whole numbers. (The whole numbers are a subset of the integers.)

b. Although –4 is greater than –5, a student could respond that either is greater depending on the context provided. Sample rationales for –4 > –5 could include these: A temperature of –4°F is warmer than a temperature of –5°F. A person is better off having a debt represented by –\$4 than a debt represented by –\$5. However, a student could argue that a debt of –\$5 is a greater debt than a debt of –\$4. Be sure to discuss all answers and students' justifications. (See the field-test comments from Deborah Regal Coller.)

c. No. Sample explanation: The quotient of two nonzero integers is an integer only when the divisor is a factor of the dividend.

3. a. Yes. Every integer is also a rational number. An integer can be written with a 1 in the denominator, such as –5/1, 0/1, and 3/1.

b. $0.\overline{3}$ can be written as the fraction 1/3.

c. π and $\sqrt{2}$ are not rational numbers.

d. There are infinitely many rational numbers between 1/3 and 2/3. Sample explanation: There are infinitely many pairs of equivalent fractions for 1/3 and 2/3. Between each fraction in those pairs are other fractions.

4. a. $\pi, \sqrt{5}$, and $\sqrt{10} \cdot \sqrt{100}$ are irrational. Note that

$$\sqrt{10} \cdot \sqrt{100} = \sqrt{1000} \approx 31.6228.$$

b. The date 3/14/15 represents the first 5 digits in the decimal expansion of pi, a nonterminating, nonrepeating decimal. This special Pi Day, 3/14/15, occurs just once each century. The next one will occur in the year 2115.

5. Many arguments are possible, including the following:
Argument 1: Begin with a fraction that is equivalent to a repeating decimal, such as

$$\frac{1}{3} = 0.\overline{3}.$$

Multiply both sides by 3 to obtain

$$\frac{3}{3} = 0.\overline{9}.$$

Because 3/3 = 1, we have $1 = 0.\overline{9}$.

Argument 2: Begin with $x = 0.\overline{9}$. Multiply both sides by 10 to obtain $10x = 9.\overline{9}$. Subtract x from both sides. Because x is equal to $0.\overline{9}$, subtract x from the left side and $0.\overline{9}$ from the right side, to obtain $9x = 9$. Divide both sides by 9 to obtain $x = 1$. Because $x = 1$, and $x = 0.\overline{9}$, we conclude that $1 = 0.\overline{9}$ (by the transitive property).

FIELD-TEST COMMENTS

My eighth-grade students were immediately engaged when they learned that this was a field test and that I valued not only how they worked through the mathematics embedded in the task but also their feedback about the task itself.

When my students argued about their responses to question 2b, I listened to their debates and asked questions to probe how they reasoned to support their evidence. What is real world? Number line? Thermometer? Altitude? Debt? Assets? My students who embraced the debtor-versus-lender scenario used correct real-world logic about their context. In much of the financial sector in the United States, the lender who holds the note with the greater debt stands to benefit most, whereas the debtor who owes the most is "in greater debt."

Given only two numbers to compare, greater than or less than in an open, real-world context may yield unexpected results. With an open invitation to create a model for a real-world situation, teachers need to be open to the claims, evidence, and reasoning that students present to teachers and peers. Students have much to share about their perceptions and experience, and adults have much to learn about how young people see the world.

To hone the question about the absolute values of –4 and –5, see this alternative: How would you visually

illustrate and explain in words why –4 is greater than –5?

Deborah Regal Coller
Pathfinder Middle School
Pinckney, Michigan

I used this activity with prealgebra and algebra 1 classes. The students found it very helpful that important words were italicized in the questions, thus helping the students think about and focus on what was being asked. As I walked around the room, I heard students say the italicized words and then discuss what the words meant. I believe that a good group activity sheet would contain a lesson on types of numbers.

Students were familiar with the various types of real numbers. After a brief class discussion of each type, I let them tackle the cartoon while they worked in groups of three. They were able to complete the activity sheet with little difficulty. For question 1b, most groups answered 89 numbers rather than 90 numbers. After I asked them to consider a smaller set of numbers, they were then able to translate that logic to the larger set and understand their mistake. For question 2a, students had not encountered the term *subset* in a while, so they needed a brief refresher on what it meant.

They puzzled over question 3d for a while, until I asked them to think of a piece of gum that is equally shared among three kids. Then, one of the kids decides to split his or her shares with the other two, and so on. Using this example, they were able to see how there could be many fractions between 1/3 and 2/3.

The challenge question stumped all but one student who then shared his solution with the other students. This was a great lead-in to my next lesson on converting repeating decimals into fractions.

