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Moving Students to "the Why?"

Justification is a critical mathematical practice that must play a role in teaching and learning at all grade levels.

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Teacher: You got 102 centimeters for the 25th figure. So why is it 102?
Student: Because I did 23 times 4, plus 10.
Teacher: Yes. Good, those are the steps you did. But *why* did you take those steps? How do you know those calculations give you the right answer?
Student: Because you always take the figure number minus two and multiply that by 4 and add 10.

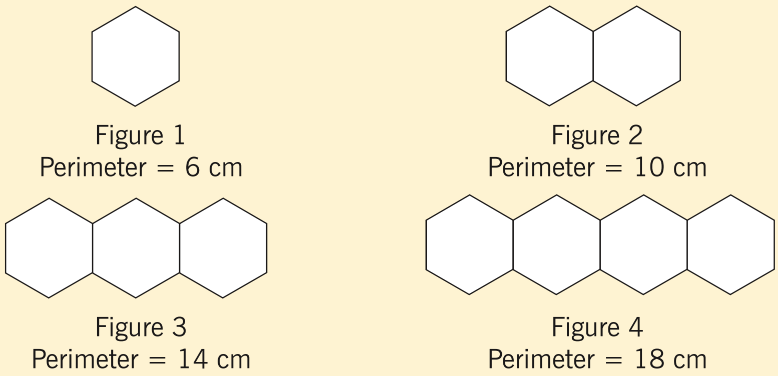
Does this scenario sound familiar? It was a fairly common occurrence in our classrooms as we started to push students to justify their results. We would press for "the why," wanting to hear more about their reasoning, and students would only give us "the how."

Having students share their reasoning and explain how they know something is true or correct is the process of justification. This mathematical practice goes by different names. NCTM (2000) describes it as part of the Process Standard of Reasoning and Proof; the Common Core State Standards for Mathematics (CCSSM) describes it in Mathematical Practice 3 as creating viable arguments and critiquing the reasoning of others (CCSSI 2010). We called this process *justification*. Justification (regardless of the exact term used) is central to doing and learning mathematics and should be incorporated across all grade levels (CCSSI 2010; NCTM 2000).

Teaching in a manner that supports students' justification, however,

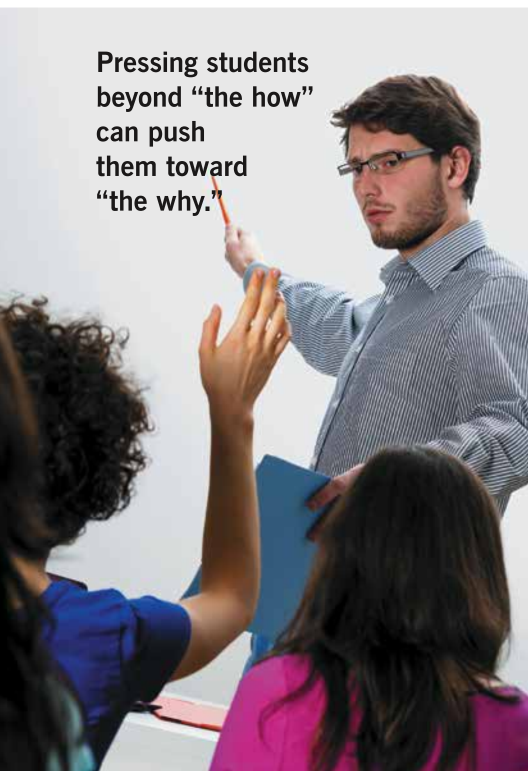
Fig. 1 The Hexagon task was assigned as a way to get students to work toward "the why."

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.



1. If the perimeter is continued, draw and find the perimeter of figure 5.
2. If the pattern of adding 1 hexagon to each figure is continued, what is the perimeter of the 25th figure in the pattern? Justify your answer.
3. Extension: How can you find the perimeter of *any* figure? (A figure with *n* hexagons?)

is extremely challenging. When justifying, students use their prior knowledge and reasoning to connect ideas or to make sense of something new. This process is cognitively demanding for students. Teachers, in turn, must make sense of students' ideas and find ways to help students' refine and build their knowledge.



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We share some of our collective learning about teaching with justification, specifically, what we learned about moving students to "the why." We (a team of teachers and researchers) worked together for two years with an NSF-funded project called Justification and Argumentation: Growing Understanding of Algebraic Reasoning (JAGUAR). The goal was to better understand what it takes to support students' engagement in justification in middle school mathematics classrooms. We implemented three justification tasks, reflected on the enactment of the tasks, and collaborated around problems of practice. We completed this entire process in two consecutive years.

