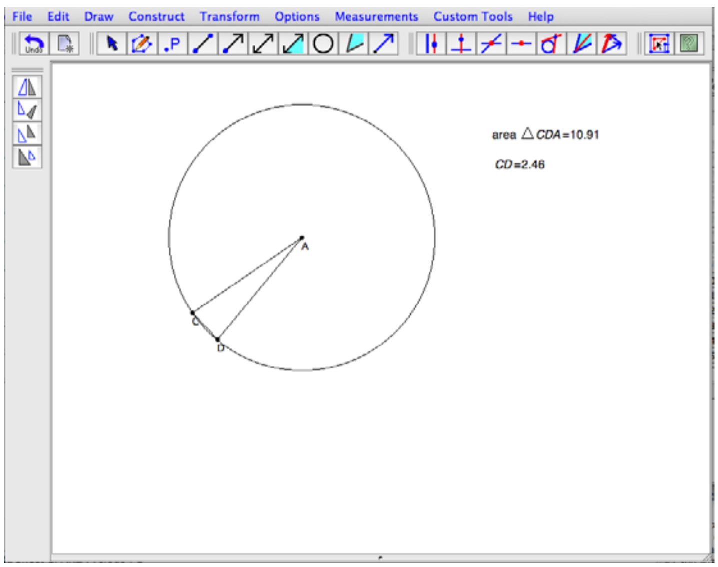
Arcavi's Problem

This lesson was inspired by tasks described in an article written by Abraham Arcavi and Nurit Hadas[1]. Students develop function models using regression and geometric relationships. They produce and use equivalent expressions to compare competing statistical and mathematics models of a mathematical phenomenon.

Begin with students producing the needed diagram.
We need an isosceles triangle that has certain properties. Here is one way to construct the triangle:

1. Construct a circle with center *A* passing through point *B* with a radius of 5 units.
2. Place points *C* and *D* on the circle.
3. Create triangle *ACD* using the polygon tool.



4. Drag point *C* or point *D*, but not point *A.*

Question to ask in examining the diagram.
→ *As you drag these points, what changes and what stays the same?*

Note: This question is inspired by **Big Idea 2** from the book *Developing Essential Understanding (EU): Geometry 9–12.* It says: **Geometry is about working with variance and invariance, despite appearing to be about theorems**. In particular, pay attention to

[1] Arcavi, Abraham, and Nurit Hadas. "Computer Mediated Learning: An Example of an Approach." *International Journal of Computers for Mathematical Learning* 5 (2000): 325–45.

"**Essential Understanding 2b.** Invariances are rare and can be appreciated only when they emerge out of much greater variation."

Students likely will notice that the triangle remains an isosceles triangle. The leg lengths do not change but the base length does.
→ *If we think about area and perimeter it makes sense that they would therefore change. Let's consider area, for example. As the length of the base increases, what happens to the area?*

Students typically respond with "Increase" or "It increases, but then I don't know what happens when the base gets really long."

Follow-up with a question of evidence:
→ *Why do you think it <repeat student claim>?*

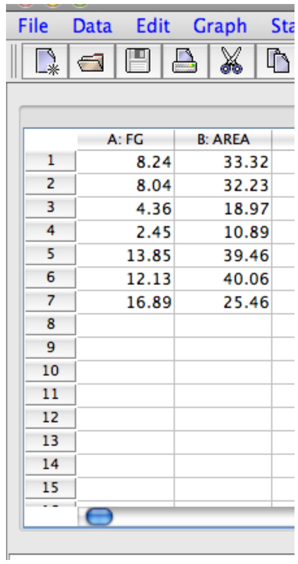
Raise the question of family of function:
→ *What family of function would model area as a function of side length?*
→ *What characteristics of that family make it a better choice than another family?*

Note: The underlying question is **Big Idea 3** from the *9–12 EU Functions* book: "Functions can be classified into different families of functions, each with its own unique characteristics. Different families can be used to model different real-world phenomena."

Note: The questioning falls within *Focus in High School Mathematics (FHSM): Reasoning with Functions* in terms of the key element of "Modeling by using families of functions."

Show students how to collect data, if needed.

5. Select triangle and measure area.
6. Select the base—constructing the line segment if needed—and measure length.
7. Open statistics window.
8. Label columns with base length and area.
9. Alternate between dragging the points and entering data, accumulating at least seven data points.