Machele Lynch
St. Patrick School
Carlisle, Pennsylvania

My eighth-grade algebra students tried this activity and loved it. The students began by completing a three-circle Venn diagram to classify numbers, using the sets of factors of 36, prime numbers less than 20, and even numbers less than 20. They had all used a two-circle diagram to compare and contrast in other subjects, but for many students, this was the first time they had used a Venn diagram in math or had considered

placing something outside the circles.

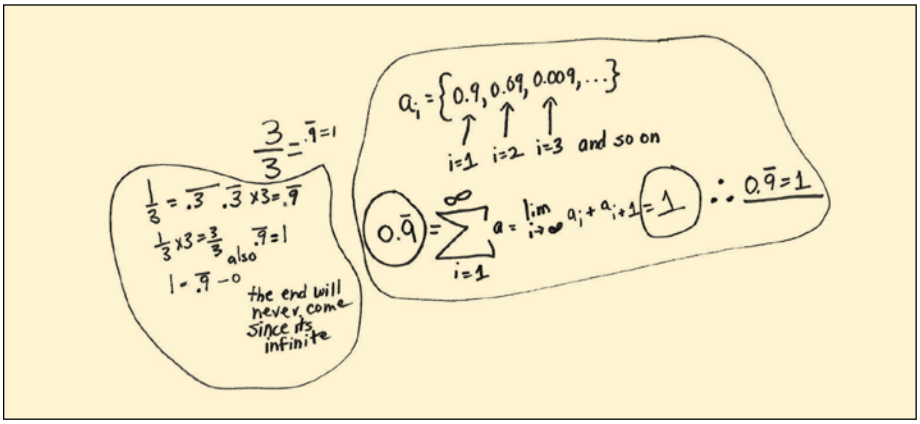
I distributed the activity, giving no directions. Students worked with a partner through all questions, without using additional resources. When everyone finished, the class discussed each question. The first question gave us an opportunity to uncover assumptions and look at things from different perspectives. As eighth graders, they are encouraged to reflect on everything from a current task to their entire middle school experience as they prepare for high school.

Some had difficulty with question 3d until a classmate suggested finding equivalent fractions. Then all agreed that there had to be an infinite number of rational numbers between the two given values. Other students wanted to confirm with their calculators that numbers such as $\sqrt{5}$ were nonterminating, nonrepeating decimals. Doing so helped them the next day when they were placing irrational numbers on a number line.

The class ended the lesson by creating a Venn diagram to show the relationships among the categories of numbers. After drawing a rectangle around rational and irrational numbers for the set of real numbers, someone asked why they are called "real." That led to the inevitable, "Does that mean some numbers are not real?" We added imaginary numbers outside the rectangle and mentioned that it would come up later in the year when they were solving quadratic equations.

Pam Haner
St. Catherine's School
Richmond, Virginia

These problems elicited great discussion among my students. My sixth-grade prealgebra students worked on the holes in Joe's thinking. They concluded that his answer was correct based on the homophone "hole," but they needed to correct his misconception of whole number.



Students were unfamiliar with the term *subset,* so we analyzed the prefix to determine the meaning of the sentence in problem 2. The relationships among natural numbers, whole numbers, integers, and rational and irrational numbers were defined, discussed, and applied to the questions throughout this activity. Real-world applications of integers ranged from negative temperatures and banking to wrong answers found on *Jeopardy!*

The Special Pi Day fascinated them. One student stated that we missed *Super* Pi Day, which would have occurred on 3/14/1592.

I included an illustration (see the student work above) of some students' explanations for 0.999 . . . = 1. It was an amazing discussion. One sixth grader wrote an entire explanation of limits.

As I looked at some students (with a "deer in the headlights" expression), I explained that they were mathematically traveling toward calculus and if they understood just a glimmer of what he was describing about limits, that was excellent. I suggested that they recall the book *A Wrinkle in Time* by Madeleine L'Engle. In it, Meg is trying to understand the explanation of a *tesseract* (traveling in the fourth dimension) and gets a glimmer of understanding.

Judy Kraus
Hyde Park School
Las Vegas, Nevada

OTHER IDEAS

- The word *rational* comes from the word ratio. Have students discuss how that relates to rational numbers.
- Have students investigate how to tell (without dividing) whether a fraction is equal to a terminating decimal or a repeating decimal.
- Ask students to prepare a graphic organizer depicting the relationships among whole numbers, integers, rational numbers, and irrational numbers.
- The Cartoon Corner that appeared in the March 2013 issue of *MTMS* deals with various aspects of pi. For a wealth of suggestions for celebrating Pi Day, go to the Math Forum at www.mathforum.org/t2t/faq/faq.pi.html.
- Check out *The Number Devil: A Mathematical Adventure* by Hans Magnus Enzensberger. The fourth chapter discusses, among other topics, converting fractions to decimals, repeating decimals, irrational numbers, square roots, a proof of $\sqrt{5}$, and the Pythagorean theorem.