More of Exploring combinations with apples and bananas

Teacher reflection
This year Tammy Sanford and I are piloting a kindergarten classroom focused on cognitively guided, student-centered mathematics instruction. We chose this problem, which is adapted from the Seven Peas and Carrots problem in the Investigations in Number, Data, and Space series, grade 1, because it incorporated many number-sense and Process Standard ideas on which her students had been focusing, such as the following:

- **Counting** and quantity (keeping track of a set of seven)
- **More and fewer** (combining apples and bananas to make seven)
- **Relationship** of parts to wholes (changing the ratio of apples to bananas to make a new combination)
- **Problem** solving (finding all solutions)
- **Representation** (choosing appropriate materials to show the solutions)
- **Communication** (using words to adequately explain work)

We also chose this problem because of its integration of number, operations, and algebra.

Classroom discourse
We were pleasantly surprised at the level of discussion this task brought out. Initially, much of the talk centered on comparing two pieces of student work to evaluate whether the solutions were accurate. Students were concerned with whether there were seven pieces of fruit total and whether bowls had more or fewer apples or bananas. To some, such talk may seem trivial, but kindergartners struggle with these concepts when working on them individually, much less when dealing with them together. And yet, bigger math ideas quickly emerged in the discussion:

[Student 1] If there are more apples, you will always have less bananas. You will never have the same amount. [We see an opening to explore odd and even numbers.]

[Student 2] I see six, two times. I see six apples and one banana here. I see one apple and six bananas here [showing the class how the six apples are on one side of the chart, whereas the six bananas are on the opposite of the chart]. But there are still seven.

[Student 3] I see four, two times. I see four apples here and four bananas here. [Although this task is not an appropriate one for doing so, we see an opening to explore the commutative property.]

[When asked if they thought they had found all the combinations of fruit in the bowl, student 2 rubbed her hands up and down the edges of the chart.] We're missing a 7. Look, there is a 1, a 2, a 3, and a 4 like a pattern [referring to the bananas on the right side]. The 3 and the 4 come back over here [referring to the numbers representing the quantity of apples on the left]. So we don't have them all [the combinations].

At this point, Walker had stepped in.

Next steps
Sanford and I were overwhelmed with how much math came out of this task. When discussing our next instructional move, only one thing was clear. Students fell along every point on the learning continuum. Some were still working on one-to-one correspondence; others understood the idea of compensation.

We agreed that the next step was to try a similar problem, this time using the number 6. I thought that by choosing 6, the task would allow students to do the following:

- **Manage** fewer “pieces” when they have counting issues. Every student can anchor to and decompose 5. In choosing 6, I hoped...
they could better manipulate the two parts while having to retain the whole of 6.
• **Explore** more, fewer, and the same, now with an example that had an equal number of items (addends would be the same).
• **Have** another experience to investigate the commutative property and turnaround facts.
• **See** the pattern of the combinations, perhaps finding them all with a smaller number.
• **Obtain** individualized attention if they needed it, because their peers are now familiar with the task and following their own learning paths.

I can say for sure that when Sanford committed her classroom to cognitively guided, student-centered math instruction, she never realized how messy math teaching would become. No longer would each student learn the same concept according to a pacing guide. When a good task is chosen, more learning happens. As evident by this task, however, more math is happening with these five-year-olds three months into school than had happened at the same time the year before.