We discuss three pedagogical foci that helped us better support students in understanding what it meant to offer a justification, as well as getting them started on this process. The three elements are helping students—

- understand what it means to justify;
- learn what makes a good justification;
- generate initial ideas, and then develop those into a justification.

We then discuss some ideas for getting started on growing the practice of justification in the classroom. Although challenging, we have found that incorporating justification in the classroom is well worth the effort.

HELPING STUDENTS UNDERSTAND WHAT IT MEANS TO JUSTIFY

When we began our work on justification, as the opening dialogue suggests, we found that students did not seem to understand justification or lacked the tools to respond to prompts such as "justify your answer," "why are you doing what you are doing?" or "how do you know your answer correct?" Instead of explaining why a calculation was warranted or why a relationship existed, students explained their steps or provided evidence to show that the relationship held (e.g., plugging in numbers to show "it worked").

To illustrate the difference, we consider student responses to the Hexagon task (see **fig. 1**), one of the justification problems that we implemented each year as part of this project. Columns 1 and 2 of **table 1** offer examples of typical student responses that we do not consider justifications. Rather, these responses (a) recount steps and calculations, showing how one arrives at an answer (column 1); or (b) provide supporting evidence that the relationship holds, without demonstrating why it must hold (column 2). (See, also, Lannin, Barker, and Townsend 2006.) We wanted responses that more closely resembled those in column 3—responses that we consider

Table 1 These typical student responses to question 2 of the Hexagon task were categorized by what they accomplished. Revisions for "the why" are shown in red.

Explanation, Not Justification		Justifications (Partial or Full)
Students articulate methods for finding the perimeter without explaining why the method is appropriate or correct.	**Students give evidence that a relationship holds, but do not explain why the relationship must hold.**	**Students offer a mathematical reason for why their method is correct.**
A. To find the perimeter, take away 2 from the figure number and multiply by 4. Then you add in 10 for the 2 you took away. So for figure 25, we do 23×4 plus 10, which is 102.	D. We saw it's always 4 times the figure number plus 2 because every time you take a figure number and multiply it by 4, and add 2, you get the perimeter. We tested it on all the values we had. So the perimeter of figure 25 is $4(25) + 2 = 102$.	F. (A, revised) To find the perimeter, take away 2 from the figure number and multiply by 4. You do this because each of the "interior" hexagons gives you 4 toward the perimeter. Then you add in 10 for the 2 you took away, because each of those 2 end hexagons give 5 each to the perimeter. So for figure 25, we do 23×4 plus 10, which is 102.
B. In our table, we saw that it goes up by 4 every time. So to get the 25th figure, you find 20 times 4, which is 80, and add it to 22 cm to get 102 cm, which is the perimeter of figure 5.	E. We got $4n + 2$ because we noticed it went up by 4 each time, but $4n$ didn't work. It was always 2 too low. For example, figure 3 was 14, but $4(3) = 12 \neq 14$. So we added 2 and got $4n + 2$, which always worked. So $25(4) + 2 = 102$.	G. (D, revised) We saw it's always 4 times the figure number plus 2 because every time you take a figure number and multiply it by 4, and add 2, you get the perimeter. We know multiplying by 4 every time is right because for every hexagon, you have 4 sides—the 2 tops and the 2 bottoms—that are part of the perimeter. We know you have to add 2 in the end because there are 2 sides—the left end and the right end—that are not counted by the tops and bottoms and are part of the perimeter. So the perimeter of figure 25 is $4(25) + 2 = 102$.
C. Figure 5 has a perimeter of 22 cm. To find the perimeter of figure 25, you multiply figure 5 by 5. So $22 \times 5 = 110$ cm, which is the perimeter of figure 25.		

justifications for students' answers to question 2. One strategy we used to address this challenge was to elongate the question we asked students. For example, instead of asking "why?" we asked, "Why does it make sense that. . . ?" This revised question prompted students to focus on sense making and reasoning about the relationship. In the case of the Hexagon task, we asked, "Why does it make sense that the perimeter of the hexagon chain is 4 times the figure number plus 2? Why would that be?" A student whose response included that "we saw that it goes up by 4 every time" might be asked, "Why does it make sense that the perimeter goes up by 4 every time?" An additional follow-up question might include a more directive prompt, "Can you show me that in the diagram?"