	A: FG	B: AREA
1	8.24	33.32
2	8.04	32.23
3	4.36	18.97
4	2.45	10.89
5	13.85	39.46
6	12.13	40.06
7	16.89	25.46
8		
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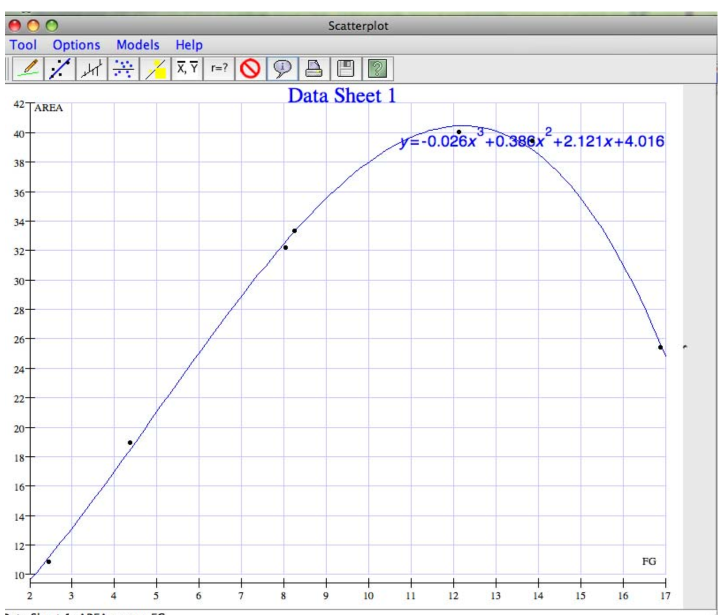
Before fitting the anticipated functions:
→ *Which function appears to be the best choice? Why?*

Note: This question draws to some extent on the *9–12 EU Statistics* book's "**Essential Understanding 1c:** Statistical models are evaluated by how well they describe data and whether they are useful."



Taking a statistical approach to this problem led us to an idea of the kind of function that would be a good model. It was based on graphs.

→ *What if we use what we know about geometry? What geometry ideas could we use to develop a symbolic rule for the function?*

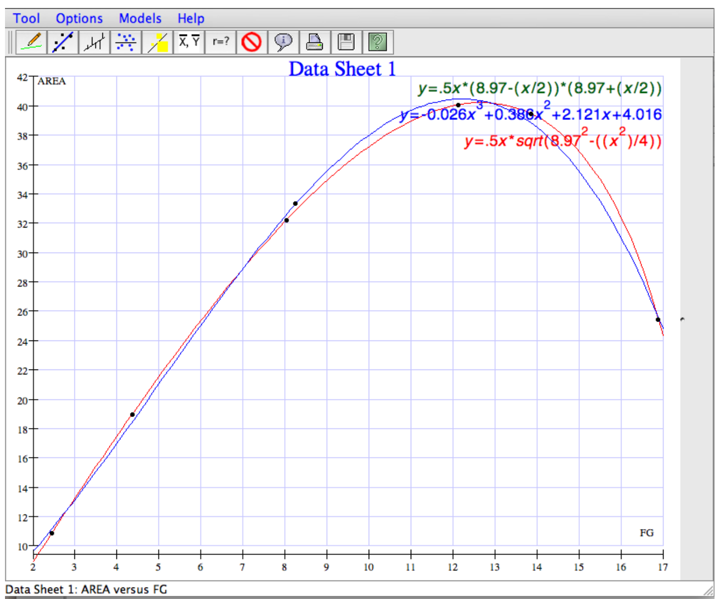
Possible ideas include the area formula. Students working in groups typically can figure out a function rule, but typically leave it in the form shown here.

$$A = bh$$

$$A = CD * sqrt(25 - (CD\wedge 2/4))$$

Graphing the derived function with the data provides additional evidence that the resulting symbol rule represents a reasonable model.

Note: The move from graphical to symbolic and from symbolic to graphical fit with the FHSM key element of "Using multiple representations of function."



→ *The fitted cubic seemed to work well. Our derived function's graph is close to that of the cubic function. How can that be when our function rule includes a square root?*

The purpose here is for students to reinterpret the symbolic rule and use a common factoring skill to rewrite the derived function as a product of three linear binomials—thus representing, much to their surprise, a cubic function.

Note: The question addresses the CCSSM standard cluster "Seeing Structure in Expressions" and specific standards such as "A-SSE-2. Use the structure of an expression to identify ways to rewrite it."