In this and other tasks, we note that students often would see a pattern and would rely on noticing a pattern on a small number of examples as evidence that this pattern always held. Although useful, noting a pattern was not a justification because it did not reveal why that pattern existed or held for all examples or cases. We preferred, "Why does it make sense that the pattern goes up by 4 each time?" to "How do you know that it continues like that?" (which was another common follow-up question) because students tended to think that it was self-evident that the pattern would continue. A question about sense making pushed them into a new area.

HELPING STUDENTS LEARN WHAT MAKES A GOOD JUSTIFICATION

In tandem with helping students understand what "counted" in response to a "why" question or the prompt "justify your answer," we found it was important to have strategies to help students understand what was useful or valuable about one justification relative to another. In other words, what made a good justification? We had many of these conversations among project team members, as well.

One tool we used was a rubric, either given to students to guide their work or co-developed with students based on conversations about class work. **Figure 2** shows an example of a CLEAR rubric that was given to

Fig. 2 A CLEAR rubric can be used to assess students' justifications.

Rubric for Assessing a CLEAR Response

Acronym	Score Point	0	1
C	Calculations	• No work is shown. • Some work is missing.	• Calculations show mathematical ideas involved. • Answer includes calculations and/or tables, graphs, or pictures.
L	Labels	• No labels are included. • Items are incorrectly labeled.	• Calculations are correctly labeled.
E	Evidence	• Calculations do not support the decision made. • Evidence is missing for some part of the problem.	• Calculations support the decision made. • Evidence is provided for all parts of the problem.
A	Answers the question	• Answer is inaccurate. • Answer does not answer the question being asked.	• Answers the question asked using a complete sentence (capitalization and punctuation). • Answer is accurate.
R	Reasons why	• Mathematical reasoning is not given for the procedure, or the explanation is not given. • The response shows confusion about content ideas and concepts.	• Procedure is identified. • Procedure is explained and what it means. • Clear understanding is shown of content ideas and concepts.

Fig. 3 THE RACE rubric is able to support student justification across many different areas.

Answering Open Response Questions

RACE:

Reword: Reword the question into a statement to begin your answer.

Answer: Answer the question that you were asked to answer.

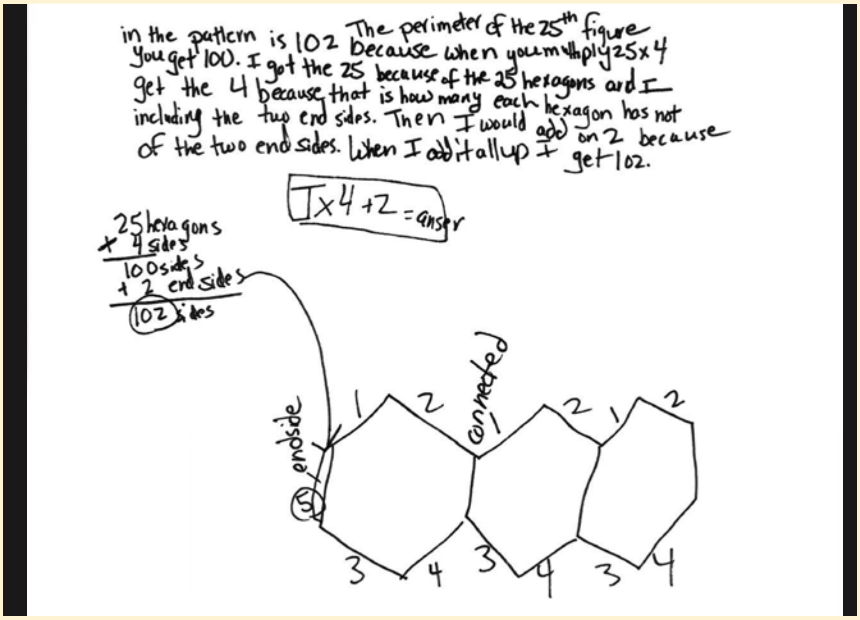
Cite: Cite examples from your life, text, previous investigations that relate to your answer and to your explanation.

Explain: Explain how you arrived at your answer (your thinking), and how what you cited relates to your answer.

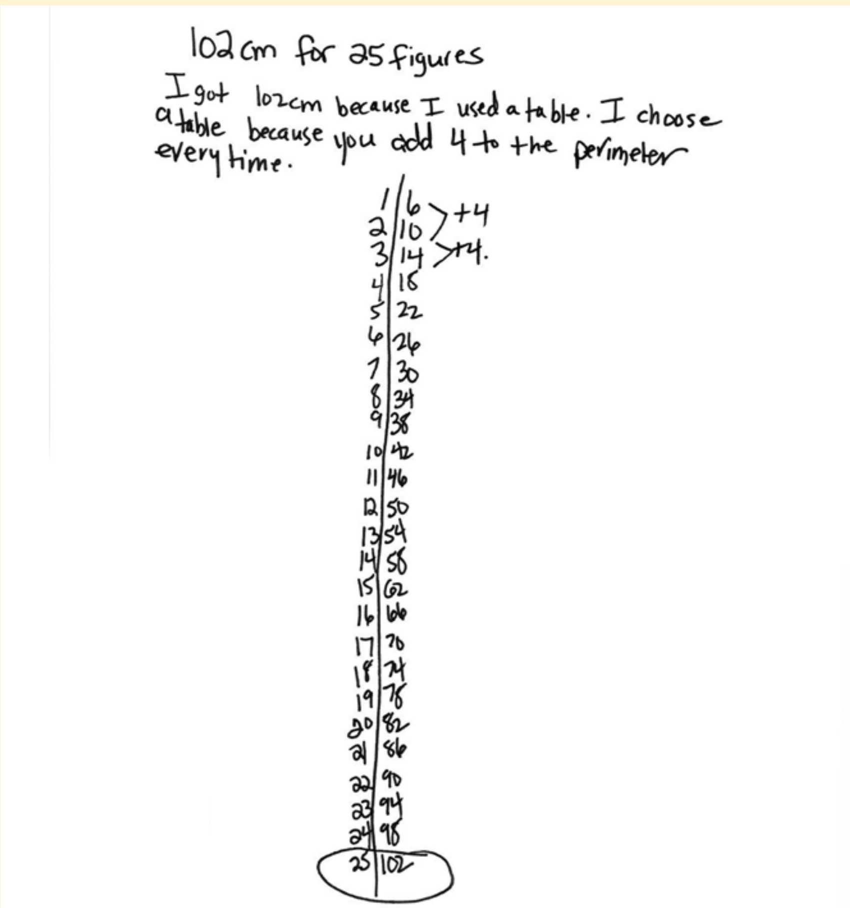
seventh-grade students. Notice that E (evidence) and R (reasons why) get to the heart of a justification. The other categories focus on communicating ideas clearly. **Figure 3** shows the RACE rubric, which was a grade-level team rubric used to guide persuasive writing. It was used across subjects, such as English/language arts, science, social studies, and math. When used with mathematics, we focused on fleshing out what those criteria looked like in a math class. Other rubrics captured similar features. For example, see Vazquez (2008) for an A-E-I-O-U and always Y rubric. These rubrics were used throughout the year.

A second strategy we used to develop an understanding of a "good" justification was to discuss sample justifications. These class discussions provided the opportunity to work together for the purposes of creating a shared meaning of justification and establishing the criteria of a "good" justification. Part of this process included discussing how to make

Fig. 4 Two student work samples showing justifications of the perimeter of figure 25 were presented for analysis.



(a)



(b)

justifications even better. For example, we presented student work samples (see **fig. 4**) to the whole class (real or teacher-created) and asked students to make sense of the justification, decide whether they agreed or disagreed, and suggest ways to improve the justification. Among other discussion points, comparing these work samples can highlight the value of various components of the CLEAR rubric, such as labeling, what evidence each student was using, and the student's reasoning. A related strategy we used was peer review, in which students shared their justifications with one another and gave feedback from using a class rubric (e.g., CLEAR or RACE). When guided by a rubric, these discussions helped students develop an understanding of what each category in the rubric might mean in relation to a justification.

Both of these processes (using peer review and discussing sample responses) orient students to an audience beyond the teacher. This brings up the important point that producing a good justification relies on communicating and representing one's ideas to others who will critically evaluate whether a chain of reasoning makes sense and shows something to be true. The peer-review process creates an authentic situation to press on the dual purpose for a justification: to show why something is true to another and to communicate it in a way that another can access.

A third strategy we used to help students learn what makes a good justification was setting justification goals for lessons. Like content objectives or language objectives (see Echevarría, Vogt, and Short 2008), these goals were "justification objectives," developed to help us think about what our students needed to learn about justifying and guide our planning to help them. Some of us shared these with students; others used them primarily for planning purposes. Here are some examples of

justification goals. Students will be able to—

- explain why they cannot use examples to show that something is always true but use an example (or counterexample) to show that something is false;
- connect the plus-four pattern to the diagram (showing which four sides are added in each time) as evidence that the plus-four pattern continues indefinitely; and
- analyze diagrams and determine whether the drawing represents a specific case or is generalized.

Proficiency with justification must be developed deliberately over time. Setting justification goals, much like mathematical content goals, helped keep us on track and helped us think about, and break down, how students develop in their abilities to generate and express justifications over time.

GENERATING AN INITIAL IDEA AND DEVELOPING A JUSTIFICATION

Justifying may be new for many students. We found that initially it was important for us to simply have students share some ideas or thoughts, which then could be developed into clearer and more rigorous justifications. There are many ways to help students access a task and generate some initial ideas. For example, we focused on several strategies: clearly introducing the task, making sure that unfamiliar vocabulary was explained, providing "private think time," and allowing students to represent their ideas in multiple ways (e.g., making drawings, using words, pointing toward the board, moving manipulatives). Similarly, many strategies are possible to help students share or express an initial idea even if not yet well formed, for example, using think-pair-share routines; asking students to write an idea down (either before or after sharing); using public visuals to have students "show" what they mean; and emphasizing that the audience for their reasoning is the class, not just the teacher.

Our work was most effective when we built on and developed students' thinking, even when it did not match how we were thinking, or what we thought was the "best" approach. If we did not consistently work to build on students' ideas, it would undermine efforts to get students to generate and then develop their own ideas toward a more complete justification.

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This work required careful listening by the teacher and making deliberate efforts to develop students' ideas. The main point is that justification is about reasoning: It cannot be the teacher's reasoning; it has to be the students' reasoning. The commitment to building on students' thinking involves managing mistakes (see also Hoffman, Breyfogle, and Dressler 2009); finding what is productive in what students do; and figuring out how much to "give" students to support, but not override, their thinking.

GETTING STARTED

Getting students to justify is not something that happens simply by asking them to justify. After two years of work on this project, we are still in the process of developing our pedagogical strategies to support students' process of justification. In addition to the strategies we discussed (helping students understand what it means to justify, learning what makes a good justification, and generating initial ideas and developing them), we offer the following tips from our experiences.

Pick a lesson and start small: Choose a lesson that you are comfortable with and design a small task that includes a "why" question, but not as the first question. Have the students work in pairs to solve the problem and share their solution (answers and reasoning) with the class. This gets them working together, communicating with one another, and conveys that we value what they do and think.

Link justification in math class with students' other experiences: The written part of the justification process can easily link to what the students do in language arts when they write a five-paragraph persuasive essay. Their language arts teacher asks them to be sure to include supporting details, and we are looking for the same thing with a written justification in math.

Interweave justification opportunities throughout the year: If you have started focusing on justification in a few, key lessons (as most of us did), a next step is to make justification a daily presence in your classroom. If you do not already, ask "why" questions during a warm-up, even in relation to procedural topics (e.g., how do we know that 2/7 is the same as 4/14?). Elicit more than one response. Listen carefully to, and build on, students' reasoning. Do not simply listen for the one correct answer.

Be patient with yourself: Sometimes lessons will exceed your expectations, revealing the wonderful thinking of your students. Other times, a lesson may be hit-and-miss, and you may be disappointed with the results. Be patient with yourself and know that you, like your students, are learning. Reflect on the lesson, try to diagnose the issue, and try another lesson. We have found that lessons "fly" when they are not overly structured, thus allowing room for student thinking, but not understructured, making the target unclear or producing such divergent thinking from students that it is hard to find common ground and have students analyze arguments. Finding this balance takes time.

If you have a colleague interested in doing this work, collaborate: It is always more fun and productive to work with a colleague. Co-plan lessons in which you brainstorm key questions to prompt justification and anticipate the variety of solutions that students might offer. You can also use NCTM articles with student work to help you anticipate students' approaches and how you might respond to develop the ideas. (See Smith et al. 2009 for a helpful approach.) It is impossible to anticipate all student responses, but the more you can anticipate, the more prepared you might feel to draw out students' partially formed thinking.

KEEPING YOUR EYE ON THE PRIZE

With the constant push for skill development and urgency of meeting standardized testing goals, it is easy to question how you can take the time to allow students to justify their thinking in math class. It is important to remember that if we are teaching mathematics, we must include mathematical reasoning in our daily lessons. Mathematics is a tool we use to analyze, explore, and build new understandings from discoveries made by others. Those understandings and the process of justification are not only interconnected and lasting but also allow knowledge to continually be developed, revised, and extended. What could be more important?

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Any thoughts on this article? Send an email to **mtms@nctm.org.**—*Ed.*





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